

# A Model Reference Robust Control and its Application to Autopilot Control Law Design\*

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## Abstract

In this paper, a new model reference robust control (MRRC) scheme is proposed and is applied to autopilot control law design. By using our scheme, both the tracking performance and the control effort can be improved significantly. The simulation results of a pitch-attitude-hold autopilot design show that, even with the existence of plant uncertainty, nonlinear disturbance and unmodeled dynamics, the steady-state error of the step response is no more than 0.3 percent of the final value and, the amplitude of the control signal is only about 35 percent of that by using the traditional zero-pole placement control scheme.

## 1 Introduction

In the design of autopilot control systems, it is important to take plant uncertainty, nonlinear disturbance and unmodeled dynamics into account. The classical control does not consider the problem at the initial stage of the design and therefore, the parameter modulation must be used to counteract their effects once the initial design is completed [1,2]. Though some simple techniques, such as *zero-pole placement*, can be used at the initial design, the parameter modulation is always complicated. Furthermore, the higher the demands of security and reliability presented to the advanced aircraft autopilot design are, the more complicated this modulation becomes. This motivates the search for new robust control schemes at the beginning of the autopilot design so as to alleviate the burden of parameter modulation.

In [3], a model reference robust control (MRRC) was proposed to overcome the influence of parameter uncertainty, nonlinear input disturbance and unmodeled dynamics related to the plant output, which combines the advantages of both robust control and model reference adaptive control (MRAC) and has been extended to MIMO, non-minimum phase plants [4]-[6]. For plants with relative degree greater than one, this method can be considered as an extension of the *backstepping* design [7,8], where the final control signal was obtained through a series of *fictitious control signals* designed recursively. We note that theoretically, a higher tracking precision can be achieved by using

the MRRC. From a practical point of view, however, the tracking performance of the MRRC may be unacceptable. In fact, as the authors themselves pointed out, the control signal may change rapidly and have large amplitude by using their scheme if a higher tracking precision is needed.

Given the motivation above, the objective of this paper is to modify the MRRC and to apply it to an autopilot control with nice performance and reasonable control effort. To this end, we first introduce a special transformation to the tracking system, and then a new *Lyapunov* function as well as a modified design algorithm is given. It can be shown that by using the new algorithm, both the tracking performance and the control effort can be improved significantly. Furthermore, the strictly positive real (SPR) condition required by the earlier research can be removed. We then apply the new MRRC algorithm to the autopilot design. In contrast to the classical methods, the main advantage of the new scheme is that the effect of the plant uncertainty, nonlinear disturbance and unmodeled dynamics can be taken into account even at the beginning of the design and therefore, the burden of parameter modulation is alleviated.

The paper is organized as follows. Section 2 gives the basic assumptions and the control structure. In Section 3, a new control law, which modifies the previous design technique given by [3], is proposed to improve both the tracking performance and the control effort of the MRRC. Section 4 discusses a pitch-attitude-hold autopilot control law design problem by applying the new design method. The results are illustrated by simulation.

## 2 System and Assumptions

The plant to be controlled is

$$y = G_p(s)[u + d] = \frac{k_p B_p(s)}{A_p(s)}[u + d], \quad (2.1)$$

where  $y$  and  $u$  are the plant output and input, respectively,  $d = d(y, t)$  represents disturbance and unmodeled dynamics related to output  $y$ ,  $B_p(s)$  and  $A_p(s)$  are monic coprime polynomials with  $\deg(A_p(s)) = n$  and  $\deg(B_p(s)) = m$ , respectively.

