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A Direct Construction of Nonlinear Discrete-Time Observer with Linearizable Error Dynamics

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Abstract— We provide a direct method for the construction of a change of coordinates for the design of nonlinear discrete-time observers. An explicit expression of a change of coordinates is given. Some simulations for chaotic systems, such as Lozi system and Hénon system, are provided to illustrate the method. The proposed approach can be applied to continuous-time systems once the corresponding discretized structures are determined.

I. Introduction

We consider the problem of estimating the current state $x(k)$ of a nonlinear discrete-time dynamical system, described by a system of first-order difference equations

$$\begin{aligned} x(k+1) &= f(x(k)) \\ y(k) &= h(x(k)), \end{aligned} \tag{I.1}$$

from the past observations $y(s), s \leq k$, where $k \in \{0, 1, 2, \ldots\}$. The vector fields $f : \mathbb{R}^n \to \mathbb{R}^n$, and $h : \mathbb{R}^n \to \mathbb{R}^p$ are continuous with $p \leq n$. The variable $x \in \mathbb{R}^n$ stands for the state of the system, while the variable $y \in \mathbb{R}^p$ represents the measurement output.

An observer is a system driven by the observations

$$\hat{x}(k+1) = \hat{f}(\hat{x}(k), y(k)) \tag{I.2}$$

such that the error $\tilde{x}(k) = x(k) - \hat{x}(k)$ goes to zero as $k \to \infty$. A local observer is one that converges for small difference of $x(0)$ and $\tilde{x}(0)$ (in Euclidean norm sense).

One of typical approaches for constructing an observer for nonlinear systems (see, e.g., [7], [1], [5], [10], [13], [11], [9] and references therein) is to seek a change of state coordinates $z = \theta(x)$ which can transform the nonlinear system (I.1) to a linear system:

$$z(k+1) = Az(k) + \beta(y), \tag{I.3}$$

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where the term $\beta(y)$ is called the output injection. Then an observer is constructed in the z-coordinate as

$$\begin{aligned} \hat{z}(k+1) &= A\hat{z}(k) + \beta(y) \\ \hat{x}(k) &= \theta^{-1}(\hat{z}(k)). \end{aligned} \tag{I.4}$$

Under such a construction the error dynamics $\tilde{z} := z - \hat{z}$ becomes linear in z-coordinates, and satisfies the equation:

$$\tilde{z}(k+1) = A\tilde{z}(k). \tag{I.5}$$

If the eigenvalues of A are chosen to be strictly inside the unit disk, then the error dynamics $\tilde{z}(k) \to 0$ exponentially as $k \to \infty$.

Krener and Xiao [8], Xiao, Kazantzis, Kravaris, and Krener [13] show that either in discrete-time (or in continuous-time) case, an observer given by (I.2) whose error dynamics is linear in the transformed coordinates must be in the form of (I.4). If such a change of coordinates $z = \theta(x)$ exists, it is easy to see that θ satisfies the functional equation

$$\theta(f(x)) = A\theta(x) + \beta(h(x)). \tag{I.6}$$

When the method of a change of coordinates mentioned above is used, the convergent domain of an observer fully depends on the valid region of the corresponding change of the coordinates. It is quite challenge to determine or to estimate the domain of θ, even the solution of (I.6) exists, since (1.6) usually does not have a closed-form solution. It is known that in some cases (for both continuous- and discrete-time systems) when the output injection function β is appropriately chosen the functional equation (I.6) admits a global solution or a semi-global solution, but there is no general theory to guarantee such cases.

A common approach is to consider the linear term of $\theta(x)$. Let us here assume f, h, and β are C^1 functions. Furthermore, without loss of generality, we may assume that $x = 0$ is an equilibrium of

(I.1), that is $f(0)=0$, and $h(0)=0$. Let $\theta(x) = Tx + o(|x|^2)$. Then T can be obtained uniquely by solving the matrix equation:

$$TF = AT + BH \tag{I.7}$$

where

$$\frac{\partial f}{\partial x}(0) = F, \qquad \frac{\partial \beta}{\partial x}(0) = B, \qquad \frac{\partial h}{\partial x}(0) = H$$

provided that $\sigma(F) \cap \sigma(A) = \emptyset$, where $\sigma(F)$ and $\sigma(A)$ stand for the spectrums of F and A, respectively. If T is invertible (by choosing B and A appropriately), then $\theta(x)$ is invertible in a neighborhood of the equilibrium $x=0$. The size of such a neighborhood is usually unknown, but it might be quite small since we here essentially make use of the linear approximation for the Jacobi matrix function $\frac{\partial \theta}{\partial x}(x)$ around the equilibrium.

