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Stability and Feedback Control of Wireless Networked Systems

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***Abstract*—This study analyzes data losses and their effect on wireless networked system stability/performance using a simple communication scheme and a stochastic 2-state Markov network model. An architecture utilizing event-driven control and clock-driven, co-located sensing and actuation serves as the base framework. Data losses are treated in a switched system scheme where two discrete dynamics are available. When the network transmits data successfully, the closed loop dynamics are active; when packets are dropped, zero control is applied and open loop dynamics exist. Using analysis tools for stochastic stability of Markov jump linear systems, operating conditions in a probability space can be identified for given plant dynamics that ensure second moment stability (SMS). Additionally, switched system H_∞ norms can be used to identify regions of network operation in the probability space with expected levels of performance. Results from these tools and experiments involving wireless hardware suggest a scheme for decentralized controller variation that rebalances network loading for the control loops when communication disturbances occur in the network. This scheme varies the sampling period of the individual loops based on network condition measurements, system stability/performance requirements, and computation/bandwidth limits of the hardware.**

I. INTRODUCTION

CLOSED loop control over wireless networks has become an increasingly popular research topic over the past several years. While wireless network control loops may be currently infeasible for high bandwidth and mission critical systems, they are still quite attractive for a variety of applications due to their simple, low cost setup and their compatibility with high controller or actuator mobility. However, closing the loop wirelessly provides many challenging control problems including management of data loss and delays, communication scheduling, bandwidth allocation, protocol design and selection, performance evaluation, stability analysis, and loop coordination.

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When examining wireless networks, treatment of losses becomes a more important consideration than for wired networks due to much higher levels of data dropout and corruption. Initial wireless control research included experimental and proof of concept tests where successful real-time control was implemented for small size systems (few nodes) using Bluetooth and UDP/IP over wireless Ethernet [1],[2]. These works showed that even with much higher rates of data loss and corruption, successful control was possible. Recent study has focused on overcoming wireless network limitations by using estimators and predictors for dropout compensation [3], appropriate medium access control selection [4], and packing more control information per transmission to make more efficient use of available bandwidth [5]. Wireless network analysis has also become popular. By modeling losses or inter-transmission times as stochastic processes, or placing constraints on consecutive dropouts, stability and performance analysis of wireless network systems is possible [6]-[9].

This work focuses on the analysis of stability and performance of a basic wireless communication strategy drawing on previous results in the areas of switched systems, stochastic stability [10], and H_∞ disturbance-to-output gains [6]. A sample time variation strategy based on the stability results and experimental measurements of wireless network behavior is also proposed. The strategy responds to network disturbances and changes in network loading by estimating the network state in terms of a 2-state Markov model and shifting to a more favorable network operating point through sample period variation.

A more thorough problem description is given in Section II focusing on the communication strategy, stability tests, and H_∞ disturbance-to-output gains. Section III describes a network control strategy involving decentralized sample time variation that improves stability under varying network conditions. Section IV provides experimental results using this strategy on a wireless network control system, while Section V provides conclusions and future directions.

II. Problem Formulation

A. Communication

The underlying transmission framework comes from our previous communication method for wireless control using the user datagram protocol (UDP) [11]. A collocated sensor and actuator node samples a plant at a particular sampling frequency and transmits this feedback information wirelessly to a remote controller node. Due to the collocation, the time between the sampling and transmission is assumed negligible compared to the communication delays. The feedback is received by the controller which processes the information, computes the appropriate actuator output, and transmits the output back to the controller. The sensor/actuator node receives this control action, and waits to apply the output at the next sampling instant. In this framework, the system can be modeled discretely with period *Ts* and a one step delay in actuation, as long as no transmissions are lost and the roundtrip communication time is less than the sampling period,

$$Ts \geq \tau_{feedback} + \tau_{computation} + \tau_{forward} = \tau_{roundtrip}\,. \quad (1)$$

Using a state feedback controller, the resulting system becomes:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ u(k) &= Kx(k-1) \end{aligned} \quad (2)$$

