$\mathcal{H}^\infty$ Mixed Sensitivity Minimization for Stable Infinite-Dimensional Plants Subject to Convex Constraints

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Index Terms: $\mathcal{H}^\infty$ mixed sensitivity, convex optimization, infinite dimensional, time domain constraints

***Abstract*— This paper shows how convex optimization may be used to design near-optimal finite-dimensional compensators for stable linear time invariant (LTI) infinite dimensional plants. The infinite dimensional plant is approximated by a finite dimensional transfer function matrix. The Youla parameterization is used to parameterize the set of all stabilizing LTI controllers and formulate a weighted mixed-sensitivity $\mathcal{H}^\infty$ optimization that is convex in the Youla Q-Parameter. A finite-dimensional (real-rational) stable basis is used to approximate the Q-parameter. By so doing, we transform the associated optimization problem from an infinite dimensional optimization problem involving a search over stable real-rational transfer function matrices in $\mathcal{H}^\infty$ to a finite-dimensional optimization problem involving a search over a finite-dimensional space. In addition to solving weighted mixed sensitivity $\mathcal{H}^\infty$ control system design problems, it is shown how subgradient concepts may be used to directly accommodate time-domain specifications (e.g. peak value of control action) in the design process. As such, we provide a systematic design methodology for a large class of infinite-dimensional plant control system design problems. In short, the approach taken permits a designer to address control system design problems for which no direct method exists. Illustrative examples are provided.**

I. INTRODUCTION

This paper addresses the problem of designing finite-dimensional controllers for infinite-dimensional LTI plants subject to $\mathcal{H}^\infty$ mixed-sensitivity performance objectives and convex constraints. While the methods apply to unstable plants, the paper focusses on stable plants.

Approximate/Design Approach. The plant is first approximated by a finite-dimensional model [1], [2]. A finite-dimensional controller based on this model is then designed. While design methods exist to address such problems (e.g. $\mathcal{H}^\infty$, $\mathcal{H}^2$, $\mathcal{L}^1$ mixed sensitivity optimization), methods which systematically address non-differentiable time domain constraints and more general convex constraints (e.g. overshoots) were not available until the seminal work of [3]. The work presented also builds upon the fundamental convex optimization ideas presented in [3], [4].

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In this work, we are primarily concerned with control system designs that optimize $\mathcal{H}^\infty$ mixed-sensitivity objectives and satisfy convex constraints. Such (nondifferentiable) problems may be convexified using the celebrated Youla parameterization [5], [6]. This is significant because (1) analytic methods for such problems are currently unavailable and (2) very efficient interior point methods exist to solve such (nonlinear convex) optimization problems [7], [8], [9], [10], [11].

ACCPM Algorithm. When appropriate subgradient information is available, cutting plane methods are easy to code for nondifferentiable problems [3], [8]. In this work, we use the *analytic center cutting plane method (ACCPM)*, which combines the simplicity of cutting plane methods with the speed of interior point methods [7], [8], [9].

In addition to solving standard weighted mixed sensitivity $\mathcal{H}^\infty$ control system design problems, it is shown how subgradient concepts may be used to directly accommodate convex specifications in the design process. It is also shown that if the plant is appropriately approximated, near-optimal finite-dimensional controllers may be directly designed on the basis of the finite-dimensional plant approximants. Moreover, given an *a priori* specified optimality tolerance, it is possible to specify the required fidelity of the plant approximants; e.g. approximation order. In this sense, the method presented is very systematic. Illustrative examples are provided. Relevant work on control system design for infinite-dimensional plants includes [12], [13], [14], [15], [16]. Relevant work on convex optimization includes [3], [4], [7], [17], [18], [19], [20], [21], [22].

II. PROBLEM STATEMENT

In this section, we summarize results for $\mathcal{H}^\infty$ mixed-sensitivity minimization for stable infinite-dimensional plants [1].The following assumption will be made.