To guarantee the invertibility of $\theta(x)$ around the equilibrium of systems may not be enough for a lot of nonlinear systems, in particular, for those systems whose solutions eventually get attracted to a set that is neither an equilibrium nor a periodic orbit, such as for the chaotic systems.

In this paper we propose a direct method for constructing a nonlinear observer, in which the valid domain of a change of coordinates can be verified directly. More specifically, we construct a change of variable such that an observer has a form:

$$\hat{x}(k+1) = \hat{f}\left(\hat{x}(k), y(k), y(k+n)\right) \tag{I.8}$$

where n is the dimension of the system (I.1). The difference between (I.8) and most of approaches in literature is that the proposed observer (I.8) is required to have two inputs: $y(k)$ and $y(k+n)$.

The paper is organized as follows. Section 2 is to introduce a new way for the construction of a change of coordinates, which depends on the structure of a given system. Section 3 studies the situation when there is an external input. The paper ends with final remarks.

II. Construction of a Change of Coordinates

Let $\beta : I\!R^p \to I\!R^n$ be a continuous function and A be an $n \times n$ invertible matrix. Now we define a continuous function on $I\!R^n$ as

$$\begin{aligned}\theta(x) &= \beta(h(f^{(0)}(x))) + A^{-1}\beta(h(f^{(1)}(x))) \\ &\quad + \cdots + A^{-(n-1)}\beta(h(f^{(n-1)}(x))) \\ &= \sum_{i=0}^{n-1} A^{-i}\beta(h(f^{(i)}(x)))\end{aligned} \tag{II.9}$$

where $f^{(0)}(x) = x, f^{(1)}(x) = f(x)$, $f^{(i)}(x) = f\left(f^{(i-1)}(x)\right)$ for $i \geq 2$. Clearly, if f, h are continuous function on $I\!R^n$ then θ is well-defined and continuous on $I\!R^n$. Notice that the positive integer n appeared in (II.9) is the dimension of the states, which is fixed once a system is given. One can verify that if (H, F) is observable and (A, B) is controllable, then the map defined in (II.9) is invertible in a neighborhood of the origin provided that f and h are smooth functions. The further motivation for such a construction is illustrated in [15].

Now we let

$$z(k) := \theta(x(k))$$

for $k = 0, 1, 2, \ldots$. We have the following theorem, whose proof can be found in [15].

Theorem 2.1: If we introduce a change of variable $z = \theta(x)$, then the dynamics in the new variable z satisfies

$$z(k+1) = Az(k) + A^{-(n-1)}\beta\left(y(k+n)\right) - A\beta(y(k)) \tag{II.10}$$

for $k = 0, 1, 2, \ldots$, where n is the dimension of the original state x.
Thus if $\theta : I\!R^n \to I\!R^n$ is a homeomorphism. Then one can construct an observer

$$\begin{aligned}\hat{z}(k+1) &= A\hat{z}(k) + A^{-(n-1)}\beta\left(y(k+n))\right) - A\beta(y(k)) \\ \hat{x}(k) &= \theta^{-1}(\hat{z}(k))\end{aligned} \tag{II.11}$$

whose error $\tilde{z} = z - \hat{z}$ dynamics is linear in transformed coordinates

$$\tilde{z}(k+1) = A\tilde{z}(k). \tag{II.12}$$

Remark.

