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Adaptive Output Feedback Design for Actuator Failure Compensation Using Dynamic Bounding: Output Tracking and An Application *

Xidong Tang and *Gang Tao*
Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22903

Suresh M. Joshi
Mail Stop 132
NASA Langley Research Center
Hampton, VA 23681

Abstract

A new adaptive output feedback design using dynamic bounding is presented for actuator failure compensation of a class of nonlinear systems with unknown nonlinearities bounded by output-dependent functions and dynamic signals. The new features are that desired objectives can be achieved via a Lyapunov based design without using high gain type controllers and that it handles a larger class of systems. An adaptive scheme dealing with unknown actuator failures as well as system uncertainties is developed to achieve desired output tracking and closed-loop stability in spite of actuator failures.

1 Introduction

Compensation for actuator failures that are often unknown in terms of failure patterns, times and values is an important area in control. To design an effective adaptive control scheme achieving a desired objective in the presence of unknown actuator failures, the structure of systems and controllers, the choice of actuation schemes, and the stability of zero dynamics have to be considered with the uncertainties caused by actuator failures. There have been significant efforts in developing adaptive control algorithms for systems with failures [1], [2], [9], [10]. On the other hand, effective nonlinear feedback designs are needed to handle enlarged classes of nonlinear systems, especially for output feedback. Recently, progress has been made in nonlinear control using dynamic bounding [6]. This paper presents new results in these two aspects.

In [7], adaptive actuator failure compensation using output feedback design is studied for a class of nonlinear systems with unknown state-dependent nonlinearities bounded by a nonlinear function of the output. This paper addresses the actuator failure compensation problem using output feedback design for the class of nonlinear systems with unknown state-dependent nonlinearities, which are bounded not only by some linear parameterized static functions dependent on the output, but also by some dynamic signals generated by some linear dynamics whose inputs are functions of the output. By employing dynamic bounding, a new robust adaptive scheme is presented to solve the actuator failure problem with the stability established based on a new backstepping design and Lyapunov analysis. Compared with the control scheme designed in [7], the new control scheme achieves better results in the sense that we avoid using a high gain type controller to ensure the stability and that we can handle a larger class of systems at the same time.

2 Problem Statement

Consider the nonlinear system in the form of

$$\begin{aligned} \dot{x}_i &= x_{i+1}+\phi_i(x,t)+\varphi_i(y),\ i=1,2,\ldots,\rho-1 \\ \dot{x}_\rho &= x_{\rho+1}+\phi_\rho(x,t)+\varphi_\rho(y)+\sum_{j=1}^{m} b_{n^*,j}\beta_j(y)u_j,\ \ldots, \\ \dot{x}_{n-1} &= x_n+\phi_{n-1}(x,t)+\varphi_{n-1}(y)+\sum_{j=1}^{m} b_{1,j}\beta_j(y)u_j \\ \dot{x}_n &= \phi_n(x,t)+\varphi_n(y)+\sum_{j=1}^{m} b_{0,j}\beta_j(y)u_j \qquad (2.1)\end{aligned}$$

where $u_j,\ j=1,2,\ldots,m$, are the inputs that may fail during system operation, $x=[x_1,x_2,\ldots,x_n]^T$ is the vector of unmeasured states, $y=x_1$ is the output, $b_{r,j},\ r=0,1,\ldots,n^*=n-\rho,\ j=1,2,\ldots,m$, are unknown constants, $\varphi_i(y),\ i=1,2,\ldots,n$, and $\beta_j(y),\ j=1,2,\ldots,m$, are smooth nonlinear functions and known, $\beta_j(y)\neq 0,\ \forall y\in R$, and $\phi_i(x,t)$, $i=1,2,\ldots,n$, are unknown functions that are not necessarily smooth. The following assumption is made on $\phi(x,t)=[\phi_1(x,t),\phi_2(x,t),\ldots,\phi_n(x,t)]^T$:

(A1) There exists a vector of functions $\psi(y)=[\psi_1(y),\psi_2(y),\ldots,\psi_{l_1}(y)]^T\in R^{l_1}$, $\psi_i(y)\geq 0,\ i=1,2,\ldots,l_1$, a diagonal matrix of functions $\Psi(y)=\mathrm{diag}\{\Psi_1(y),\Psi_2(y),\ldots,\Psi_{l_2}(y)\}\in R^{l_2\times l_2}$, $\Psi_i(y)\geq 0,\ i=1,2,\ldots,l_2$, and a vector of signals $\rho(t)=[\rho_1(t),\rho_2(t),\ldots,\rho_{l_2}(t)]^T\in R^{l_2}$ with $\rho_i(t)=F_i(s)[\eta_i(y)]$, $\eta_i(y)\geq 0$, $F_i(s)$ is stable and strictly proper, and $F_i(s-\delta_i)$ with some $\delta_i>0$ is satble, $i=1,2,\ldots,l_2$, such that

$$\|\phi(x,t)\|\leq \theta_1+\theta_2^T\psi(y)+\theta_{30}^T\Psi(y)\rho(t), \qquad (2.2)$$

where $\psi(y)$, $\Psi(y)$, and $\eta(y)=[\eta_1(y),\eta_2(y),\ldots,\eta_{l_2}(y)]^T\in R^{l_2}$ are smooth and known, a positive constant δ_0 such that $\delta_0\leq\min\{\delta_i\}$ is known, and $\theta_1\in R$, $\theta_2\in R^{l_1}$, and $\theta_{30}\in R^{l_2}$ are unknown constants with $\theta_1\geq 0$, $\theta_{2i}\geq 0$, $i=1,2,\ldots,l_1$, and $\theta_{30i}\geq 0$, $i=1,2,\ldots,l_2$.

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As in [7], the actuator failures are modeled as

$$u_j(t) = \bar{u}_j,\ t \geq t_j,\ j \in \{1,2,\ldots,m\}, \tag{2.3}$$

where the failure value $\bar{u}_j$, the failure time instant t_j, and the failure pattern, that is, the index j are unknown. More general failures modeled as $u_j(t) = \bar{u}_j + \sum_{i=1}^{n_j} \bar{d}_{ji} f_{ji}(t)$, $j = 1,2,\ldots,m$, with unknown constants $\bar{u}_j$ and $\bar{d}_{ji}$ and known signals $f_{ji}(t)$, can also be handled similarly.

Suppose that there are p_k actuators failed at a time instant t_k, $k = 1,2,\ldots,q$, and $t_0 < t_1 < t_2 < \ldots < t_q < \infty$. Due to up to $m-1$ failures, it follows that $\sum_{k=1}^{q} p_k \leq m-1$. In other words, at time $t \in (t_k, t_{k+1})$, $k = 0,1,\ldots,q$, with $t_{q+1} = \infty$, there are $p = \sum_{i=1}^{k} p_i$ failed actuators, that is, $u_j(t) = \bar{u}_j$, $j = j_1,\ldots,j_p$, $0 \leq p \leq m-1$, and $u_j(t) = v_j(t)$, $j \neq j_1,\ldots,j_p$, where $v_j(t)$, $j = 1,2,\ldots,m$, are applied control inputs to be designed.