If a transmission between controller and actuator/sensor is lost in either direction, the information is corrupted, or the round trip delay exceeds the sampling period, no new useful control information will reach the actuator/sensor node during the sampling period. Several choices exist for handling these occurrences including outputting zero control or using a hold (zero-order, first order, etc.) Many researchers currently are investigating how to most effectively deal with this lost data including sending more than one control in each packet based on prediction, estimation, or multi-rate sampling [5]. These extra estimated control values would be used when data are lost. In the current work, zero control is applied at the actuator when data are lost because it provides an appropriate framework for analysis, though other strategies could be examined similarly. When data are lost, the system dynamics are simply open loop and given by

$$x(k+1) = Ax(k)\,. \quad (3)$$

With this strategy, the networked control system naturally fits into the framework of a switched system with two discrete dynamic modes of operation, A_{OL} and A_{CL}, as follows:

$$\tilde{x}(k+1) = A_{\sigma(k)}\tilde{x}(k), \quad where \quad (4)$$

$$\tilde{x}(k+1) = \begin{pmatrix} x(k+1) \\ x(k) \end{pmatrix}, \quad \sigma(k) \in \{OL, CL\},$$

$$A_{OL} = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix}, \quad A_{CL} = \begin{bmatrix} A & BK \\ I & 0 \end{bmatrix}$$

This system representation is convenient because it allows application of stability analysis techniques for stochastically switched systems.

B. Networked System Stability

The two-state system operation described above fits naturally with common erasure network models. In wireless networks, it is common for transmissions to exhibit bursty behavior where the likelihood of losing a packet after a lost packet transmission is higher than after a successful packet transmission. This behavior is frequently modeled using a two-state Markov chain (Figure 1) where for our notation $p_{i,j} = \Pr[\sigma(k+1) = j \mid \sigma(k) = i]$ for $i,j \in \{OL, CL\}$.

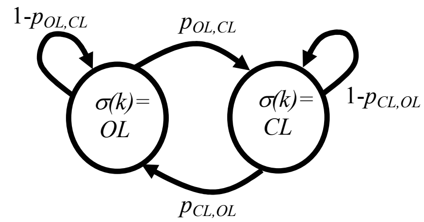

Fig. 1. Two-state Markov chain network model

When presented in this framework, stability results for stochastically switched systems apply and several probabilistic types of stability may be deduced for a particular set of network conditions given by $p_{OL,CL}$ and $p_{CL,OL}$. Three types of stability that can be tested are described in [10] which proves equivalence between mean-square stability, stochastic stability, and exponential mean square stability for discrete-time Markov Jump Linear Systems (MJLS):

Mean-square stability:

$$\forall(x_0,\sigma_0), \quad \lim_{k\to\infty} E[\|x(k)\|^2 \mid x_0,\sigma_0] = 0 \quad (5)$$

Stochastic stability:

$$\forall(x_0,\sigma_0), \quad \sum_{k=0}^{\infty} E[\|x(k)\|^2 \mid x_0,\sigma_0] < \infty \quad (6)$$

Exponential mean square stability:

$$\forall(x_0,\sigma_0)\ \exists \text{ constants } 0<\alpha<1, \beta>0$$
$$\text{s.t. } \forall(x_0,\sigma_0), \quad E[\|x(k)\|^2 \mid x_0,\sigma_0] < \beta\alpha^k\|x_0\|^2\,. \quad (7)$$

These equivalent notions for MJLS are referred to collectively as second moment stability (SMS) [10]. To check SMS stability in general for a set of *N* different Markov switched dynamics $\{A_1, A_2, \ldots A_N\}$, several distinct but equivalent methods have been developed including checking if there exist $\{G_i\} > 0$ such that

$$A_i^T\left[\sum_{j=1}^{N} p_{ij} G_j\right] A_i - G_i < 0, \quad i = 1, 2, \ldots N \tag{8}$$

is satisfied [10]. Alternatively, we can consider an eigenvalue condition [14], testing if

$$\rho\left(\left(\Pi' \otimes I_n\right)\left[diag\left(A_i \otimes A_i\right)_{i=1,2,\ldots,N}\right]\right) < 1 \tag{9}$$

where ρ is spectral radius and $\otimes$ represents the Kronecker product operation [12]. In the situation considered in this work, N=2 corresponding to OL and CL operation. By testing either of the conditions in (8) or (9) for the entire space of network conditions, $p_{OL,CL} \in [0,1]$ and $p_{CL,OL} \in [0,1]$, regions of stability and instability can be identified that are convenient to visualize graphically.