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For the controlled system, we make the following assumptions:

**(A1)**  $G_p(s)$  is minimum phase and, the order  $n$  and the relative degree  $n^* := n - m$  are known and constant; **(A2)** The coefficients of  $G_p(s)$  are unknown but belong to a known compact set; **(A3)** The sign of the high frequency gain  $k_p (\neq 0)$  is known and, without loss of generality, it is assumed that  $k_p > 0$ ; **(A4)** The term  $d$  is bounded by a known continuous function  $\rho(y, t)$  as, for all  $(y, t) \in \mathbb{R} \times \mathbb{R}^+$

$$|d(y, t)| \leq \rho(y, t). \quad (2.2)$$

Let the desired behavior of the closed-loop system be specified by the following reference model:

$$y_m = G_m(s)[r] = \frac{k_m B_m(s)}{A_m(s)}[r], \quad (2.3)$$

where  $y_m$  is the reference output,  $r$  is a piece-wise continuous and bounded reference input and,  $A_m(s)$  and  $B_m(s)$  are monic, coprime Hurwitz polynomials with  $\deg(A_m(s)) = n_m$  and  $\deg(B_m(s)) = m_m$ , respectively. Further, it is assumed that the matching condition [9]  $n_m - m_m = n^*$  is satisfied in this paper.

The objective is to determine the control signal  $u$  to ensure the tracking error

$$e := y - y_m \quad (2.4)$$

to converge to a residual set that can be made arbitrarily small by properly choosing some design parameters, where

$$u := \theta^T w + u_R, \quad (2.5)$$

where  $u_R$  is the nonlinear control to be designed, the constant vector  $\theta$  is defined as

$$\theta := [k \quad \theta_0 \quad \theta_1^T \quad \theta_2^T]^T \in \mathbb{R}^{2n}, \quad (2.6)$$

and  $w$ , the regressor vector, is given as

$$w := [r \quad y \quad w_1^T \quad w_2^T]^T \in \mathbb{R}^{2n}, \quad (2.7)$$

where  $w_1, w_2 \in \mathbb{R}^{n-1}$  are generated by input/output filters according to

$$\begin{aligned} \dot{w}_1 &= A_0 w_1 + B_0 u, w_1(0) = 0, \\ \dot{w}_2 &= A_0 w_2 + B_0 y, w_2(0) = 0, \end{aligned} \quad (2.8)$$

where  $A_0 \in \mathbb{R}^{(n-1) \times (n-1)}$ ,  $B_0 \in \mathbb{R}^{n-1}$  such that  $\det(sI - A_0)$  is a Hurwitz polynomial and  $(A_0, B_0)$  is a controllable pair. It is well known [10] that under the above assumptions, there exists a unique constant vector

$$\theta^* = [k^* \quad \theta_0^* \quad (\theta_1^*)^T \quad (\theta_2^*)^T]^T \in \mathbb{R}^{2n}, \quad (2.9)$$

such that, modulo exponentially decaying terms due to initial conditions,

$$y = G_p(s)[(\theta^*)^T w] = G_m(s)[r] = y_m. \quad (2.10)$$

Letting

$$\tilde{\theta} = \theta - \theta^* \quad (2.11)$$

and taking the disturbance  $d$  into consideration, we have

$$y = G_m(s)[r + (\tilde{\theta}^T w + d_f + u_R)/k^*], \quad (2.12)$$

where

$$d_f := (1 - d_1(s))[d],$$

$$d_1(s) := (\theta_1)^T (sI - A_0)^{-1} B_0. \quad (2.13)$$

From (2.5) to (2.13), we can write (2.4) in the following form for the case of  $n^* > 1$ :

$$e = G_m(s)L(s)[\tilde{\theta}^T \bar{w} + d_L + \bar{u}_R]/k^*, \quad (2.14)$$

where

$$L(s) := s^{n^*-1} + \alpha_1 s^{n^*-2} + \dots + \alpha_{n^*} \quad (2.15)$$

and is assumed to be a Hurwitz polynomial such that  $G_m(s)L(s)$  is of relative degree one, and  $\bar{w}$ ,  $d_L$  and  $\bar{u}_R$  are defined, respectively, as

$$\bar{w} := L^{-1}(s)[w],$$

$$d_L := L^{-1}(s)[d_f],$$

$$\bar{u}_R := L^{-1}(s)[u_R]. \quad (2.16)$$

### 3 Performance Improvement of the MRRC

As mentioned above, the MRRC introduces a special backstepping control, where *bounding functions* are used to replace the update law. However, in the MRRC, the control signal for each step includes the following term:

$$\frac{-[e_i g_i |e_i g_i|^{\tau_i}]}{(|e_i g_i|^{\tau_i+1} + \varsigma_i^{\tau_i+1})g_i}, \quad i = 1, 2, \dots, n^*, \quad (3.1)$$

where  $e_i$  can be considered as an ‘‘auxiliary error’’,  $\tau_i \geq 0$  and  $\varsigma_i > 0$  are adjustable parameters, and  $g_i$  is the bounding function of the preceding fictitious control signal. It is clear that if  $\varsigma_i$  tends to zero, (3.1) will tend to  $-g_i \operatorname{sgn}(e_i)$ , which implies that a smaller  $\varsigma_i$  may cause chattering and more seriously, may cause the amplitude of the final control signal to be extremely high due to the recursive manner of the control and the fact that the bounding function for each step must deal with the derivative of the preceding control signal. This situation is illustrated in Fig.1 below, where we choose the same plant as that in the example 2 of [3] and, to simplify the analysis, the model is chosen as  $G_m(s) = 2/(s+1)(s+5)$ . Let  $L(s) = s+5$ ; hence  $G_m(s)L(s) = 2/(s+1)$  is obviously a SPR function. The input signal of the system, say,  $r$ , is a square wave with unit amplitude and 5 period of units of time. We choose the design parameters  $\varsigma_1 = \varsigma_2 = 5$  and  $\varsigma_1 = \varsigma_2 = 0.5$ , respectively, where  $\varsigma_1$  and  $\varsigma_2$  were used in [3] to control the tracking precision, *i.e.*, the smaller the values of  $\varsigma_1$  and  $\varsigma_2$  are, the higher the tracking precision is. Figs.1-1 and 1-2 show that the tracking error is, in general, large, while from Figs.1-3 and 1-4, it is clear that the amplitude of the control signal is high due to the smaller  $\varsigma_1$  and  $\varsigma_2$  though the

tracking precision is better. It also should be pointed out that the simulation of example 2 of [3] is incorrect since it can be checked that  $G_m(s)L(s)$  is not a SPR function when  $L(s) = s + 10$ .

**Remark 1:** The bounding function of a signal  $f$ , say,  $\text{BND}(|f|)$  is a known, continuous and nonnegative function that bounds the magnitude of  $f$ . Readers may refer to [3] for detail about the definition.

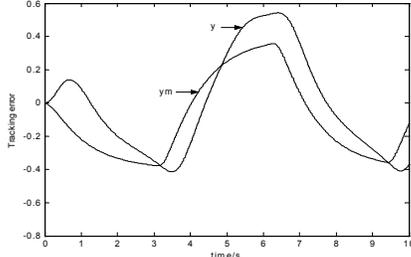


Fig. 1-1. Tracking error

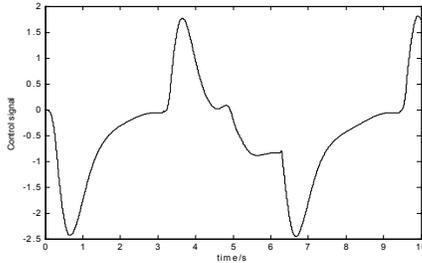


Fig. 1-2. Control signal

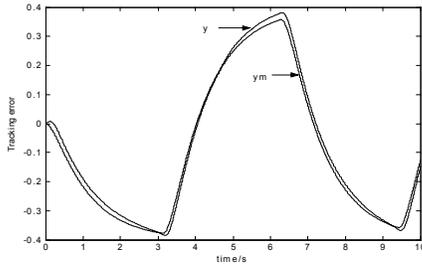


Fig. 1-3. Tracking error

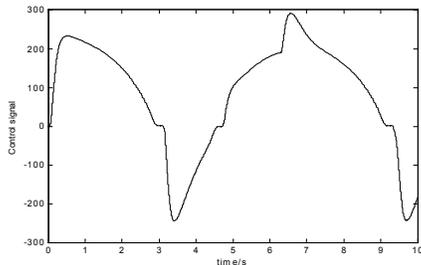


Fig. 1-4. Control signal

To overcome the above-mentioned flaw of the MRRC, a modified MRRC is proposed in this paper. The design procedure of the new scheme is as follows.