The control objective is to design an output feedback scheme for the nonlinear plant (2.1) with p unknown actuator failures, when p changes at time instants t_k, $k = 1,2,\ldots,q$, such that the plant output $y(t)$ tracks a given reference signal $y_r(t)$ with up to ρth order derivative bounded as close as possible and all closed-loop signals are bounded despite unknown plant parameters and state-dependent nonlinearities.

3 Adaptive Compensation Control

The zero dynamics of (2.1) with p actuator failures are dependent on the failure pattern. For closed-loop stability, zero dynamics need to be stable for any possible failure pattern. We specify the following condition with which a stable adaptive scheme can be developed to achieve the control objective.

(A2) The polynomials $\sum_{j\neq j_1,\ldots,j_p} \mathrm{sign}[b_{n^*,j}] B_j(s)$ are stable, $\forall \{j_1,\ldots,j_p\} \subset \{1,2,\ldots,m\}$, $\forall p \in \{0,1,2,\ldots,m-1\}$, where

$$B_j(s) = b_{n^*,j} s^{n^*} + b_{n^*-1,j} s^{n^*-1} + \cdots + b_{1,j} s + b_{0,j}. \tag{3.1}$$

For an adaptive design, another assumption is required:

(A3) The sign of $b_{n^*,j}$ is known, for $j = 1,2,\ldots,m$.

We apply a proportional actuation scheme

$$v_j = \mathrm{sign}[b_{n^*,j}] \frac{1}{\beta_j(y)} v_0,\ j = 1,2,\ldots,m, \tag{3.2}$$

where v_0 is a control signal generated via a backstepping design procedure to be given next.

Applying the actuation scheme (3.2) and defining

$$\begin{aligned}
\varphi(y) &= [\varphi_1(y), \varphi_2(y), \ldots, \varphi_n(y)]^T, && (3.3)\\
k_{1,r} &= \sum_{j\neq j_1,\ldots,j_p} \mathrm{sign}[b_{n^*,j}] b_{r,j},\ r = 0,1,\ldots,n^*, \\
k_{2,rj} &= b_{r,j}\bar{u}_j,\ r = 0,1,\ldots,n^*,\ j = j_1,\ldots,j_p, \\
k_{2,rj} &= 0,\ r = 0,1,\ldots,n^*,\ j \neq j_1,\ldots,j_p, && (3.4)
\end{aligned}$$

we express the system (2.1) with p actuator failures as

$$\begin{aligned}
\dot{x} &= Ax + \phi(x,t) + \varphi(y) + \sum_{r=0}^{n^*} e_{n-r} \sum_{j=1}^{m} k_{2,rj} \beta_j(y) + \sum_{r=0}^{n^*} e_{n-r} k_{1,r} v_0 \\
y &= c^T x, && (3.5)
\end{aligned}$$

where $A \in R^{n\times n}$ is a canonical form describing a chain of n integrators, $c = [1,0,\ldots,0]^T$, and e_i is the ith coordinate vector in R^n. It follows from Assumption (A2) that the polynomial $k_{1,n^*} s^{n^*} + k_{1,n^*-1} s^{n^*-1} + \cdots + k_{1,1} s + k_{1,0}$ is stable, and, in addition, from (3.2), that $k_{1,n^*} > 0$.

3.1 State Observation

Choose a vector $l \in R^n$ such that $A_o = A - lc^T$ is stable. With knowledge of $k_{1,r}$ and $k_{2,rj}$, $j = 1,2,\ldots,m$, $r = 0,1,\ldots,n^*$, we have a nominal state observer for x as

$$x^* = \xi + \xi^* + \sum_{r=0}^{n^*} \sum_{j=1}^{m} k_{2,rj} \zeta_r + \sum_{r=0}^{n^*} k_{1,r} \mu_{rj}, \tag{3.6}$$

with the filters defined as

$$\begin{aligned}
\dot{\xi} &= A_o \xi + ly + \varphi(y),\ \dot{\xi}^* = A_o \xi^* + \phi(x,t), \\
\dot{\zeta}_{rj} &= A_o \zeta_{rj} + e_{n-r} \beta_j(y),\ 0 \leq r \leq n^*,\ 1 \leq j \leq m, \\
\dot{\mu}_r &= A_o \mu_r + e_{n-r} v_0,\ 0 \leq r \leq n^*. && (3.7)
\end{aligned}$$

With $\varepsilon = x - x^*$, it follows from (3.5), (3.6) and (3.7) that $\dot{\varepsilon} = A_o \varepsilon$ and $\lim_{t\to\infty} \varepsilon(t) = 0$ exponentially. Hence x^* is the nominal observation of x in the sense that the error $x(t) - x^*(t)$ converges to zero exponentially. However, the vector ξ^* is not available, as the system state x in $\phi(x,t)$ is not available, while the second component of ξ^*, i.e., ξ_2^* appears in the backstepping design procedure (e.g., added to $\phi_1(x,t)$ in (3.16)).

To deal with the unavailable component ξ_2^*, we introduce dynamic bounding signals $\chi_0(t) \in R^{l_2}$ and $\chi(t) \in R^L$, $L = l_1 + l_2 + 1$, which are generated from

$$\dot{\chi}_0(t) = -\delta_0 \chi_0(t) + \eta(y),\ \dot{\chi}(t) = -\bar{\delta}\chi(t) + \Phi, \tag{3.8}$$

where $\Phi = [1, \psi^T(y), \chi_0^T(t)\Psi(y)]^T$, and $\bar{\delta} > 0$ is chosen such that $A_o + \bar{\delta} I$ is stable. Recall Assumption (A1) and notice that $\delta_0 \leq \min\{\delta_i\}$ such that $F_i(s - \delta_0)$, $i = 1,2,\ldots,l_2$, are stable. It can be concluded from Lemma 2.6 in [8] that

$$|\rho_i(t)| \leq \delta_{fi} \frac{1}{s+\delta_0} \eta_i(y) = \delta_{fi} \chi_{0i}(t),\ i = 1,2,\ldots,l_2, \tag{3.9}$$

where $\delta_{fi} = \|f_{i\delta_0}(\cdot)\|_1$ with $f_{i\delta_0}(t)$ denoting the impulse response function of $F_i(s - \delta_0)s$, and $\chi_{0i}(t)$ is the ith component of $\chi_0(t)$. From (3.9), it in turn follows that

$$\theta_{30}^T \Psi(y) \rho(t) \leq \theta_3^T \Psi(y) \chi_0(t), \tag{3.10}$$

where $\theta_3 \in R^{l_2}$ is an unknown constant vector whose components $\theta_{3i} = \theta_{30i}\delta_{fi} \geq 0$, $i = 1,2,\ldots,l_2$.