C. H_∞ disturbance-to-output gain and stability

Information beyond binary determination of whether the networked system is stable or unstable is available by calculating the H_∞ norm of the discrete-time MJLS. As with deterministic linear systems, the H_∞ norm for MJLS is defined as the maximum input-output gain with appropriately defined 2-norms. However, the stochastic norm computation requires using a bounded real lemma for MJLS that can be found in [13].

Given a stochastic system P,

$$\begin{bmatrix} x(k+1) \\ z(k) \end{bmatrix} = \begin{bmatrix} A_{\sigma(k)} & B_{\sigma(k)} \\ C_{\sigma(k)} & D_{\sigma(k)} \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} \tag{10}$$

where $x(k) \in R^{n_x}$ is the state vector, $d(k) \in R^{n_d}$ is the disturbance vector and $z(k) \in R^{n_x}$ is the error vector, if P is weakly controllable, then P is SMS and $\|P\|_\infty < \gamma$ if and only if there exist matrices $\{G_i\} > 0$ that satisfy $\{R_i\} < 0$ where

$$R_i := \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}^T \begin{bmatrix} \sum_{j=1}^{N} p_{ij} G_j & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} - \begin{bmatrix} G_i & 0 \\ 0 & \gamma^2 I \end{bmatrix}, \tag{11}$$
$$i \in \{1, 2, \ldots N\}$$

By evaluating (11) over the space of possible network conditions, the expected effect of a disturbance and the variation in performance as network conditions change can be presented graphically.

The surface also displays the relationship between the H_∞ gain and the stability conditions in (8) and (9). The curve in the network space where the disturbance-to-output gain approaches infinity corresponds directly with the curve dividing the stable and unstable regions. A comparison of the SMS stability condition and an H_∞ gain surface for a sample inverted pendulum system model [11] is shown in Figures 2 and 3. The surface plot in Figure 3 shows the stochastic system gain from an impulsive force disturbance applied to the pendulum arm to the pendulum arm position θ. The gain surface is cutoff at a gain level of 0.2 where the slope in the surface is nearly vertical and a small perturbation in the network parameters would cause the gain to jump to infinite values. By comparing Figures 2 and 3, the equivalence between the SMS stability line and the limiting position where the H_∞ gain surface grows towards infinity is clear for the inverted pendulum model.

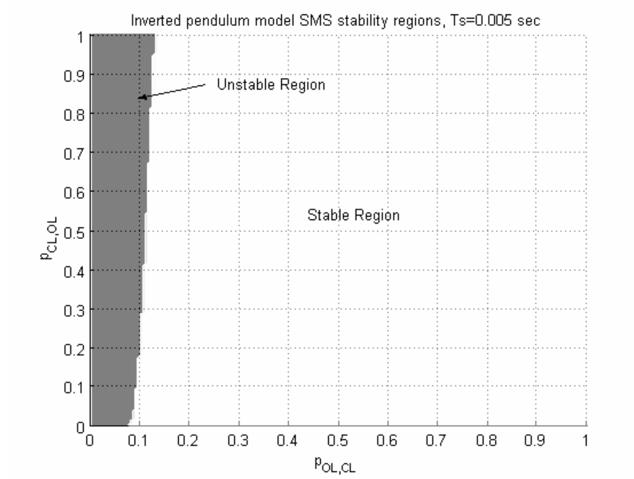

Fig. 2. Inverted pendulum model theoretical SMS stability regions using LQR control and Ts=5 ms