Assumption 2.1: (Plant, Approximants, Weights)
Let $P \in \mathcal{H}^\infty$ denote a stable transfer function matrix for an infinite dimensional plant. Let $\{P_n\}_{n=1}^\infty \in R\mathcal{H}^\infty$ denote a sequence of stable finite-dimensional approximants for P such that $\lim_{n\to\infty} \|P_n - P\|_{\mathcal{H}^\infty} = 0$. We also assume that the mixed-sensitivity weighting transfer function matrices satisfy $W_1, W_2, W_2^{-1} \in R\mathcal{H}^\infty$. ■

Definition 2.1: (Optimal Performance)

$$\mu_{opt}(\gamma) \triangleq \inf_{\substack{K \\ stabilizing}} \left\{ \left\| \begin{bmatrix} W_1[I+PK]^{-1} \\ W_2K[I+PK]^{-1} \end{bmatrix} \right\|_{\mathcal{H}^\infty} | C < \gamma \right\} \quad (1)$$

where C is a convex constraint on the associated closed loop maps. ∎

It is assumed that a near-optimal controller exists. Because such a controller (in general) is infinite-dimensional and we seek a finite-dimensional controller, we pose the following purely finite-dimensional problem.

Definition 2.2: (Expected Performance)

$$\mu_n(\gamma) \triangleq \inf_{\substack{K_n, FD \\ stabilizing}} \left\{ \left\| \begin{bmatrix} W_1[I+P_nK_n]^{-1} \\ W_2K_n[I+P_nK_n]^{-1} \end{bmatrix} \right\|_{\mathcal{H}^\infty} | C < \gamma \right\} \quad (2)$$

where C is a convex constraint on the associated closed loop maps. ∎

Efficient (interior point) convex programming algorithms exist for this problem [3], [7]. This problem results in a finite-dimensional controller K_n that is evaluated with the actual infinite-dimensional plant P as follows:

Definition 2.3: (Actual Performance)

$$\tilde{\mu}_n(\gamma) \triangleq \left\| \begin{bmatrix} W_1[I+PK_n]^{-1} \\ W_2K_n[I+PK_n]^{-1} \end{bmatrix} \right\|_{\mathcal{H}^\infty} \quad (3)$$

∎

This, of course, makes sense only if K_n internally stabilizes P. The following theorem ensures this as well as performance guarantees.

Theorem 2.1: (Result for Unconstrained Problem [1]) Under assumption 2.1 (with no constraints, i.e. $\gamma = \infty$), it follows that

$$\lim_{n\to\infty} \mu_n = \mu_{opt} \quad (4)$$

$$\lim_{n\to\infty} \tilde{\mu}_n = \mu_{opt}. \quad (5)$$

More precisely, let $\varepsilon_d > 0$ denote a desired performance tolerance. Choose $\varepsilon > 0$ such that

$$\varepsilon \leq \frac{\varepsilon_{\rm d}}{\|W_1\|_{\mathcal{H}^\infty} + 3 + \varepsilon_{\rm d}} \quad (6)$$

and $n_0 \in Z_+$ such that

$$\|P_n - P\|_{\mathcal{H}^\infty} \leq \frac{\min\{\frac{\varepsilon}{\|W_1\|_{\mathcal{H}^\infty}}, 1, \varepsilon\}}{\|W_2^{-1}\|_{\mathcal{H}^\infty}(\|W_1\|_{\mathcal{H}^\infty} + \varepsilon)} \quad (7)$$

for all $n > n_0$. Suppose that the finite-dimensional controller K_n achieves an expected performance $\mu_n + \varepsilon$ for all $n > n_0$. It then follows that K_n internally stabilizes the infinite-dimensional plant P and

$$|\mu_n - \mu_{opt}| < 2\varepsilon \quad (8)$$

$$\mu_{opt} \leq \tilde{\mu}_n < \mu_{opt} + \varepsilon_{\rm d} \quad (9)$$

for all $n > n_0$. ∎

Under additional assumptions, it can be shown using (Arzela-Ascoli) equicontinuity concepts that the resulting closed loop maps (e.g. sensitivity, etc.) converge uniformly on compact subsets of the imaginary axis. Illustrative examples will be presented.

III. SOLUTION TO CONSTRAINED PROBLEM

Achieving Convexity: It is well known that the above optimization problems may be convexified by exploiting the Youla-Bongiorno-Jabr-Kucera Q-Parameterization [5], [6], [23] for the set of all controllers which stabilize a plant. When P is LTI and finite-dimensional, this parameterization takes the form

$$K(Q) = Q(I - PQ)^{-1} \quad (10)$$

where $Q \in R\mathcal{H}^\infty$.