From the dynamic equations of ξ^* in (3.7), it follows that

$$\xi_2^*(t) = H_1^*(s)[\phi_1(x,t)] + \cdots + H_n^*(s)[\phi_n(x,t)], \tag{3.11}$$

where $H_i^*(s) = \frac{N_i(s)}{D(s)}$ is the transfer function from $\phi_i(x,t)$ to ξ_2^*, $i = 1,2,\ldots,n$, with $D(s) = \det(sI - A_o)$. Since $A_o + \bar{\delta} I$ is stable, we have that $D(s - \bar{\delta})$ is stable. It is also noted that $H_i^*(s)$ is strictly proper. With the definition that $H_i(s) =$

$(s+\bar{\delta})H_i^*(s) = \frac{N_i(s)(s+\bar{\delta})}{D(s)}$ for $i = 1,2,\ldots,n$, the signal ξ_2^* in the input-to-output form (3.11) is rewritten as

$$\xi_2^*(t) = H_1(s)\frac{1}{s+\bar{\delta}}[\phi_1(x,t)] + \cdots + H_n(s)\frac{1}{s+\bar{\delta}}[\phi_n(x,t)], \quad (3.12)$$

where $H_i(s)$ is a proper transfer function for $i = 1,2,\ldots,n$.

Based on Lemma 2.6 in [8], it follows from (3.12) that

$$|\xi_2^*(t)| \le \delta\frac{1}{s+\bar{\delta}}\left(\theta_1 + \theta_2^T\psi(y) + \theta_{30}^T\Psi(y)\rho(t)\right) \le \delta\theta^T\chi(t), \quad (3.13)$$

with $\theta = [\theta_1, \theta_2^T, \theta_3^T]^T = [\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(L)}]^T \in R^L$, and $\delta = \|h_{1\bar{\delta}}(\cdot)\|_1 + \|h_{2\bar{\delta}}(\cdot)\|_1 + \cdots + \|h_{n\bar{\delta}}(\cdot)\|_1$, where $h_{i\bar{\delta}}(t)$ is the impulse response function of $H_i(s-\bar{\delta})$, $i = 1,2,\ldots,n$.

3.2 Adaptive Design for Output Tracking

The backstepping technique [4] is applied with a design procedure consisting of ρ steps to derive an adaptive actuator failure compensation scheme for the output tracking objective.

Define unknown constant $k = [k_{1,n^*}, k_1^T, k_2^T]^T$ and k_3 with

$$\begin{aligned} k_1 &= [k_{1,0}, k_{1,1}, \ldots, k_{1,n^*-1}]^T, \\ k_2 &= [k_{2,01}, k_{2,02}, \ldots, k_{2,0m}, k_{2,11}, \ldots, k_{2,1m}, \ldots, k_{2,n^*m}]^T, \\ k_3 &= [\theta_1^2, \theta_{21}^2, \theta_{22}^2, \ldots, \theta_{2l_1}^2, \theta_{31}^2, \theta_{32}^2, \ldots, \theta_{3l_2}^2]^T, \end{aligned} \quad (3.14)$$

and $\kappa = \frac{1}{k_{1,n^*}}$, and vectors of signals $\upsilon_i = [\mu_{n^*,i}, \omega_i^T, \varepsilon_i^T]^T$, ς with

$$\begin{aligned} \omega_i &= [\mu_{0,i}, \mu_{1,i}, \ldots, \mu_{n^*-1,i}]^T, \; i = 1,2,\ldots,n, \\ \varepsilon_i &= [\zeta_{01,i}, \zeta_{02,i}, \ldots, \zeta_{0m,i}, \zeta_{11,i}, \ldots, \zeta_{1m,i}, \ldots, \zeta_{n^*m,i}]^T, \\ \varsigma &= [\chi_1^2 + 1, \chi_{21}^2 + \psi_1^2(y), \ldots, \chi_{2l_1}^2 + \psi_{l_1}^2(y), \\ &\quad \chi_{31}^2 + \Psi_1^2(y)\chi_{01}^2, \ldots, \chi_{3l_2}^2 + \Psi_{l_2}^2(y)\chi_{0l_2}^2]^T, \end{aligned} \quad (3.15)$$

where $\mu_{r,i}$ and $\zeta_{rj,i}$, $i = 1,2,\ldots,n$, are the ith component of μ_r and ζ_{rj}, $r = 0,1,\ldots,n^*-1$, $j = 1,2,\ldots,m$. Let $\hat{\kappa}$, $\hat{k}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_3$ denote the estimates of κ, k, k_{1,n^*}, k_1, k_2, and k_3.

For the issue of robustness, assume that the failure value $\bar{u}_j$ is bounded by a known U_j, i.e., $|\bar{u}_j| \le U_j$, $j = 1,2,\ldots,m$, and $0 < b_{r,j}^a \le |b_{r,j}| \le b_{r,j}^b$, where $b_{r,j}^a$ and $b_{r,j}^b$ are known, for $r = 1,2,\ldots,n^*$, $j = 1,2,\ldots,m$, and $\|\theta\|_\infty \le \Theta$ with Θ known. It follows that $|\kappa| \le M_\kappa$, $\|k\| \le M$, and $\|k_3\| \le M_3$, where $M_\kappa = \frac{1}{\min_{j=1,\ldots,m}\{b_{n^*,j}^a\}}$, $M = \max\{\sum_{j=1}^m b_{n^*,j}^b, \sum_{j=1}^m \sum_{r=1}^{n^*-1} b_{r,j}^b, \sum_{j=1}^m U_j \sum_{r=1}^{n^*} b_{r,j}^b\}$, $M_3 = (l_1 + l_2 + 1)\Theta^2$.