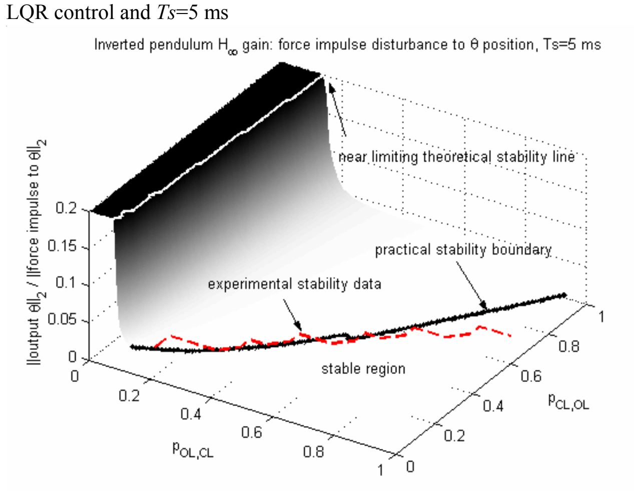

Fig. 3. H_∞ gain surface for inverted pendulum vertical arm force disturbance to θ position output with comparisons of practical stability region, experimental stability region, and theoretical stability region

Using the idea of stochastic disturbance-to-output H_∞ gain, a notion of practical stability can be defined. Modeling error and noise cause a real networked system to behave differently than its ideal linear model. As a result, the stability region derived from the model overestimates the true size. Experimental results suggest that the boundary of network conditions where the actual system is stable may be estimated by the intersection of a plane at level $\gamma_{practical}$ with the maximum allowable disturbance-to-output gain. This region R would be described by:

$$R = \{(p_{OL,CL}, p_{CL,OL}) : \frac{\|y\|_2}{\|d\|_2} < \gamma_{practical}(A, B, C, D, \Delta, Ts, noise)\} \qquad (12)$$

Results for an experiment comparing an inverted pendulum plant model with an actual system [11] are shown in Figure 3. The dashed line represents network conditions below which the actual physical system was stable. The region was determined by initially setting the closed loop to execute with a nominal sampling period of 5 ms. Artificial network losses were injected at the actuator node using the Markov model to simulate different network parameters in increments of 0.1 for both $p_{OL,CL}$ and $p_{CL,OL}$. This dashed experimental line corresponds reasonably well with a practical stability region defined by:

$$\frac{\|output\ \theta\|_2}{\|force\ impulse\ on\ \theta\|_2} < 0.03525 \qquad (13)$$

that is also shown in Figure 3. The relationships between this notion of practical stability and stochastic H_∞ gain are an ongoing area of research.

III. Network Control Policy Using Sample Time Variation

In the wireless communication framework used, packet losses are easily detected at the actuator/sensor node. Because that node executes its actions with regular periodicity, at each sampling instant it can check whether feedback was received from the controller. If either the feedback transmission or the control transmission were lost or damaged, no new control information would have been received. In this manner there is a data source to evaluate the overall closed loop transmission quality. By keeping statistics about the losses, the operating point in the two-state Markov model framework can be estimated using:

$$\hat{p}_{OL,CL} = \frac{\text{\# of transitions from OL to CL}}{\text{\# of OL instances}} \qquad (14)$$

$$\hat{p}_{CL,OL} = \frac{\text{\# of transitions from CL to OL}}{\text{\# of CL instances}}. \qquad (15)$$

Estimating the evolution of $p_{OL,CL}$ and $p_{CL,OL}$ over time using a moving window allows detection of network disturbances and prediction of how the system should operate in terms of stability and disturbance rejection based on the results in Section II. To manage network loading and improve performance during network disturbances, a strategy that varies the sampling period of individual control loops based on estimated network conditions is proposed. The rationale is that many systems are capable of being controlled over some range of sampling frequencies. Additionally, experimental observation has shown that when performing closed loop control over a wireless network, small changes in the sampling period can drastically affect the network behavior, especially when operating near the maximum available bandwidth. Typical trends for average network parameter estimates versus sampling period are shown in Figures 4 and 5. The variances about these average values can be fairly large, but the shape and trend with *Ts* is very repeatable.