1) The observer is operated in the following way: the observer starts to run after the output information $(y(n), y(n-1), \cdots, y(1))$ is avaliable. At each iteration k, the output information $y(n+k-1)$ is considered to be the most current output, and the n information $(y(n+k-1), y(n+k-2), \cdots, y(k))$ needs to be stored for next iteration $k+1$. The positive integer n is the dimension of the states.
2) The first part of the observer (II.11) depends only on the output, and there is no restriction on the output injection function β. Thus we can focus on the selection of β such that θ is invertible, which corresponds to the requirement of the second part of (II.11).
3) According to (II.11), it is not necessary to require θ to be a homeomorphism in the

entire $I\!R^n$. We only need θ to be invertible on the discrete set $\{\hat{z}(k)\}$.

□

Example 1. Lozi Map. We consider the following discrete-time dynamical system:

$$\begin{aligned} x_1(k+1) &= 1 + x_2(k) - a|x_1(k)| \\ x_2(k+1) &= bx_1(k). \end{aligned} \tag{II.13}$$

It is known that for $b = 0.5$ Lozi map is chaotic for a to be a number between 1.51 and 1.7 [2]. It has two fixed points $(\frac{1}{1+a-b}, \frac{b}{1+a-b})$ and $(\frac{1}{1-a-b}, \frac{b}{1-a-b})$. Numerical simulations show that almost all solutions eventually get attracted to a set, called Lozi attractor, which is neither a fixed point nor a periodic orbit. The Lozi attractor is shown in Fig.1. Lozi map is often used in applications for algorithm tests, such as in data compression, and for chaotic synchronization [12].

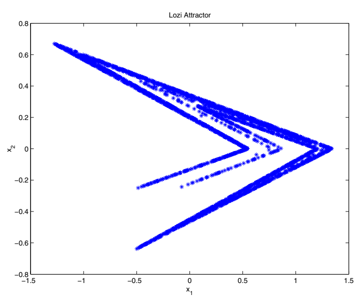

Fig. 1. Lozi attractor with parameters $a = 1.7, b = 0.5$.

Following [3], we let $a = 1.7$ and $b = 0.5$. Let us assume the output measurement is given by

$$y = x_1 + 5x_2. \tag{II.14}$$

Thus $h(x) = x_1 + 5x_2$. Let β be a linear functional of y

$$\beta(y) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} y \tag{II.15}$$

and

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}. \tag{II.16}$$

Now we construct a mapping $\theta : I\!R^2 \to I\!R^2$ according to (II.9):

$$\begin{aligned} &\theta(x) = \beta(h(x)) + A^{-1}\beta(h(f(x))) \\ &= \begin{pmatrix} x_1 + 5x_2 \\ -(x_1 + 5x_2) - 2(1 + x_2 - a|x_1| + 5bx_1) \end{pmatrix}. \end{aligned} \tag{II.17}$$

It can be shown that $\theta : I\!R^2 \to I\!R^2$ is a homeomorphism, and its inverse is given by

$$\theta^{-1}(z) = \begin{bmatrix} -\dfrac{7z_1 + 5z_2 + 10}{-2 - 10a + 50b} \\ \dfrac{(1 - 2a + 10b)z_1 + z_2 + 2}{-2 - 10a + 50b} \end{bmatrix}$$

if $7z_1 + 5z_2 + 10 \le 0$ and

$$\theta^{-1}(z) = \begin{bmatrix} -\dfrac{7z_1 + 5z_2 + 10}{-2 + 10a + 50b} \\ \dfrac{(1 + 2a + 10b)z_1 + z_2 + 2}{-2 + 10a + 50b} \end{bmatrix}$$

if $7z_1 + 5z_2 + 10 > 0$, where $z = [z_1, z_2]^T$. Hence a

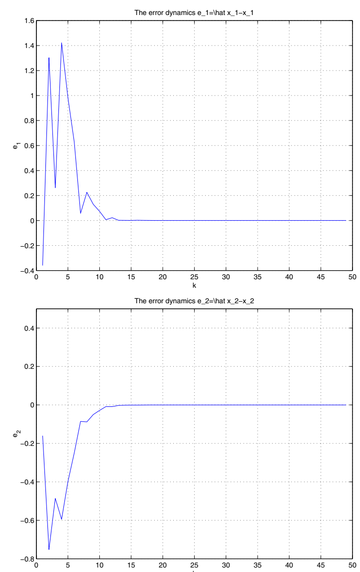

Fig. 2. Error dynamics $e_1 = \hat{x}_1 - x_1, e_2 = \hat{x}_2 - x_2$ with initial value $(\hat{z}_1(0), \hat{z}_2(0)) = (0, 0)$.