- First, using the technique introduced by [11], we can transform (2.14) as

$$e = \left( \frac{1}{s + \lambda} \right) k_p [\tilde{\theta}^T \bar{w} + d_L - \frac{\rho(s)}{k_p L(s)} [e] + L^{-1}(s)[u_R]], \quad (3.2)$$

where the polynomial  $\rho(s)$  satisfies  $B_m(s) = (s + \lambda)L(s) + \rho(s)$  with  $\deg(\rho(s)) < \deg(L(s))$  and can be obtained *a priori*. In particular, if  $G_m(s)L(s)$  is SPR, we have  $\lambda > 0$  [12]. Eq. (3.2) is equivalent to

$$\dot{e} = -\lambda e + k_p (\tilde{\theta}^T \bar{w} + d_L - \frac{\rho(s)}{k_p L(s)} [e] + L^{-1}(s)[u_R]). \quad (3.3)$$

Thus, we have transformed (2.14) into a first order differential equation, which plays a key role in the next step.

- Based on the transformation, a new *Lyapunov* function is introduced as follows:

$$V = \frac{1}{2} e^2 + \frac{1}{2} k_p \sum_{i=1}^{n^*-1} (z_i - V_i)^2, \quad (3.4)$$

where  $V_i$ 's are the control signals to be designed in a recursive manner, and

$$z_1 := \bar{u}_R = L^{-1}(s)[u_R], \quad (3.5)$$

whose controllable canonical form is

$$\begin{aligned} \dot{z}_1 &= z_2; \quad \dot{z}_i = z_{i+1}, \quad i = 2, \dots, n^* - 2; \\ \dot{z}_{n^*-1} &= -\alpha_1 z_{n^*-1} - \dots - \alpha_{n^*-1} z_1 + u_R. \end{aligned} \quad (3.6)$$

- Finally, the new backstepping control law is given as follows,

$$\begin{aligned} V_1 &:= -\sigma e - \frac{\mu_1 |\mu_1|^{\tau_1}}{|\mu_1|^{\tau_1+1} + \zeta_1^{\tau_1+1}} g_1, \\ u_R = V_2 &:= -\gamma(z_1 - V_1) - e + \alpha_1 z_1, \\ &\quad - \frac{\mu_2 |\mu_2|^{\tau_2}}{|\mu_2|^{\tau_2+1} + \zeta_2^{\tau_2+1}} g_2, \quad \text{if } n^* = 2, \\ V_2 &:= -\gamma(z_1 - V_1) \\ &\quad - e - \frac{\mu_2 |\mu_2|^{\tau_2}}{|\mu_2|^{\tau_2+1} + \zeta_2^{\tau_2+1}} g_2, \quad \text{if } n^* > 2, \\ V_i &:= -\gamma(z_{i-1} - V_{i-1}) - (z_{i-2} - V_{i-2}), \\ &\quad - \frac{\mu_i |\mu_i|^{\tau_i}}{|\mu_i|^{\tau_i+1} + \zeta_i^{\tau_i+1}} g_i, \quad i = 3, \dots, n^* - 1, \\ u_R = V_{n^*} &:= -\gamma(z_{n^*-1} - V_{n^*-1}) \\ &\quad - (z_{n^*-2} - V_{n^*-2}) \\ &\quad + (\alpha_1 z_{n^*-1} + \dots + \alpha_{n^*-1} z_1), \quad (3.7) \\ &\quad - \frac{\mu_{n^*} |\mu_{n^*}|^{\tau_{n^*}+1}}{|\mu_{n^*}|^{\tau_{n^*}+1} + \zeta_{n^*}^{\tau_{n^*}+1}} g_{n^*} \end{aligned}$$