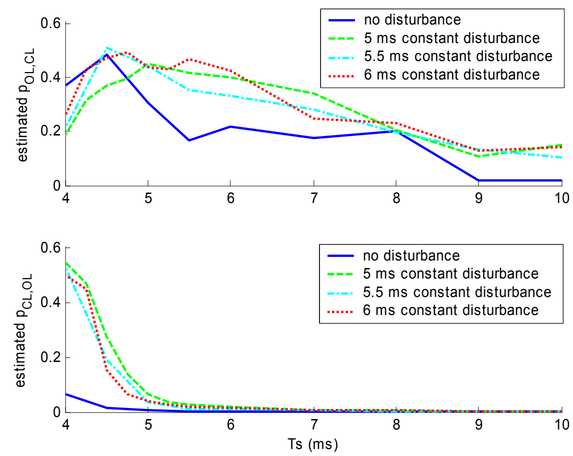

Fig. 4. Individual effect of sample time variation on $\hat{p}_{OL,CL}$ and $\hat{p}_{CL,OL}$ for 4 different network loading conditions – no disturbances, and constant competing disturbance traffic with 5 ms, 5.5 ms, or 6 ms sampling periods.

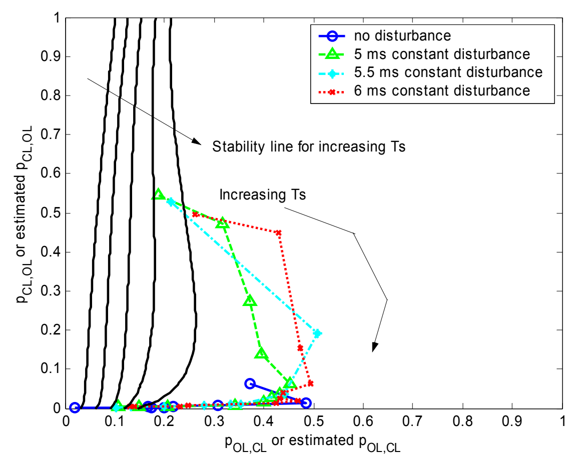

Fig. 5. 2D effect of sample time variation on $p_{CL,OL}$ and $p_{OL,CL}$ for 4 different network loading conditions including corresponding shift of stable region.

While the parameter $\hat{p}_{OL,CL}$ has a generally decreasing trend with increasing *Ts*, $\hat{p}_{CL,OL}$ has a more consistent decrease and is more sensitive to changes in *Ts* $\left(\left|\frac{\partial p_{CL,OL}}{\partial Ts}\right| >> \left|\frac{\partial p_{OL,CL}}{\partial Ts}\right|\right)$ when the loop is operating near the network's bandwidth limits. As a result, $\hat{p}_{CL,OL}$ is a better choice of feedback variable. From experiments, $\hat{p}_{CL,OL}$ approximately follows a model relationship of the form:

$$\hat{p}_{CL,OL} = f(Ts,d) = \frac{K(d)}{(Ts - Ts_{\min}(d))^{n(d)}},\ n(d) \geq 1 \quad (16)$$

where $Ts_{min}(d)$ represents the minimum possible sampling period and $K(d)$ and $n(d)$ are fitting constants that depend on a disturbance vector d that is intended to capture the effect of uncontrolled factors. These factors include variations in computation/communication time, level of network traffic, distance between nodes, interference and noise on the communication channel, etc. that affect the model. While the precise nature of the disturbance d is unknown, experiments have shown a decreasing trend for $\hat{p}_{CL,OL}$ with increasing Ts ($\partial f/\partial Ts < 0$) under a wide range of network conditions.

If the function f were known precisely, if an inverse function $f^{-1}(p_{CL,OL}, d)$ could be found, and if d could be measured, an open loop policy shown schematically in Figure 6 could be used to regulate $p_{CL,OL}$. Ideally $p_{CL,OL}$ would track a value that ensures stability and a minimum level of performance based on the SMS and H_∞ norm results from the networked system model.