global observer can be constructed as follows:

$$\begin{aligned} \hat{z}(k+1) &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \hat{z}(k) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} y(k+2) \\ &\quad - \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} y(k) \\ \hat{x}(k) &= \begin{pmatrix} -\frac{7\hat{z}_1(k)+5\hat{z}_2(k)+10}{-2-10a+50b} \\ \frac{(1-2a+10b)\hat{z}_1(k)+\hat{z}_2(k)+2}{-2-10a+50b} \end{pmatrix} \\ &\quad \text{if} \quad 7\hat{z}_1(k) + 5\hat{z}_2(k) + 10 \le 0 \\ &= \begin{pmatrix} -\frac{7\hat{z}_1(k)+5\hat{z}_2(k)+10}{-2+10a+50b} \\ \frac{(1+2a+10b)\hat{z}_1(k)+\hat{z}_2(k)+2}{-2+10a+50b} \end{pmatrix} \\ &\quad \text{if} \quad 7\hat{z}_1(k) + 5\hat{z}_2(k) + 10 > 0. \end{aligned} \tag{II.18}$$

Remark. At each iteration k, the above observer requires to store two inputs $y(k+1)$ and $y(k)$ for iteration since the state space is two-dimensional. □

Example 2. Hénon map. Consider the discrete-time dynamical system

$$\begin{array}{rcl} x_1(k+1) & = & 1 + x_2(k) - a x_1^2(k) \\ x_2(k+1) & = & b x_1(k) \end{array} \tag{II.19}$$

where both a and b are parameters. The map

$$\left[\begin{array}{c} x_1 \\ x_2 \end{array} \right] \rightarrow \left[\begin{array}{c} 1 + x_2 - a x_1^2 \\ b x_1 \end{array} \right] \tag{II.20}$$

is called the Hénon map which was suggested by Michael Hénon in 1976 to be used as a simplified model of the Lorenz system [4]. The Hénon map is a commonly studied map in dynamical systems. Its global dynamics are quite complicated. With $a = 1.4$ and $b = 0.3$, Hénon discovered through numerical experiments that in a region of the plane almost all solutions eventually get attracted to a set-the Hénon attractor-that is neither a fix point nor a periodic orbit (see Fig.3). With $a = 1.4$ and

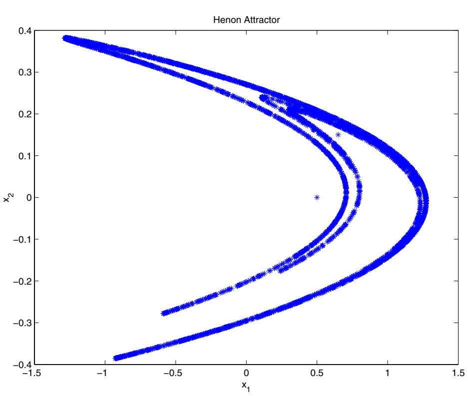

Fig. 3. Hénon attractor with parameters $a = 1.4, b = 0.3$.

$b = 0.3$, there are two fixed points given by

$$x_1^0 = \frac{-(1-b) \pm \sqrt{(1-b)^2 + 4a}}{2a}, \quad x_2^0 = b x_1^0. \tag{II.21}$$

One of the fixed points

$$x_1^0 = 0.63135448..., \qquad x_2^0 = 0.18940534... \tag{II.22}$$

is a saddle point with eigenvalues

$$\lambda_1 = 0.15594632..., \qquad \lambda_2 = -1.92373886.... \tag{II.23}$$

Now suppose that we have a system:

$$\begin{array}{rcl} x_1(k+1) & = & 1 + x_2(k) - a x_1^2(k) \\ x_2(k+1) & = & b x_1(k) \\ y(k) & = & x_1(k) \end{array} \tag{II.24}$$

with both parameters a and b being unknown. Applying our proposed method, we are going to reconstruct the state x_2 as well as to estimate the parameters a and b via the output measurement y.