Step 1: Defining the tracking error $z_1 = y - y_r$, we have

$$\dot{z}_1 = \varepsilon_2 + \upsilon_2^T k + \xi_2 + \xi_2^* + \phi_1(x,t) + \varphi_1(y) - \dot{y}_r. \quad (3.16)$$

Choose the auxiliary signal

$$\begin{aligned} & z_2 = \mu_{n^*,2} - \hat{\kappa}\dot{y}_r - \alpha_1, \; \alpha_1 = \hat{\kappa}\bar{\alpha}_1, \\ & \bar{\alpha}_1 = -c_1 z_1 - d_1 z_1 - \omega_2^T\hat{k}_1 - \varepsilon_2^T\hat{k}_2 - \lambda_1 z_1\varsigma^T\hat{k}_3 - \xi_2 - \varphi_1(y). \end{aligned} \quad (3.17)$$

Substituting (3.17) into (3.16) results in

$$\begin{aligned} \dot{z}_1 &= -c_1 z_1 - d_1 z_1 - \lambda_1 z_1\varsigma^T k_3 + \varepsilon_2 + \xi_2^* + \phi_1(x,t) \\ &\quad + \hat{k}_{1,n^*} z_2 - k_{1,n^*}\tilde{\kappa}(\dot{y}_r + \bar{\alpha}_1) + \tilde{k}_{1,n^*}(\mu_{n^*,2} - \hat{\kappa}\dot{y}_r - \alpha_1) \\ &\quad + \omega_2^T\tilde{k}_1 + \varepsilon_2^T\tilde{k}_2 + \lambda_1 z_1\varsigma^T\tilde{k}_3, \end{aligned} \quad (3.18)$$

where $\tilde{k}_{1,n^*} = k_{1,n^*} - \hat{k}_{1,n^*}$, $\tilde{k}_1 = k_1 - \hat{k}_1$, $\tilde{k}_2 = k_2 - \hat{k}_2$, $\tilde{k}_3 = k_3 - \hat{k}_3$, and $\tilde{\kappa} = \kappa - \hat{\kappa}$ and c_1, d_1, and λ_1 are some positive constants to be chosen.

Consider the partial Lyapunov function $V_1 = \frac{1}{2}z_1^2 + \frac{k_{1,n^*}}{2\gamma_1}\tilde{\kappa}^2$, where $k_{1,n^*} > 0$ due to Assumption (A2) and $\gamma_1 > 0$ is given. Choose the adaptive law for $\hat{\kappa}$ as

$$\dot{\hat{\kappa}} = -\gamma_1 z_1(\dot{y}_r + \bar{\alpha}_1) - \gamma_1\sigma_\kappa(\hat{\kappa})\hat{\kappa}, \quad (3.19)$$

where $\sigma_\kappa(\hat{\kappa})$ is the switching signal defined as

$$\sigma_\kappa(\hat{\kappa}) = \begin{cases} 0 & \text{if } |\hat{\kappa}| < M_\kappa \\ (|\hat{\kappa}| - M_\kappa)^{2\rho} & \text{if } |\hat{\kappa}| \ge M_\kappa, \end{cases} \quad (3.20)$$

and the tuning functions for $\hat{k}$ and $\hat{k}_3$ as

$$\tau_1 = z_1[(\mu_{n^*,2} - \hat{\kappa}\dot{y}_r - \alpha_1), \omega_2^T, \varepsilon_2^T]^T, \; \varpi_1 = \lambda_1 z_1^2\varsigma. \quad (3.21)$$

Then the time-derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 - d_1 z_1^2 - \lambda_1 z_1^2\varsigma^T k_3 + z_1\varepsilon_2 + z_1\xi_2^* + z_1\phi_1(x,t) \\ &\quad + \hat{k}_{1,n^*} z_1 z_2 + k_{1,n^*}\sigma_\kappa(\hat{\kappa})\hat{\kappa}\tilde{\kappa} + \tau_1^T\tilde{k} + \varpi_1^T\tilde{k}_3, \end{aligned} \quad (3.22)$$

where $\tilde{k} = k - \hat{k}$, $\hat{k}_{1,n^*} z_1 z_2$ will be cancelled at the next step and $z_1\varepsilon_2$, $z_1\xi_2^*$, $z_1\phi_1(x,t)$ will be handled later on.

Step $i = 2,3,\ldots,\rho$: Define z_i, $i = 2,3,\ldots,\rho$, as

$$z_i = \mu_{n^*,i} - \hat{\kappa}y_r^{(i-1)} - \alpha_{i-1}. \quad (3.23)$$

Choose the stabilizing function α_i as

$$\begin{aligned} \alpha_i &= -a_i z_{i-1} - c_i z_i - d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i - \lambda_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i\varsigma^T\hat{k}_3 + l_i\mu_{n^*,1} \\ &\quad + \left(y_r^{(i-1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\kappa}}\right)\dot{\hat{\kappa}} + \sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(q-1)}}y_r^{(q)} + \frac{\partial\alpha_{i-1}}{\partial\chi}(-\bar{\delta}\chi + \Phi) \\ &\quad + \frac{\partial\alpha_{i-1}}{\partial\chi_0}(-\delta_0\chi_0 + \eta(y)) + \frac{\partial\alpha_{i-1}}{\partial y}(\upsilon_2^T\hat{k} + \xi_2 + \varphi_1(y)) \\ &\quad + \sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\xi_q}(\xi_{q+1} - l_q\xi_1 + l_q y + \varphi_q(y)) \\ &\quad + \sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\upsilon_q}(\upsilon_{q+1} - l_q\upsilon_1) + \frac{\partial\alpha_{i-1}}{\partial\hat{k}}(\Gamma\tau_i - \Gamma\sigma(\hat{k})\hat{k}) \\ &\quad - \sum_{q=2}^{i-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\upsilon_2 z_q + \frac{\partial\alpha_{i-1}}{\partial\hat{k}_3}(\Gamma_3\varpi_i - \Gamma_3\sigma_3(\hat{k}_3)\hat{k}_3) \\ &\quad + \sum_{q=2}^{i-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_3}\Gamma_3\lambda_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i\varsigma z_q, \end{aligned} \quad (3.24)$$

where $a_2 = \hat{k}_{1,n^*}$ and $a_i = 1$, $i = 3,4,\ldots,\rho$, $c_i > 0$, $d_i > 0$, $\lambda_i > 0$, $i = 2,3,\ldots,\rho$, $\Gamma = \Gamma^T > 0$ and $\Gamma_3 = \Gamma_3^T > 0$ are the adaptive gains, $\sigma(\hat{k})$ and $\sigma_3(\hat{k}_3)$ are the switching signals given by

$$\begin{aligned} \sigma(\hat{k}) &= \begin{cases} 0 & \text{if } |\hat{k}| < M \\ (\|\hat{k}\| - M)^{2\rho} & \text{if } |\hat{k}| \ge M, \end{cases} \\ \sigma_3(\hat{k}_3) &= \begin{cases} 0 & \text{if } \|\hat{k}_3\| < M_3 \\ (\|\hat{k}_3\| - M_3)^{2\rho} & \text{if } \|\hat{k}_3\| \ge M_3, \end{cases} \end{aligned} \quad (3.25)$$

and τ_i and ϖ_i, $i=2,3,\ldots,\rho$, are the tuning functions:

$$\tau_i=\tau_{i-1}-\frac{\partial\alpha_{i-1}}{\partial y}\omega_2 z_i,\ \varpi_i=\varpi_{i-1}+\lambda_i(\frac{\partial\alpha_{i-1}}{\partial y})^2 z_i^2\varsigma. \quad (3.26)$$