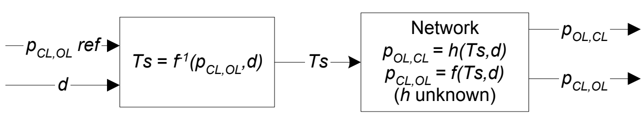

Fig. 6. Idealized open loop network condition control policy

Because d and f are unknown, and the network state parameters are estimates that are stochastically changing, a heuristic control policy (G_{Ts}) is proposed to accommodate these unknowns and inherent variations. A three level operation strategy is designed that coarsely behaves as f^{-1} would for a fixed level of disturbance. It also incorporates feedback from the system to accommodate unknown disturbances and changing network conditions. A schematic of this control arrangement is shown in Figure 7.

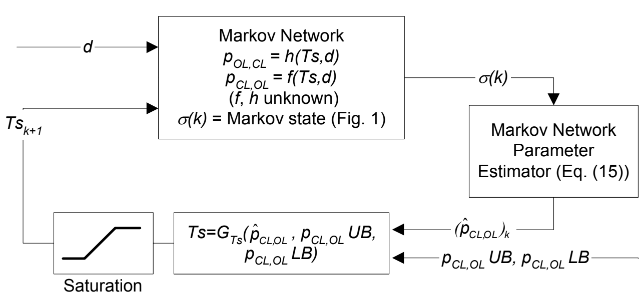

Fig. 7. Diagram of $\hat{p}_{CL,OL}$ band-based network control policy using Ts variation

Three levels of network operation are defined including a poor network condition band that is entered due to a disturbance or excessive external traffic, an acceptable band of network conditions defined by an upper and lower bound ($p_{CL,OL}\_UB$ and $p_{CL,OL}\_LB$ respectively), and an excessively conservative band of network operation. A static plot of a typical stability region and band levels would appear as shown in Figure 8.

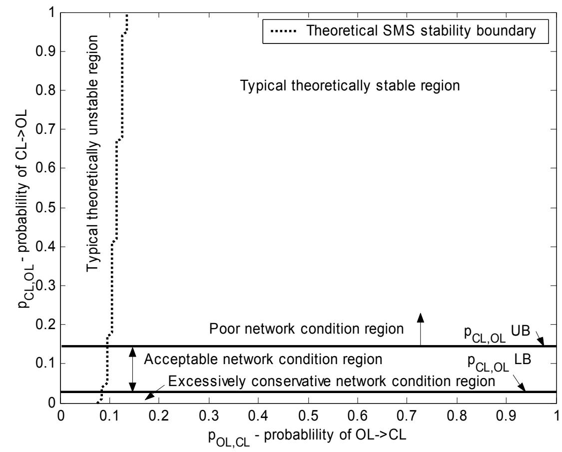

Fig. 8. Typical network condition regions for $p_{CL,OL}$ in Markov model stability space

A policy to vary Ts according to estimated network conditions and the preset desirable bound levels is given by:

$$Ts_{k+1} = G_{Ts}(\hat{p}_{CL,OL}, p_{CL,OL}\_UB, p_{CL,OL}\_LB) = \begin{cases} Ts_k - \delta T^- & if\ (p_{CL,OL} < p_{CL,OL}\_LB) \\ & and\ k - (k_{switch})_{q-1} > k_{dwell} \\ Ts_k + \delta T^+ & if\ (p_{CL,OL} > p_{CL,OL}\_UB) \\ & and\ k - (k_{switch})_{q-1} > k_{dwell} \\ Ts_k & else \end{cases} \quad (17)$$

When the $\hat{p}_{CL,OL}$ estimate exceeds the upper threshold $p_{CL,OL}\_UB$ moving into the poor network condition band, Ts is increased by δT^+ to bring $\hat{p}_{CL,OL}$ back into the acceptable region. This new sample period is held for k_{dwell} iterations without another switch (indexed by q) to allow the effect of the change to be measured. When conditions improve, or if too large of a correction were made, $\hat{p}_{CL,OL}$ may decrease into the excessively conservative region below $p_{CL,OL}\_LB$. The sampling period Ts will then be decreased by δT^- with a hold of k_{dwell} for each change until $\hat{p}_{CL,OL}$ has moved back into the acceptable operation band. Prolonged operation above $p_{CL,OL}\_UB$ or below $p_{CL,OL}\_UB$ is undesirable. The levels for $p_{CL,OL}\_UB$ and $p_{CL,OL}\_LB$ are system specific. They are set to ensure that too small a sampling period is not used which could result in poor robustness to network condition variation, or too large a sampling period is used which could destabilize the system by excessively reducing the stability region size (Figure 5).