In this case we have

$$h(x) = \left[\begin{array}{cc} 1 & 0 \end{array} \right] x. \tag{II.25}$$

Let β be a linear functional

$$\beta(x) = \left(\begin{array}{c} b_1 \\ b_2 \end{array} \right) x \tag{II.26}$$

and

$$A = \left[\begin{array}{cc} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{array} \right]. \tag{II.27}$$

Now we define a mapping $\theta(x)$:

$$\begin{array}{l} \theta(x) = \beta(h(x)) + A^{-1}\beta(h(f(x))) \\ = \left(\begin{array}{c} b_1 \\ b_2 \end{array} \right) x_1 + \left(\begin{array}{cc} 2 & 2 \\ 0 & 2 \end{array} \right) \left(\begin{array}{c} b_1 \\ b_2 \end{array} \right) (1 + x_2 - a x_1^2) \\ = \left(\begin{array}{c} b_1 x_1 + 2(b_1 + b_2)(1 + x_2 - a x_1^2) \\ b_2 x_1 + 2 b_2 (1 + x_2 - a x_1^2) \end{array} \right). \end{array} \tag{II.28}$$

If we set $b_1 + b_2 = 0$ with $b_1 \neq 0$ and $b_2 \neq 0$, then we have

$$\theta(x) = \left(\begin{array}{c} b_1 x_1 \\ b_2 x_1 + 2 b_2 (1 + x_2 - a x_1^2) \end{array} \right) \tag{II.29}$$

which is a global diffeomorphism on $\mathbb{R}^2$. For convenience, we let $b_1 = 1$ and $b_2 = -1$, then we have

$$\theta(x) = \left(\begin{array}{c} x_1 \\ -x_1 - 2x_2 - 2 + 2a x_1^2 \end{array} \right) \tag{II.30}$$

and

$$\theta^{-1}(z) = \left(\begin{array}{c} z_1 \\ -\frac{z_1 + z_2}{2} + a z_1^2 - 1 \end{array} \right). \tag{II.31}$$

Hence a global observer (for the states) can be constructed as follows:

$$\begin{array}{rcl} \hat{z}(k+1) & = & \left[\begin{array}{cc} \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{array} \right] \hat{z}(k) - \left(\begin{array}{c} 0 \\ 2 \end{array} \right) y(k+2) \\ & & - \left(\begin{array}{c} 1 \\ -\frac{1}{2} \end{array} \right) y(k) \\ \hat{x}(k) & = & \left(\begin{array}{c} \hat{z}_1(k) \\ -\frac{\hat{z}_1(k) + \hat{z}_2(k)}{2} + a \hat{z}_1^2(k) - 1 \end{array} \right). \end{array} \tag{II.32}$$

Of course, the above observer cannot be run yet since a is unknown. Since the eigenvalues of A are chosen to be inside of the unit disk, we know that

$$\hat{x}(k) \rightarrow x(k) \qquad \text{as} \quad k \rightarrow \infty \tag{II.33}$$

if a were known. Thus when k is large enough, we should have $\hat{x}_2(k+1) \approx b\hat{x}_1(k)$, or equivalently,

$$-\frac{\hat{z}_1(k+1)+\hat{z}_2(k+1)}{2} + a\hat{z}_1^2(k+1) - 1 \approx b\hat{z}_1(k). \tag{II.34}$$

This allows us to define $\hat{a}(k)$ such that

$$\frac{-\frac{\hat{z}_1(k+1)+\hat{z}_2(k+1)}{2} + \hat{a}(k)\hat{z}_1^2(k) - 1}{\hat{z}_1(k)} = \frac{-\frac{\hat{z}_1(k+2)+\hat{z}_2(k+2)}{2} + \hat{a}(k)\hat{z}_1^2(k+2) - 1}{\hat{z}_1(k+1)}.$$