where  $V_1, V_2, \dots, V_{n^*-1}$  are the so called fictitious control signals, and  $u_R = V_{n^*}$  is the actual control signal;  $\sigma > 0$ ,  $\tau_1 \geq 0$ ,  $\zeta_1 > 0$ ,  $\tau_j \geq 0$ ,  $\zeta_j > 0$  ( $j = 2, \dots, n^*$ ) and  $\gamma$  are design parameters;

$$\gamma \geq \lambda + k_p \sigma := \gamma_1, \quad (3.8)$$

where  $\underline{k}_p$  is a lower bound of  $k_p$ ; the bounding functions  $g_1$ ,  $g_i$ , and the functions  $\mu_1$ ,  $\mu_i$  are designed, respectively, as follows:

$$g_1 = \text{BND}(\dot{\theta}^T \bar{w} + d_L), \mu_1 = e g_1, \\ g_j = \text{BND}(\dot{V}_{j-1}), \mu_j = (z_{j-1} - V_{j-1}) g_j. \quad (3.9)$$

We summarize the main result of this section in the following theorem whose proof and the reason to choose the control signals in the form of (3.7) may refer to [13] for detail.

**Theorem 3.1** Suppose the MRRC system given by (2.14) satisfies the assumptions (A1)-(A4). Let the control signal  $u_R$  be defined by (3.7). Then all the signals of the closed loop system are uniformly bounded, and the  $L_\infty$  bound of the tracking error satisfies

$$|e| \leq \sqrt{2 \exp(-2\gamma_1 t) V(0) + k_p \varsigma / \gamma_1}, \quad \forall t \geq 0, \quad (3.10)$$

where,  $\varsigma = \sum_{i=1}^{n^*} \varsigma_i$ .

**Proof:** See [13].

In comparison to Theorem 2 of [3], the main feature of the modification is that we can transform (2.14) into a first-order differential equation as shown in (3.2), which paves the way for the introduction of the new *Lyapunov* function and the design parameters  $\sigma$  and  $\gamma$  as shown in (3.4) and (3.7), respectively. Since  $\sigma$ ,  $\gamma$  as well as  $\gamma_1$  are at designer's disposal, the theorem states that the tracking error converges to a residual set that is a decreasing function of  $\gamma_1$  and therefore, the tracking performance improvement can be achieved. In fact, from (3.10), we have  $\lim_{t \rightarrow \infty} |e| = \sqrt{k_p \varsigma / \gamma_1}$ , which implies that we can choose a larger  $\gamma_1$  to improve the tracking performance while maintaining a reasonable control effort. This is the main difference between our modification and that of the earlier research.

Another interesting feature of the modification is that the SPR assumption of  $G_m(s)L(s)$  can be removed. In fact, if  $G_m(s)L(s)$  is not SPR, the method of [3] cannot be applied. Note that by using our modification, we can choose  $\sigma$  such that  $\gamma_1$  in (3.8) is greater than zero even when  $\lambda \leq 0$ . The proof of the theorem given in [13] shows that  $\gamma_1 > 0$  is essential to guarantee the stability of the MRRC system.

#### 4 Autopilot Control Law Design with the Modified MRRC

In this section, we will apply the modified MRRC to a pitch-attitude-hold autopilot control law design. The controlled variable is pitch attitude  $\vartheta$  and the input signal is elevator deflection  $\delta_e$ . The output of a reference

model gives the ideal pitch attitude  $\vartheta_m$ . Fig.2 shows the block diagram of this control system.

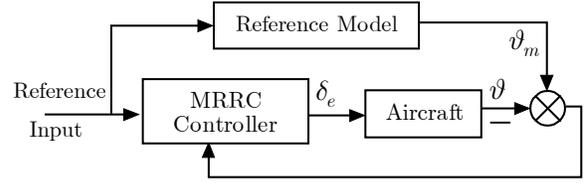


Fig.2 A pitch-attitude-hold autopilot by using the modified MRRC

A classical control law design of a pitch-attitude-hold autopilot is shown in [1], where traditional zero-pole placement method is used, and the simulation results show that the step response has a large steady-state error and eventually settles at about 0.77. Though a dynamic compensator can be used to reduce the steady-state pitch error, the control effort would be large [1].