A possible concern regarding this strategy is whether the sample period variation and real-time changes in controller gain could destabilize the system due to switching transients even in the absence of communication losses. One solution is to choose k_{dwell} sufficiently large. Because the controlled system is stable under perfect network conditions for all permissible choices for *Ts*, the state vector can be guaranteed to decrease with each switch if the dwell time is sufficiently long. For known dynamics, a bound on the minimum number of iterations per switch $(k_{dwell})_{min}$ can be found using:

$$(k_{dwell})_{min} = \{\min k : \overline{\sigma}(A^{k}_{d,CL}(Ts)) < 1, Ts_{min} < Ts < Ts_{max}\} \quad (18)$$

where $A_{d,CL}$ in this relation is the appropriate discretized closed-loop dynamics at *Ts* and $\overline{\sigma}$ is the maximum singular value. Alternatively, a family of *Ts* parameterized controllers could be designed that share a common closed loop Lyapunov function which would yield stability regardless of the sample period applied.

IV. Experimental Results

The effect of sample time variation on performance is examined using an inverted pendulum plant with LQ control described in [11]. For the inverted pendulum plant, the control objective is regulation of all states to zero. Due to system nonlinearities including friction and a motor deadzone, the plant exhibits significant limit cycle behavior about the regulated equilibrium point.

Experimental results show that the sample time variation strategy produces better performance robustness when a network disturbance occurs than fixed sample period control. In the fixed sampling case, the period is set to 4.25 ms (235 Hz), while in the variable sampling case, the period is allowed to vary in increments of 0.25 ms between 4 ms and 10 ms based on the switching rules given by (17) and the constants listed in the Appendix. In both cases, an external network disturbance operating for 20 seconds with a period of 8 ms is introduced after 65 seconds of a 100 second run with results presented in Figures 9 and 10.

Figure 9 compares how $p_{CL,OL}$ and the average closed loop communication success rate are affected when a disturbance loop is introduced. The dotted lines show how for a fixed sample period of 4.25 ms, $p_{CL,OL}$ jumps from a nominal level of about 0.05 to 0.30 and the average success completion drops from 95% to 75% during the disturbance. The solid lines show the effect of the sampling period variation algorithm on $p_{CL,OL}$ and the average packet success rate. Changing the sampling period regulates the $p_{CL,OL}$ level within the acceptable band during the disturbance and reduces the disturbance effect on the percentage of lost packet transmissions.

Figure 10 shows the control strategy effect on regulation performance with the same disturbance. The regulation of pendulum arm position (θ) and horizontal arm position (ϕ) are compared with and without sample time variation. Sample time variation improves both the θ and ϕ regulation when the disturbance is present. Both states have smaller amplitude limit cycles during the disturbance when *Ts* switching is allowed.

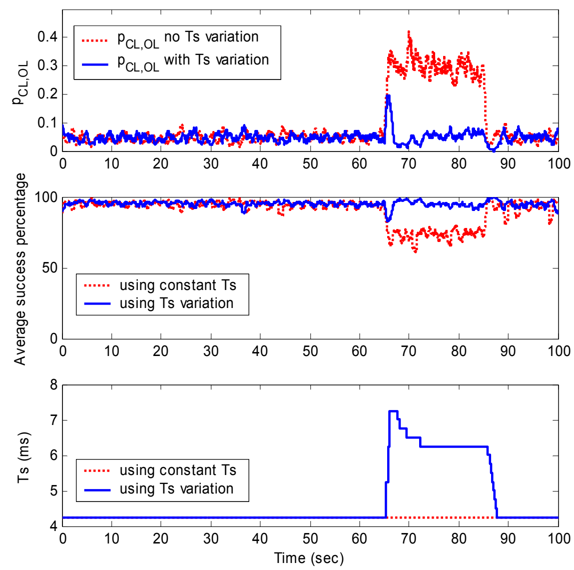

Fig. 9. Experimental results showing the effect of sample time variation on estimated network condition $\hat{p}_{CL,OL}$ and average successful transmission percentage with network disturbance.