Thus an observer can be constructed as follows:

$$\begin{array}{rcl} \hat{x}_1(k) & = & \hat{z}_1(k), \hat{x}_1(k+1) = \hat{z}_1(k+1), \\ & & \hat{x}_1(k+2) = \hat{z}_1(k+2) \\ \hat{a}(k) & = & \dfrac{\frac{\hat{z}_1(k+2)+\hat{z}_2(k+2)+2}{2z_1(k+1)} - \frac{\hat{z}_1(k+1)+\hat{z}_2(k+1)+2}{2\hat{z}_1(k)}}{\frac{\hat{z}_1^2(k+2)}{\hat{z}_1(k+1)} - \frac{\hat{z}_1^2(k+1)}{\hat{z}_1(k)}} \\ \hat{x}_2(k) & = & -\frac{\hat{z}_1(k)+\hat{z}_2(k)}{2} + \hat{a}(k)\hat{z}_1^2(k) - 1 \\ \hat{b}(k) & = & \frac{\hat{x}_2(k)}{\hat{z}_1(k)}. \end{array} \tag{II.35}$$

The error dynamics for states and the dynamics for identification of parameters a and b are shown in Fig.4.

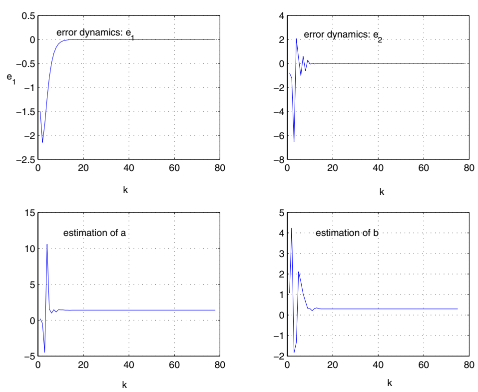

Fig. 4. Error dynamics $e_i = \hat{x}_i - x_i \to 0, i = 1, 2$ and dynamics for $\hat{a} \to a = 1.4, \hat{b} \to b = 0.3$.

III. Systems with Inputs

The proposed method can be applied to systems with inputs. Let us consider the problem of estimating the current state $x(k)$ of

$$\begin{array}{rcl} x(k+1) & = & f(x(k), u(k)) \\ y(k) & = & h(x(k)), \end{array} \tag{III.36}$$

from the past observations $y(s), s \leq k$, where $k \in \{0, 1, 2, \ldots\}$. The vector fields $f : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$, and $h : \mathbb{R}^n \to \mathbb{R}^p$ are assumed to be continuous with $p \leq n$. The variable $u \in \mathbb{R}^q$ is the external input.

Let $\beta : \mathbb{R}^p \to \mathbb{R}^n$ be a continuous function and A be an $n \times n$ invertible matrix. Now we define a continuous function on $\mathbb{R}^n \times \mathbb{R}^q$

$$\begin{array}{l} \theta(x,u) := \beta(h(f^{(0)}(x,u))) + A^{-1}\beta(h(f^{(1)}(x,u))) \\ + \cdots + A^{-(n-1)}\beta(h(f^{(n-1)}(x,u))) \\ = \sum_{i=0}^{n-1} A^{-i}\beta(h(f^{(i)}(x,u))) \end{array} \tag{III.37}$$

where $f^{(0)}(x,u) = x, f^{(1)}(x,u) = f(x,u)$, $f^{(i)}(x,u) = f\left(f^{(i-1)}(x,u)\right)$ for $i \geq 2$.

Similar to previous case, if we define a dynamics

$$z(k) := \theta(x(k), u(k))$$

for $k = 0, 1, 2, \ldots$, then $z(k)$ satisfies

$$z(k+1) = Az(k) + A^{-(n-1)}\beta(y(k+n)) - A\beta(y(k))$$

for $k = 0, 1, 2, \ldots$. Suppose that for a given input u, θ is invertible in terms of x, we denote it by $\theta_u^{-1}(x)$, then we can construct an observer

$$\begin{array}{l} \hat{z}(k+1) = A\hat{z}(k) + A^{-1}\beta(y(k+n))) - A\beta(y(k)) \\ \hat{x}(k) = \theta_u^{-1}(\hat{z}(k)) \end{array} \tag{III.38}$$

whose error $\tilde{z} = z - \hat{z}$ dynamics is linear in transformed coordinates

$$\tilde{z}(k+1) = A\tilde{z}(k). \tag{III.39}$$

Example 3. Consider the following systems:

$$\begin{array}{rcl} x_1(k+1) & = & x_2(k) \\ x_2(k+1) & = & -\frac{1}{2}x_1(k)x_3(k) - \frac{1}{2}x_1(k) \\ & & +\frac{1}{2}(x_2(k)+1)^2u(k) \\ x_3(k+1) & = & \frac{1}{2}x_3(k) + (x_2(k)+1)u(k) \\ y(k) & = & x_1(k). \end{array} \tag{III.40}$$