Considering the partial Lyapunov candidate function $V_i=V_{i-1}+\frac{1}{2}z_i^2$, with $z_{i+1}=\mu_{n^*,i+1}-\hat{\kappa}y_r^{(i)}-\alpha_i$, we have

$$\begin{aligned}\dot{V}_i &= -\sum_{q=1}^{i}c_q z_q^2-d_1z_1^2+z_1\varepsilon_2-\lambda_1 z_1^2\varsigma^T k_3+z_1\xi_2^* \\ &+z_1\phi_1(x,t)-\sum_{q=2}^{i}d_q(\frac{\partial\alpha_{q-1}}{\partial y})^2 z_q^2-\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial y}\varepsilon_2 \\ &-\sum_{q=2}^{i}\lambda_q(\frac{\partial\alpha_{q-1}}{\partial y})^2 z_q^2\varsigma^T k_3-\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial y}\xi_2^* \\ &-\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial y}\phi_1(x,t)+z_iz_{i+1}+k_{1,n^*}\sigma_\kappa(\hat{\kappa})\hat{\kappa}\tilde{\kappa} \\ &+\varpi_i^T\tilde{k}_3+\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial\hat{k}_3}(\Gamma_3\varpi_i-\Gamma_3\sigma_3(\hat{k}_3)\hat{k}_3-\dot{\hat{k}}_3) \\ &+\tau_i^T\tilde{k}_1+\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial\hat{k}}(\Gamma\tau_i-\Gamma\sigma(\hat{k})\hat{k}-\dot{\hat{k}}). \quad (3.27)\end{aligned}$$

Controller. The control signal $v_0(t)$ in (3.2) is designed as

$$v_0=\alpha_\rho+\hat{\kappa}y_r^{(\rho)}+\sum_{j=1}^{m}\frac{\partial\alpha_{\rho-1}}{\partial\zeta_{n^*j,\rho}}\beta_j(y)-\mu_{n^*,\rho+1}, \quad (3.28)$$

such that $v_j=\text{sign}[b_{n^*,j}]\frac{1}{\beta_j(y)}v_0$, $j=1,2,\ldots,m$, and the adaptive laws for $\hat{\kappa}$, $\hat{k}$, and $\hat{k}_3$ as

$$\begin{aligned}\dot{\hat{\kappa}} &= -\gamma_1 z_1(\dot{y}_r+\bar{\alpha}_1)-\gamma_1\sigma_\kappa(\hat{\kappa})\hat{\kappa}, \\ \dot{\hat{k}} &= \Gamma\tau_\rho-\Gamma\sigma(\hat{k})\hat{k},\ \dot{\hat{k}}_3=\Gamma_3\varpi_\rho-\Gamma_3\sigma_3(\hat{k}_3)\hat{k}_3. \quad (3.29)\end{aligned}$$

Consider a Lyapunov function

$$V=V_\rho+\frac{1}{2}\tilde{k}^T\Gamma^{-1}\tilde{k}+\frac{1}{2}\tilde{k}_3^T\Gamma_3^{-1}\tilde{k}_3+\sum_{i=1}^{\rho}\frac{1}{2d_i}\varepsilon^T P\varepsilon, \quad (3.30)$$

where $P=P^T>0$ satisfying the Lyapunov equation

$$PA_o+A_o^TP=-I. \quad (3.31)$$

From (3.13) and Assumption (A1), it follows that

$$\begin{aligned}\dot{V} &\le -\sum_{i=1}^{\rho}c_iz_i^2-\sum_{i=1}^{\rho}d_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i^2-\sum_{i=1}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial y}\varepsilon_2 \\ &-\sum_{i=1}^{\rho}\lambda_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i^2\varsigma^T k_3+\sum_{i=1}^{\rho}|z_i\frac{\partial\alpha_{i-1}}{\partial y}||\xi_2^*| \\ &+\sum_{i=1}^{\rho}|z_q\frac{\partial\alpha_{i-1}}{\partial y}||\phi_1(x,t)|+k_{1,n^*}\sigma_\kappa(\hat{\kappa})\hat{\kappa}\tilde{\kappa} \\ &+\sigma(\hat{k})\hat{k}^T\tilde{k}+\sigma_3(\hat{k}_3)\hat{k}_3^T\tilde{k}_3-\sum_{i=1}^{\rho}\frac{1}{2d_i}\varepsilon^T\varepsilon \\ &\le -\sum_{i=1}^{\rho}c_iz_i^2-\sum_{i=1}^{\rho}\frac{1}{4d_i}\|\varepsilon\|^2+\lambda+k_{1,n^*}\sigma_\kappa(\hat{\kappa})\hat{\kappa}\tilde{\kappa} \\ &+\sigma(\hat{k})\hat{k}^T\tilde{k}+\sigma_3(\hat{k}_3)\hat{k}_3^T\tilde{k}_3, \quad (3.32)\end{aligned}$$

where $\lambda=\sum_{i=1}^{\rho}\frac{L(\delta^2+1)}{4\lambda_i}$ is a positive constant.

3.3 Stability Analysis

The proposed adaptive scheme has the following properties:

Theorem 3.1 *The adaptive output feedback control scheme consisting of the controller (3.28) and the filters (3.7) along with the parameter update laws (3.29) applied to the system (2.1), based on Assumptions (A1), (A2) and (A3), ensures global boundedness of all closed-loop signals and desired output tracking performance:* $\int_{t_1}^{t_2}(y(t)-y_r(t))^2dt\le\frac{\lambda}{c_1}(t_2-t_1)+\gamma_0$*, for any* $t_2>t_1\ge 0$ *and some constant* $\gamma_0>0$*, and the tracking error can be made as small as desired by choosing sufficiently large* c_1 *and* λ_i*,* $i=1,2,\ldots,\rho$*.*

Proof: For each time interval (t_k,t_{k+1}), $k=0,1,\ldots,q$, we have such a V defined by (3.30) whose time-derivative $\dot{V}$ satisfies (3.32) . Starting from the first time interval, we conclude that $V(t)\in L^\infty$ for $\forall t\in[t_0,t_1)$, so that z, $\hat{\kappa}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, $\hat{k}_3$, and ε are bounded for $t\in[t_0,t_1)$. As $z_1=y-y_r$, y is bounded.