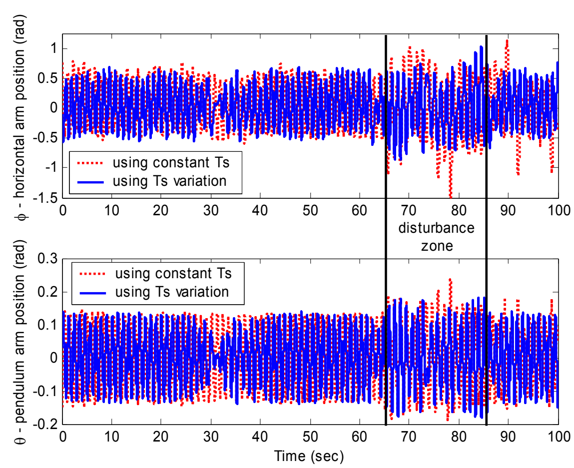

Fig. 10. Experimental results showing the effect of sample time variation on ϕ and θ regulation performance with network disturbance.

In terms of error statistics, Table 1 shows average error information from five repeated experiments with and without sample time variation. The root-mean-square (RMS) error is compared for the four different operating cases over 50 second long periods: fixed sampling with no disturbance, fixed sampling with disturbance, variable sampling without disturbance, and variable sampling with disturbance. Tracking errors in both ϕ and θ are smaller for the *Ts* variation cases than for the fixed sampling cases by more than 14%. When no network disturbances are present and conditions are favorable, *Ts* may be decreased to the lower bound allowing the variable sample rate case to achieve lower error levels than the fixed sample period case. When a network disturbance occurs, *Ts* is increased to maintain better network conditions. This action reduces the limit cycle amplitude and achieves lower RMS error levels.

Table 1. Summary of typical RMS errors when using fixed sampling or adaptive sampling for 50 seconds – average of five trials.

	RMS error ϕ	RMS error θ
Fixed Ts at 4.25 ms – no disturbance	3.167E-03	7.146E-04
Fixed Ts at 4.25 ms – with 20 sec network disturbance	4.555E-03	8.317E-04
Variable Ts – no disturbance	2.706E-03	6.102E-04
Variable Ts – with 20 sec network disturbance	3.340E-03	7.343E-04

V. Conclusion

This work examined the stability and performance of a wireless network system using an approach based on stochastic stability and H_∞ input-output gain. In the context of a full-state feedback, infinite horizon LQR control loop closed wirelessly, predictions of plant stability are possible and conveniently visualized in a Markov model network parameter space. Network conditions evolve in this space and can be associated with regions of stability or levels of performance in terms of an H_∞ disturbance-to-output gain. Early experimental results show correspondence between these level surfaces and observed stability regions for a real system with disturbances. In addition, experimental measurements of network behavior motivated a control policy for sample time variation that improves performance under varying network conditions. By actively regulating the network parameters within acceptable bounds, better robust performance is possible during unknown network traffic disturbances. Future work includes using estimation instead of zero output during packet losses as an alternate set of switched dynamics to improve stability and performance.

Appendix
Nomenclature

Symbol	Meaning	Experimental Value
$p_{CL,OL}\_UB$	upper bound of acceptable network performance	0.15
$p_{CL,OL}\_LB$	lower bound of acceptable network performance	0.02
δT^-	sample time decrement amount	0.25 ms
δT^+	sample time increment amount	1 ms
k_{dwell}	minimum number of dwell iterations after each switch	50

Symbol	Meaning
x	state vector
$\tilde{x}$	augmented state vector
k	sample period index
q	sample period switch index
$(k_{switch})_q$	sample period index of q^{th} sample period change
Ts_k	sample period of k^{th} interval
$(Ts)_{(k_{switch})_q}$	sample period after q^{th} sample period change
t	time
t_q	time at q^{th} sample period change
$p_{CL,OL}$	Markov transition probability from closed loop to open loop
$\hat{p}_{CL,OL}$	Markov transition probability from closed loop to open loop estimate

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