If one employs the Youla-Bongiorno-Jabr-Kucera Q-Parameterization, it can be shown that T_{wz} takes on the following form which is affine in Q

$$T_{wz}(Q) = F_l(T, Q) = T_1 + T_2QT_3. \quad (11)$$

where T_1, T_2, and T_3 are transfer function matrices associated with the augmented plant and the nominal controller. Given the above, we no longer have to search for a stabilizing controller K. Instead, we search over the convex set consisting of all stable transfer function matrices Q.

Definition 3.1: (Optimal Performance)

$$\mu_{opt}(\gamma) \triangleq \inf_{\substack{Q \in \mathcal{H}^\infty \\ C(T_{wz}(Q)) < \gamma}} \left\| \begin{bmatrix} W_1[I - PQ] \\ W_2Q \end{bmatrix} \right\|_{\mathcal{H}^\infty} \quad (12)$$

where C is a convex constraint on the associated closed loop maps. ∎

Let $Q_o \in \mathcal{H}^\infty$ be such that it achieves $\mu_{opt}(\gamma) + \epsilon$ in Equation 12.

Definition 3.2: (Expected Performance)

$$\mu_n(\gamma) \triangleq \inf_{\substack{Q \in R\mathcal{H}^\infty \\ C(T_{wz}(Q)) < \gamma}} \left\| \begin{bmatrix} W_1[I - P_nQ] \\ W_2Q \end{bmatrix} \right\|_{\mathcal{H}^\infty} \quad (13)$$

where C is a convex constraint on the associated closed loop maps. ∎

Let $Q_n \in R\mathcal{H}^\infty$ be such that it achieves $\mu_n(\gamma) + \epsilon$ in Equation 13.

Definition 3.3: (Actual Performance)

$$\tilde{\mu}_n(\gamma) \triangleq \left\| \begin{bmatrix} W_1[I - P_nQ_n] \\ W_2Q_n \end{bmatrix} (I - [P_n - P]Q_n)^{-1} \right\|_{\mathcal{H}^\infty}. \quad (14)$$

Assumption 3.1: (Continuity of Q_o)

If Q_o is assumed to be continuous on the extended imaginary axis, then it can be approximated uniformly by a real-rational $\tilde{Q}_o$; i.e. $\tilde{Q}_o \in R\mathcal{H}^\infty$. ∎

Assumption 3.2: (Convex Constraint)
Assume that the convex constraint, C, satisfies the following:

1) C sub-additive
2) $C(0) = 0$
3) C continuous at the origin.
4) Suppose $\|Q\| < B \in R$. Given $\epsilon > 0$ $\exists N_c(\epsilon, B)$ s.t. $C(Q(P_n - P)) < \epsilon$. ∎

This assumption will be made.

Theorem 3.1: (Result for Constrained Problem)
Under Assumptions 2.1 and 3.1, it follows that

$$\lim_{n\to\infty} \mu_n(\gamma) = \mu_{opt}(\gamma), \quad (15)$$

K_n stabilizes P for all but a finite number of n, and

$$\lim_{n\to\infty} \tilde{\mu}_n(\gamma) = \mu_{opt}(\gamma). \quad (16)$$

∎

The proof follows as in [1].

- One first proves upper semi-continuity. This uses Assumptions 2.1 and 3.1.
- One then shows that Q_n is uniformly bounded. This uses the fact that W_2 is a unit of $\mathcal{H}^\infty$ (Assumption 2.1).
- Lower semi-continuity can then be shown.
- The small gain theorem is then used to show that K_n stabilizes P for all but a finite number of n.
- Assumption 3.2 is used to ensure that the actual closed loop system obeys the constraints as $n \to \infty$.

IV. FINITE DIMENSIONAL SOLUTION

In this section, we formulate a general $\mathcal{H}^\infty$ control system design problem for which direct solutions are possible given the seminal work of [3]. We show how the problem may be posed as a convex optimization problem [4], [24].