It follows from (3.7) that ξ and ζ_{rj}, $r=0,1,\ldots,n^*$, $j=1,2,\ldots,m$, are bounded. As in [4], it can be concluded that

$$\begin{aligned}\mu_{r,i} &= e_i^T(sI-A_o)^{-1}e_{n-r}K(s)\left[\frac{d^ny}{dt}-\sum_{i=1}^{n}\frac{d^{n-i}\phi_i(x,t)}{dt}\right. \\ &\left.-\sum_{i=1}^{n}\frac{d^{n-i}\varphi_i(y)}{dt}-\sum_{r=0}^{n^*}\sum_{j=1}^{m}k_{2,rj}\frac{d^r\beta_j(y)}{dt}\right], \quad (3.33)\end{aligned}$$

which results in the boundedness of $\mu_{r,i}$, $r=0,1,\ldots,n^*$, $i=1,2,\ldots,n$, because the matrix A_o and the polynomial $k_{1,n^*}s^{n^*}+\cdots+k_{1,1}s+k_{1,0}$ are stable, and y is bounded that in turn implies that $\phi_i(x,t)$, $i=1,2,\ldots,n$, are bounded based on Assumption (A1), and $\varphi_i(\cdot)$, $i=1,2,\ldots,n$, and $\beta_j(\cdot)$, $j=1,2,\ldots,m$, are bounded too because of the smoothness. It follows from (3.6) and the boundedness of ε and ξ^* that x is bounded. According to (3.28), it can also be seen that v_0 is a bounded signal. Since $\beta_j(y)\neq 0$ for $\forall y\in R$, the boundedness of v_j is guaranteed too for $j=1,2,\ldots,m$. Therefore, all closed-loop signals are bounded for $t\in[t_0,t_1)$.

At time $t=t_1$, there occur p_1 actuator failures, which result in the abrupt change of κ, k_{1,n^*}, k_1 and k_2. Since the change of values of these parameters are finite, and z, ε, $\hat{\kappa}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_3$ are continuous, $V(t_1^+)=V(t_1^-)+\bar{V}_1$ with a finite $\bar{V}_1$ Therefore it can be concluded from (3.32) that $V(t)\in L^\infty$ for $t\in(t_1,t_2)$. By repeating the argument above, all the signals are bounded for the time interval (t_1,t_2). Continuing in the same way, finally we have that $V(t)\in L^\infty$ for $t\in(t_q,\infty)$ with $V(t_q^+)=V(t_q^-)+\bar{V}_q$ for a finite $\bar{V}_q$. Due to the finite times of actuator failures, it can be concluded that $V(t)$ is bounded, $\forall t>t_0$, and so are all the closed-loop signals.

To show the tracking performance, we consider the last time interval (t_q,∞) with a positive finite initial $V(t_q^+)$. Because $\sigma_\kappa(\hat{\kappa})\hat{\kappa}\tilde{\kappa}\le 0$, $\sigma(\hat{k})\hat{k}^T\tilde{k}\le 0$, and $\sigma_3(\hat{k}_3)\hat{k}_3^T\tilde{k}_3\le 0$, it follows from (3.32) that $\int_{t_1}^{t_2}z_1^2dt\le\frac{\lambda}{c_1}(t_2-t_1)+\gamma_0$, $\forall t_2>t_1\ge 0$ and some constant $\gamma_0>0$, where $\frac{\lambda}{c_1}$ can be made as small as desired by choosing sufficiently large c_1 and λ_i, $i=1,2,\ldots,\rho$. ∇

4 An Application to Aircraft Control

The nonlinear longitudinal dynamics of the twin otter aircraft [5] is used for our actuator failure compensation study. Choosing the velocity, angle of attack, pitch angle, and pitch rate as the states x_1, x_2, x_3 and x_4, and the elevator angles of an augmented two-piece elevator as the inputs u_1 and u_2, we write the nonlinear aircraft into the state-space form:

$$\begin{aligned}
\dot{x}_1 =& (c_1^T \varphi_0(x_2)x_1^2 + \varphi_1(x))\cos(x_2) \\
&+(c_2^T \varphi_0(x_2)x_1^2+\varphi_2(x))\sin(x_2)+d_1 g_1(x)u_1+d_2 g_1(x)u_2 \\
\dot{x}_2 =& x_4 - (c_1^T \varphi_0(x_2)x_1 + \varphi_1(x)\frac{1}{x_1})\sin(x_2) \\
&+(c_2^T \varphi_0(x_2)x_1+\varphi_2(x)\frac{1}{x_1})\cos(x_2)+d_1 g_2(x)u_1+d_2 g_2(x)u_2 \\
\dot{x}_3 =& x_4,\ \dot{x}_4 = \theta^T \phi(x) + b_1 x_1^2 u_1 + b_2 x_1^2 u_2, \qquad (4.1)
\end{aligned}$$

where $c_1 \in R^3$, $c_2 \in R^3$, $d_1 \in R$, $d_2 \in R$, $b_1 \in R$, $b_2 \in R$, $\theta = [\theta_1,\theta_2,\theta_3,\theta_4]^T \in R^4$ are constant, and

$$\begin{aligned}
\varphi_0(x_2) &= [x_2, x_2^2, 1]^T,\ \phi(x) = [x_1^2x_2, x_1^2x_2^2, x_1^2, x_1^2x_4]^T, \\
\varphi_1(x) &= p_{11} + p_{12}x_4x_1^2 - p_0\sin(x_3), \\
\varphi_2(x) &= p_{21} + p_{22}x_4x_1^2 + p_0\cos(x_3), \\
g_1(x) &= a_1x_1^2\cos(x_2) + a_2x_1^2\sin(x_2), \\
g_2(x) &= -a_1x_1\sin(x_2) + a_2x_1\cos(x_2). \qquad (4.2)
\end{aligned}$$

Assume that the velocity x_1 is measured, while x_2 and x_4 are not measured. The control objective is to design an adaptive scheme to control the elevator angles such that the pitch angle $y = x_3$ tracks a reference signal as close as desired even if any one of the elevator segments may be stuck at an unknown angle, i.e., $u_j(t) = \bar{u}_j$, $t \geq t_j$, $j = 1$ or 2 with unknown $\bar{u}_j$, t_j.

To design an adaptive compensation scheme using output feedback for the twin otter aircraft model, two issues need to be handled: zero dynamics and unmeasured states x_2 and x_4.