In this case $h(x) = x_1$. Let us set

$$A = \begin{bmatrix} 1/2 & -1/4 & 1/8 \\ 0 & 1/2 & -1/4 \\ 0 & 0 & 1/2 \end{bmatrix}, \qquad B = \begin{bmatrix} \frac{3}{4} \\ -1 \\ 1 \end{bmatrix}. \tag{III.41}$$

Now we construct the θ according to (III.37)

$$\begin{array}{l} \theta_u(x) = Bh(x) + A^{-1}Bh(f(x,u)) \\ +A^{-2}Bh(f(f(x,u))) \\ = \begin{bmatrix} \frac{3}{4}x_1 + \frac{1}{2}x_2 \\ -x_1 - x_2 \\ -x_1 + 2x_2 - 2x_1x_3 + 2(x_2+1)^2u \end{bmatrix}. \end{array} \tag{III.42}$$

If $z_1 + 2z_2 \neq 0$, one can see that

$$\theta_u^{-1}(z) = \begin{bmatrix} 4z_1 + 2z_2 \\ -4z_1 - 3z_2 \\ \frac{-6z_1 - 4z_2 - 1/2z_3 + (4z_1+3z_2-1)^2 u}{4z_1+2z_2} \end{bmatrix}. \tag{III.43}$$

Thus let $z(k) = \theta_u(x(k))$, we can construct an observer:

$$\begin{aligned} \hat{z}(k+1) &= \begin{bmatrix} 1/2 & -1/4 & 1/8 \\ 0 & 1/2 & -1/4 \\ 0 & 0 & 1/2 \end{bmatrix} \hat{z}(k) \\ &+ \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} y(k+3) - \begin{bmatrix} \frac{3}{4} \\ -\frac{3}{4} \\ \frac{1}{2} \end{bmatrix} y(k) \\ \hat{x}(k) &= \theta_u^{-1}(\hat{z}) \end{aligned} \tag{III.44}$$

The state dynamics and the error dynamics are shown in Fig.5. The input in simulations is $u(k) = \sin(k)$.

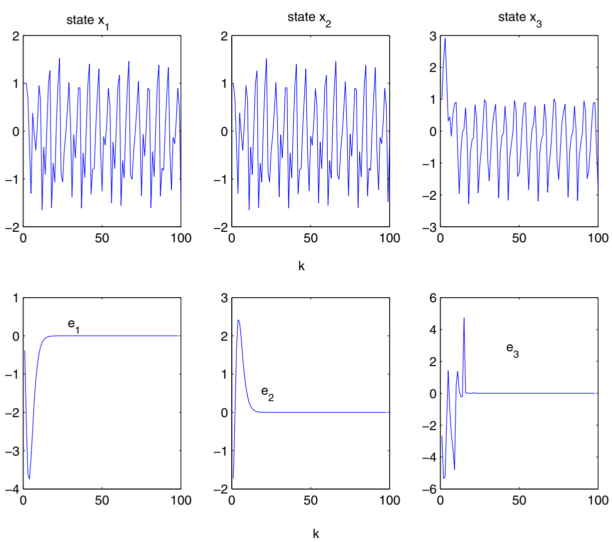

Fig. 5. System states x_i and the error dynamics $e_i = \hat{x}_i - x_i$, $i = 1, 2, 3$. The input $u(k) = \sin(k)$.

IV. Final Remarks

In this paper, we present a new and simple approach for the construction of a change of coordinates for the design of nonlinear discrete-time observer, which is different from the current approaches in literature. The proposed observer is operated under two different output injections. The advantage for such approach is that the construction of an observer is straightforward, and has no additional restriction on the systems. The proposed method can be applied to continuous-time systems once their discretized structures are determined, and a further discussion will be reported elsewhere.

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