In this paper, we consider a modified version of the general weighted $\mathcal{H}^\infty$ mixed-sensitivity optimization problem [6], [25] for which more general convex optimization methods may be employed [3]. Methods for solving the general problem - based on Riccati equations - are described in [6], [25]. To formulate a more general version of the weighted $\mathcal{H}^\infty$ mixed-sensitivity problem which accommodates additional design constraints, we consider Figure 1.
Figure 1 consists of the approximated plant P_n, finite-dimensional controller K_n, and weighting functions W_1, W_2, and W_3. The controller K_n is to be found by solving an appropriately formulated optimization problem. W_1, W_2, and W_3 are frequency-dependent weighting matrices on the variables $e \in \mathcal{R}^e$ (error signals), $u \in \mathcal{R}^u$ (controls), and $y \in \mathcal{R}^y$ (plant outputs), respectively, and are used to trade-off the properties of $S_n = [I + P_nK_n]^{-1}$ (sensitivity), K_nS_n (control sensitivity), and $T_n = I - S_n$ (complementary sensitivity). In this figure, W_k represents the k^{th} transfer function weighting matrix associated with the objective ($k = 1, 2, 3$), and $W^l_{k_c}$ is the k^{th} transfer

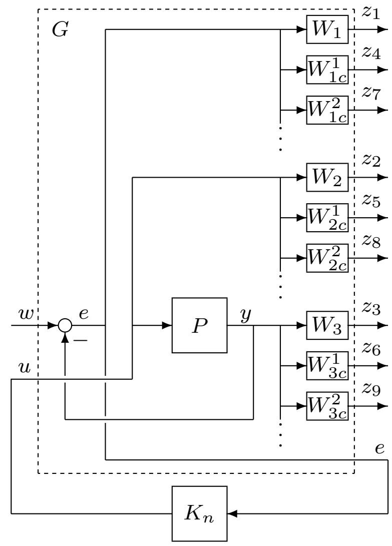

Fig. 1. System Used to Formulate Control System Design Problem

function weighting matrix associated with the l^{th} constraint ($k = 1, 2, 3,\ l = 1, 2, 3, \cdots$).

Given the above, the new optimization problem to be solved is that of finding a stabilizing finite-dimensional LTI controller K_n that minimizes the $\mathcal{H}^\infty$ norm of T_{wz}, the transfer function matrix from w to $\begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T$, while satisfying all the constraints. This may be written as follows:

$$K_n = \arg\{ \inf_{\substack{K_n \\ stabilizing}} \mu \ \left| \ \left\| \begin{bmatrix} W_1 S_n \\ W_2 K_n S_n \\ W_3 T_n \end{bmatrix} \right\|_{\mathcal{H}^\infty} < \mu, \right.$$
$$C_i \begin{pmatrix} W^i_{1c} S_n \\ W^i_{2c} K_n S_n \\ W^i_{3c} T_n \end{pmatrix} \leq \gamma \quad i = 1, 2, \ldots \ \} \quad (17)$$

where $C_i(\cdot)$ denotes the i^{th} constraint functional. It should be noted that the augmented plant, G, contains all subsystems essential to carry out the optimization.

Given the above formulation, it is important to note that the above optimization problem for K_n is nonlinear and infinite-dimensional, and no closed form solution or direct approach exists for the above problem. Concepts to address these issues are presented below.

One approach to solving the unconstrained version of the above problem is to use MathWorks' robust control toolbox [26] or the μ-tools toolbox. The theory is well developed, as are the algorithms [6], [25]. While this Riccati-based approach has been widely used, it does not lend itself to time domain constraints such as overshoot constraints. This motivates our approach [3].

One method, which is able to handle additional constraints, for solving the above general (constrained) problem is now described.

Achieving Finite Dimensionality. After achieving convexity as described in Section III, we still have an infinite-dimensional problem. To obtain a finite-dimensional problem, we express the Q as a finite linear combination of *a priori* selected stable transfer functions q_k; e.g.

$$Q_N = \sum_{k=1}^{N} X_k q_k \tag{18}$$

where

$$X_k = \begin{bmatrix} x_k^{11} & \cdots & x_k^{1n_e} \\ \vdots & & \vdots \\ x_k^{n_u 1} & \cdots & x_k^{n_u n_e} \end{bmatrix} \in \mathcal{R}^{n_u \times n_e} \tag{19}$$

Basis Used to Approximate Q. In our work, we chose [17]

$$q_k = \left(\frac{s - \alpha_a + \alpha_b}{s + \alpha_a + \alpha_b} \right)^{k-1} \quad k = 1, 2, ..., N \tag{20}$$

where both α_a and α_b are positive real numbers.
Given the above, equation (17) can be written as follows:

$$K = \arg\{ \min_{\substack{Q_N \\ \text{stabilizing}}} \mu \mid \|T_1 + T_2 Q_N T_3\|_{\mathcal{H}^\infty} < \mu, \quad \hat{C}_i(Q_N) \leq \gamma \quad i = 1, 2, \ldots \} \tag{21}$$