Zero Dynamics. The aircraft system does not have the form of the system (2.1) so that we need to obtain the inherent zero dynamics to investigate the minimum-phase property. According to the control objective, the states x_1 and x_2 in fact construct the zero dynamics. Apply a change of coordinates

$$\eta_1 = x_1\cos(x_2) - q_1x_4,\ \eta_2 = x_1\sin(x_2) - q_2x_4, \qquad (4.3)$$

where $q_1 = \frac{a_1}{k}$, $q_2 = \frac{a_2}{k}$, and $k = \frac{b_1}{d_1} = \frac{b_2}{d_2}$, and obtain the inherent zero dynamics as

$$\begin{aligned}
\dot{\eta}_1 =& f_1(\eta,\bar{x}) = c_1^T\varphi_0(x_2)x_1^2+\varphi_1(x)-x_1x_4\sin(x_2)-q_1\theta^T\phi(x) \\
\dot{\eta}_2 =& f_2(\eta,\bar{x}) = c_2^T\varphi_0(x_2)x_1^2+\varphi_2(x)+x_1x_4\cos(x_2)-q_2\theta^T\phi(x),
\end{aligned}$$

where $\eta = [\eta_1,\eta_2]^T$, $\bar{x} = [x_3,x_4]^T$. With the aircraft parameters obtained in a certain operation condition [5], we investigate the unforced zero dynamics $\dot{\eta} = f(\eta,0)$ with $\bar{x} = 0$ and $f(\eta,\bar{x}) = [f_1(\eta,x_3,x_4), f_2(\eta,x_3,x_4)]^T$. Investigate the stability of the unforced zero dynamics at the equilibrium point $\eta_e = [71.24, 1.701]^T$ by checking the matrix $[\frac{\partial(f(\eta,0)}{\partial\eta}]$ at η_e. The eigenvalues of the matrix are -1.950 and -0.05897, which indicates that the unforced zero dynamics are locally asymptotically stable. It in turn implies that the zero dynamics are locally input-to-state stable in a neighborhood of η_e with $\bar{x}$ in a neighborhood of 0 as the input [3].

Dynamic Bounding. For the unmeasured state x_2, the zero dynamics are input-to-state stable so that x_2 can be dynamically bounded by y. The zero dynamics $\dot{\eta} = f(\eta,\bar{x})$ are however highly nonlinear, which can hardly be handled. To apply the dynamic bounding design, we investigate the local property of the zero dynamics by linearizing the zero dynamics around a nominal trajectory. Since $y = x_3$ is desired to track a reference signal $y_r(t)$ and $x_4 = \dot{x}_3$, it is reasonable to take $\bar{x}_e(t) = [y_r(t), \dot{y}_r(t)]^T$ as the nominal input. We further assume that the nominal reference signal $y_r(t)$ and its time derivative $\dot{y}_r(t)$ are close to some constant $\bar{x}_e = [x_{3e}, x_{4e}]^T$, with which we consider $y_r(t) - x_{3e}$ and $\dot{y}_r(t) - y_{4e}$ as some additional small perturbation around $\bar{x}_e$.

The zero dynamics $\dot{\eta} = f(\eta,\bar{x})$ are thus linearized at $[\eta_e,\bar{x}_e]^T$, where η_e is the equilibrium point of $\dot{\eta} = f(\eta,\bar{x}_e)$. Therefore we can have x_2 dynamically bounded by y via the linear dynamics $\dot{\eta}_\Delta = A\eta_\Delta + B\bar{x}_\Delta$, where $A = \frac{\partial f}{\partial\eta}(\eta_e,\bar{x}_e)$ and $B = \frac{\partial f}{\partial\bar{x}}(\eta_e,\bar{x}_e)$, which locally approximate the nonlinear zero dynamics. Noticing that $x_4 = \dot{x}_3$, we have $\eta_\Delta = G(s)[\bar{x}] = G(s)[1,s]^T[x_3] = [G_1(s), G_2(s)]^T[y]$, where $G_1(s) = z_{12} + \frac{z_{11}s+z_{10}}{D(s)}$ and $G_2(s) = z_{22} + \frac{z_{21}s+z_{20}}{D(s)}$ with $D(s) = \det(sI-A)$. Let $G_1^*(s) = \frac{z_{11}s+z_{10}}{D(s)}$ and $G_2^*(s) = \frac{z_{21}s+z_{20}}{D(s)}$, and define a signal $\rho(t) = F(s)[\frac{y^2+1}{2}]$ with $F(s) = \frac{1}{s+\delta^*}$, where $\delta^* > 0$ is a constant such that $D(s-\delta^*)$ is stable. Define $H_i(s) = G_i^*(s)(s+\delta^*)$, $i = 1,2$. From Lemma 2.6 in [8], it follows that $|G_i^*(s)[y](t)| \leq h_{i\delta^*}\rho(t)$ where $h_{i\delta^*} = \|h_i(\cdot)\|_1$ with $h_i^*(t)$, the impulse function of $H_i(s-\delta^*)$, $i = 1,2$. Furthermore, we conclude that

$$|\eta(t)| \leq \eta_e + |\eta_\Delta| \leq \eta_e + \begin{bmatrix} |z_{12}y| + h_{1\delta^*}\rho(t) \\ |z_{22}y| + h_{2\delta^*}\rho(t) \end{bmatrix}. \qquad (4.4)$$

Consider that in normal operation $x_1 > 0$ and $|x_2| \leq \frac{\pi}{2}$ with which we have $|x_2| \leq |2\sin(x_2)|$. It follows from (4.3) that

$$|x_2| \leq 2\frac{|\eta_2|}{x_1} + 2\frac{|q_2|}{|q_1|}\frac{|\eta_1|}{x_1} + 2\frac{|q_2|}{|q_1|}. \qquad (4.5)$$

Defining $\bar{\phi}(x) = [x_1^2x_2, x_1^2x_2^2, x_1^2]^T$, we thus have Assumption (A1) satisfied by showing that

$$\begin{aligned}
|\bar{\theta}^T\bar{\phi}(x)| \leq& |\theta_1|x_1^2|x_2| + |\theta_2|x_1^2x_2^2 + |\theta_3|x_1^2 \\
\leq& \Theta_1 + \Theta_2x_1 + \Theta_3x_1^2 + \Theta_4y^2 + \Theta_5x_1y^2 + \Theta_6\rho(t) \\
&+\Theta_7x_1\rho(t) + \Theta_8y^2\rho(t) + \Theta_9\rho^2(t), \qquad (4.6)
\end{aligned}$$

where $\Theta_i > 0$, $i = 1,2,\ldots,9$, are unknown constants. Notice that in Assumption (A1) there is no a such term as $\rho^2(t)$. Since $\rho_1^2(t) < \rho_2^2(t)$ for any $|\rho_1(t)| < |\rho_2(t)|$, our dynamic bounding design is however applicable to this case.