After vectorization, Equation (21) may be written as follows

$$K = \arg\{ \min_{\substack{x_l \\ \text{stabilizing}}} \mu \mid \left\| M_o + \sum_{l=1}^{n_u \times n_e \times N} M_l \, x_l \right\|_{\mathcal{H}^\infty} < \mu, \quad \hat{\hat{C}}_i(x_l) \leq \gamma \quad i = 1, 2, \ldots \} \tag{22}$$

Order of the Controller $K(Q_N)$. It can be shown that the controller A-matrix $A_K \in \mathcal{R}^{(n+n_q)\times(n+n_q)}$ where n is the number of states of the generalized plant, G (Fig. 1), and n_q is the number of states of Q_N. Given the structure of G, it follows that

$$n = n_p + n_w \tag{23}$$

where n_p is the number of plant states and n_w is the sum of the states associated with all weighting functions (including constraint weightings). Given that Q_N is given by (18–20), it follows that

$$n_q = n_e \times (N - 1). \tag{24}$$

Hence, the number of states associated with the final controller $K(Q_N)$ is given by

$$n_k = n + n_q = n + n_e \times (N-1) \tag{25}$$

$$= n_p + n_w + n_e \times (N-1). \tag{26}$$

As N gets larger, we expect Q_N to be near optimal - as it more accurately approximates the optimal Q. As N gets larger, we therefore expect a controller which is closer to being optimal. This, however, comes at the expense of a higher order controller $K(Q_N)$. A designer must therefore use N to carefully trade-off controller optimality versus controller simplicity (i.e. order).

Further analysis on posing control system design specifications as convex constraints are given in [3, page 172], and [3, pp. 132-133, pp. 175-177]. Basics results from convex analysis and a description of *Analytic Center Cutting Plane Method (ACCPM)* used to solve problems are given in [8], [10], [9], [3, pp. 313-319], [4, pp. 79], [27], [11], [28] and [29].

TABLE I
UNCONSTRAINED CASE: μ_n and $\tilde{\mu}_n$

	$\Delta = 0.2sec$		$\Delta = 0.5sec$		$\Delta = 0.8sec$	
n	μ_n	$\tilde{\mu}_n$	μ_n	$\tilde{\mu}_n$	μ_n	$\tilde{\mu}_n$
1	.7762	.7864	.8553	.9029	.9318	1.0357
2	.7797	.7803	.8699	1.0508	.9559	1.4257
3	.7799	.7799	.8679	.9591	.9574	1.3483
4	.7799	.7799	.8682	.8826	.9588	1.0812
5			.8685	.8808	.9587	1.3702
6			.8685	.8685	.9587	1.1214
7			.8685	.8685	.9587	1.0064
8					.9587	1.1080
9					.9586	.9905
10					.9586	.9586
11					.9587	.9587
12					.9587	.9587

V. ILLUSTRATIVE EXAMPLES

This section presents illustrative examples.
In this example, we consider the infinite-dimensional plant $P = e^{-\Delta s}\frac{1}{s+1}$ and the mixed-sensitivity problem defined by the weighting functions $W_1 = \frac{0.7079s+0.5}{s+0.0005}$, $W_2 = \frac{s+10}{0.01s+1000}$. The finite dimensional approximants P_n used are as follows: $P_n = \frac{1}{s+1} \cdot \{[\text{n}, \text{n}] \text{ Padé approximation of } e^{-s\Delta}\}$. These approximants satisfy $\lim_{n \to \infty} \|P_n - P\|_{\mathcal{H}^\infty} = 0$ as required. To approximate the Q-parameter, we use the basis $q_k = \left(\frac{s-\frac{4}{\Delta}}{s+\frac{4}{\Delta}}\right)^{k-1}$ with $N = 2$ terms.

EXAMPLE 1: (Unconstrained Case)
Resulting data for the unconstrained problem is presented in Table I. for different delay values. For small delay values, convergence is quick. For larger delay values, more terms are required. This is as expected.