Observer. The unmeasured state x_4 exhibits the appearance of a linear form in $\phi(x)$. Taking advantage of this property, next we design a time-varying observer to estimate x_4. Assuming that θ_4 is known, we introduce a signal

$\beta(t) = -\theta_4 x_1^2(t)$. Since $\theta_4 < 0$, we have $\beta(t) \geq 0$, $\forall t \geq 0$. We now give a nominal state observer for $\bar{x}^* = [x_3^*, x_4^*]^T$ as $\bar{x}^* = \xi + \xi^* + k_2\zeta + k_1\mu$, with the filters given by

$$\begin{aligned}
\dot{\xi} &= A_o\xi - \beta(t)\xi + ly + [\beta(t), 0]^T y, \\
\dot{\xi}^* &= A_o\xi^* - \beta(t)\xi^* + [0,1]^T\bar{\theta}^T\bar{\phi}(x,t),\ \bar{\theta} = [\theta_1, \theta_2, \theta_3]^T \\
\dot{\zeta} &= A_o\zeta - \beta(t)\zeta + [0, x_1^2]^T,\ \dot{\mu} = A_o\mu - \beta(t)\mu + [0, v_0]^T, \quad (4.7)
\end{aligned}$$

where $l = [l_1, l_2]^T$ is chosen such that $A_o = A - lc^T$ is stable with $A \in R^{2\times 2}$ in the canonical form, and k_1 and k_2 are defined as $k_1 = b_1 + b_2$ and $k_2 = 0$, if both elevator segments work, and $k_1 = b_i$ and $k_2 = b_j\bar{u}_j$, if the ith elevator segment works and the jth elevator segment fails, $i \neq j \in \{1, 2\}$. The nominal observation error $\varepsilon = \bar{x} - \bar{x}^*$ satisfies $\dot{\varepsilon} = A_o\varepsilon - \beta(t)\varepsilon$, which implies that ε converges to zero exponentially.

For unknown ξ_2^*, we introduce dynamic bounding signals

$$\dot{\chi}_0(t) = -\delta_0\chi_0(t) + \frac{y^2+1}{2},\ \dot{\chi}(t) = -\bar{\delta}\chi(t) + \Phi, \quad (4.8)$$

where $\chi_0 \in R$ and $\chi \in R^9$, $\bar{\delta} > 0$ is chosen such that $A_o + \bar{\delta}I$ is stable, $\delta_0 > 0$ is such that $F(s - \delta_0)$ is stable, and $\Phi = [1, x_1, x_1^2, y^2, x_1y^2, \chi_0, x_1\chi_0, y^2\chi_0, \chi_0^2]^T$. It follows from (4.6) that $|\bar{\theta}^T\bar{\phi}(x)| \leq \Theta^T\Phi$, where $\Theta = [\Theta_1, \Theta_2, \ldots, \Theta_9]^T$. Although ξ^* is governed by a linear time-varying system, it is proved that Lemma 2.6 in [8] can still be applied to this case.

Adaptive Design. Define $k_3 = [\Theta_1^2, \Theta_2^2, \ldots, \Theta_9^2]^T$ and $\kappa = \frac{1}{k_1}$, and signals $\bar{\varsigma} = [\chi_1^2, \chi_2^2, \chi_3^2, \chi_4^2, \chi_5^2, \chi_6^2, \chi_7^2, \chi_8^2, \chi_9^2]^T$ and $\varsigma = [\chi_1^2 + 1, \chi_2^2 + x_1^2, \chi_3^2 + x_1^4, \chi_4^2 + y^4, \chi_5^2 + x_1^2y^4, \chi_6^2 + \chi_0^2, \chi_7^2 + x_1^2\chi_0^2, \chi_8^2 + y^4\chi_0^2, \chi_9^2 + \chi_0^4]^T$. With Assumption (A3) that the sign of k_1 is known, the adaptive controller is derived as

$$v_j = \frac{1}{x_1^2}v_0,\ j = 1, 2,\ v_0 = \alpha_2 + \hat{\kappa}\ddot{y}_r + \frac{\partial\alpha_1}{\partial\zeta_2}x_1^2, \quad (4.9)$$

and the adaptive laws are $\dot{\hat{\kappa}} = -\gamma\mathrm{sign}[k_1]z_1(\dot{y}_r + \dot{\bar{\alpha}}_1) - \gamma\sigma_\kappa(\hat{\kappa})\hat{\kappa}$, $\dot{\hat{k}} = \Gamma\tau_2 - \Gamma\sigma(\hat{k})\hat{k}$, $\dot{\hat{k}}_3 = \Gamma_3\varpi_2 - \Gamma_3\sigma_3(\hat{k}_3)\hat{k}_3$, via a backstepping procedure as shown in Section 3.2, where $\gamma > 0$, $\Gamma = \Gamma^T > 0$, and $\Gamma_3 = \Gamma_3^T > 0$ are the adaptive gains, $\sigma(\hat{\kappa})$, $\sigma(\hat{k})$, and $\sigma_3(\hat{k}_3)$ are the switching signals in the form of (3.20) and (3.25).

For simulation, the initial states are chosen as $x(0) = [62.5, -0.04, 0, 0]^T$, and $l = [2, 1]$, $\delta_0 = 0.05$, $\bar{\delta} = 0.5$, $c_1 = c_2 = d_1 = d_2 = 0.01$, $\lambda_1 = \lambda_2 = 0.000001$, $\gamma = 0.0005$, $\gamma_1 = 0.1$, $\gamma_2 = 0.001$, $\Gamma_3 = I$. Figure 1 shows the results when u_1 fails at 150 second with an unknown failure value 0.04 (rad).

5 Concluding Remarks

This paper addresses actuator failure compensation using output feedback for a class of nonlinear systems with unknown nonlinearities bounded by output-dependent functions and dynamic signals. An adaptive controller using dynamic bounding design is proposed for desired objective and an application to a nonlinear aircraft dynamics is also presented.

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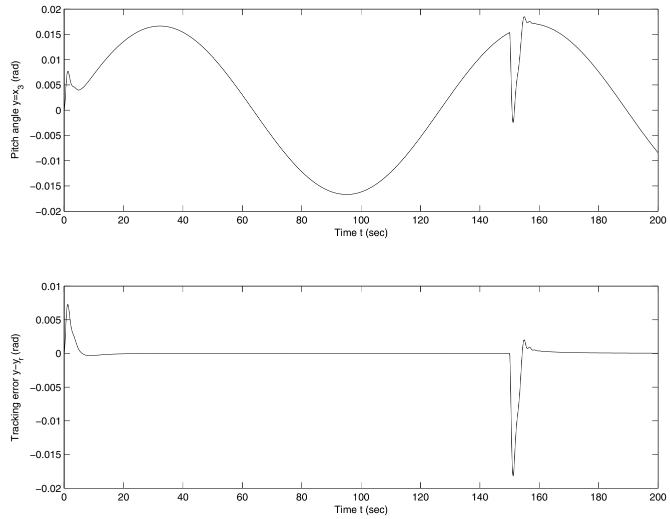

Figure 1: Output and tracking error.

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