Closed Loop Frequency Responses: Unconstrained Problem. The associated closed loop (sensitivity and reference to control) frequency responses are given in Figures 2-3 for $\Delta = 0.8$. The responses are observed to converge uniformly.
Closed Loop Time Responses: Unconstrained Problem. The associated closed loop time responses to step reference commands are given in Figures 4-5 for $\Delta = 0.8$. The responses are observed to converge uniformly.

EXAMPLE 2: (Constrained Case)
Resulting data for the constrained problem is presented in Table II. for different delay values. For small delay values, convergence is quick. For larger delay values, more terms are required. This is as expected.

Closed Loop Frequency Responses: Constrained Problem. The associated closed loop (sensitivity and reference to

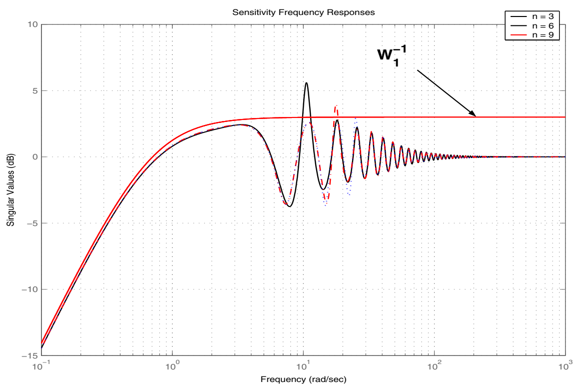

Fig. 2. Sensitivity Frequency Response, $S = \frac{1}{1+PK_n}$

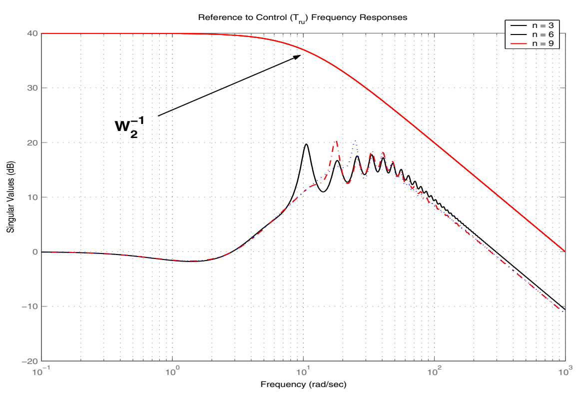

Fig. 3. Reference to Control Frequency Response, $K_n S = \frac{K_n}{1+PK_n}$

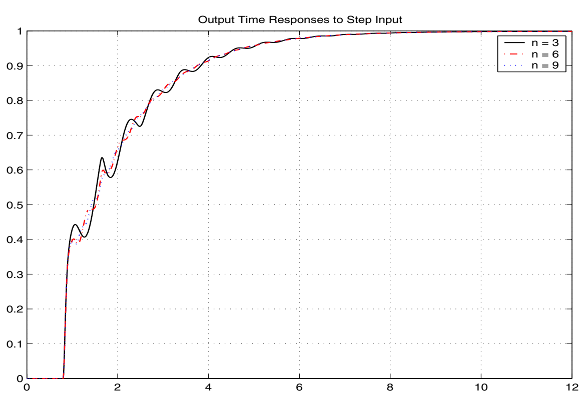

Fig. 4. Output Response to Unit Step Reference Command

TABLE II

CONSTRAINED CASE: μ_n and $\tilde{\mu}_n$

	$\Delta = 0.2sec$		$\Delta = 0.5sec$		$\Delta = 0.8sec$	
	$u(t) \leq 1.15$		$u(t) \leq 2.0$		$u(t) \leq 3.0$	
n	μ_n	$\tilde{\mu}_n$	μ_n	$\tilde{\mu}_n$	μ_n	$\tilde{\mu}_n$
1	.7845	.7931	.8686	.9095	.9484	1.0405
2	.7895	.7896	.8879	.8886	.9838	.9918
3	.7896	.7896	.8884	.8884	.9846	.9846
4	.7896	.7896	.8884	.8884	.9847	.9847
5			.8884	.8884	.9847	.9847

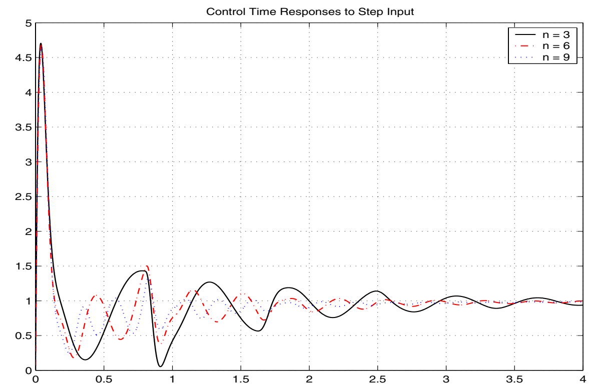

Fig. 5. Control Response to Unit Step Reference Command

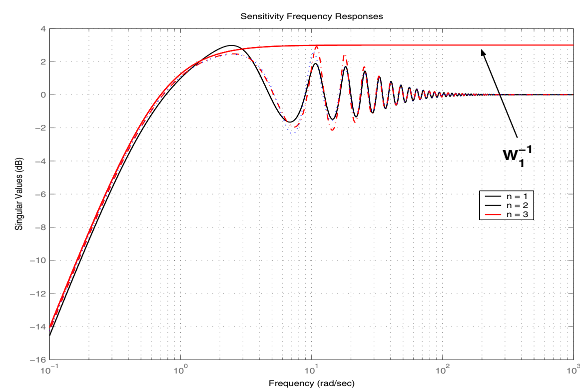

Fig. 6. Sensitivity Frequency Response of Actual Performance with Constraint

control) frequency responses are given in Figures 6-7 for $\Delta = 0.8$. The responses are observed to converge uniformly. *Closed Loop Time Responses: Unconstrained Problem.* The associated closed loop time responses to step reference commands are given in Figures 8-9 for $\Delta = 0.8$. The responses are observed to converge uniformly.

VI. CONCLUSIONS AND FUTURE WORKS

This paper has shown how convex optimization may be used to determine $\mathcal{H}^\infty$ near-optimal finite-dimensional controllers for stable plants subject to convex constraints. Future work will examine more complex infinite dimensional plants (e.g. unstable, MIMO) and convergence issues [23], [30], [31].

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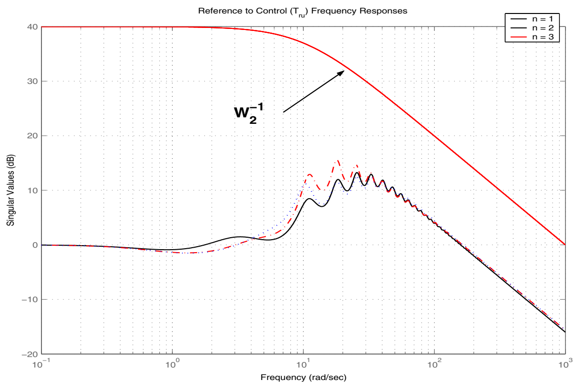

Fig. 7. T_{ru} Frequency Response of Actual Performance with Constraint

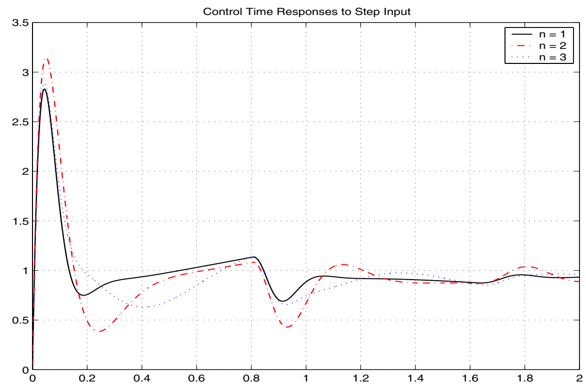

Fig. 9. Control Output of Actual Performance with Constraint

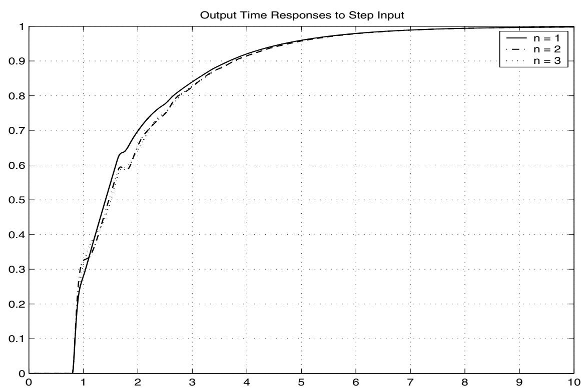

Fig. 8. Step Response of Actual Performance with Constraint

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