For fair comparison, we use the same nominal model of a transport-aircraft as that given by [1]. That is, the dynamics of the aircraft model in a level-flight cruise condition at 25000ft, 500ft/s true airspeed are given by:

$$\begin{bmatrix} \dot{v} \\ \dot{\alpha} \\ \dot{\vartheta} \\ \dot{q} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} -8.23E^{-3} & 18.94 & -32.17 & 0.0 & 5.90E^{-5} \\ -2.5617E^{-4} & -0.57 & 0.0 & 1.0 & 2.26E^{-6} \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 1.3114E^{-5} & -1.48 & 0.0 & -0.48 & -1.49E^{-7} \\ 0.0 & -500.0 & 500.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} v \\ \alpha \\ \vartheta \\ q \\ h \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.02 \\ 0 \end{bmatrix} \delta_e \\ y = \vartheta, \quad (4.1)$$

where  $v$ ,  $\alpha$ ,  $\vartheta$ ,  $q$  and  $h$  are the air speed, attack angle, pitch attitude, pitch rate and altitude, respectively, and  $\delta_e$ , the elevator deflection, is the unique input signal. The altitude pole is almost cancelled by a zero. For simplicity the altitude state has been omitted, since its effect on the design is negligible. Thus, Eq. (4.1) can be rewritten as

$$\vartheta = G_p(s)[\delta_e + d], \quad (4.2)$$

where  $G_p(s)$  is a 4th order transfer function with relative degree 2 and,  $d$  represents the nonlinear input disturbance and unmodeled dynamics and is chosen as

$$d = \frac{y}{(s+10)} + y^3 s \sin(0.3t). \quad (4.3)$$

The reference model, whose output is the desired response, is chosen as

$$\vartheta_m = G_m(s)[r] = \frac{4}{s^2 + 4s + 4}[r], \quad (4.4)$$

where  $r$  is a unit step function. Let

$$L(s) = s + \alpha_1 = s + 4, \quad (4.5)$$

where  $L(s)$  is defined by (2.15). From (2.8), we choose

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -125 & -25 & -5 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (4.6)$$

Let

$$\begin{aligned} \varsigma_1 = \varsigma_2 = 50, \sigma = 100, \gamma = 110, \\ \theta = 0, \tau_1 = 1, \tau_2 = 0, \\ \text{BND}(\tilde{k}) = 4, \text{BND}(k_p) = 4, \text{BND}(\theta_0) = 2, \\ \text{BND}(\tilde{\theta}_1) = \text{BND}(\tilde{\theta}_2) = [2 \ 2 \ 2]^T. \end{aligned} \quad (4.7)$$

From (4.3), the bounding function of the disturbance can be chosen as

$$\begin{aligned} |d(y, t)| \leq \int_0^t \exp(-10(t - \tau)) |y(\tau)| d\tau \\ + |y|^3 \leq \frac{1}{10} \|y_t\|_\infty + |y|^3 := \rho(y, t). \end{aligned} \quad (4.8)$$

From (3.7), and noting that  $k_p < 0$ , the control signals are

$$V_1 = \sigma e + \frac{\mu_1 |\mu_1|}{\mu_1^2 + \varsigma_1^2} g_1 = \sigma e + \frac{e |e| g_1^3}{e^2 g_1^2 + \varsigma_1^2}, \quad (4.9)$$

and

$$\begin{aligned} u_R = V_2 = -\gamma(z_1 - V_1) + e \\ + \alpha_1 z_1 - \frac{\mu_2 g_2}{|\mu_2| + \varsigma_2}, \end{aligned} \quad (4.10)$$

respectively, where

$$\begin{aligned} g_1 = \text{BND}(|\tilde{\theta}^T \bar{w} + d_L|) \\ = \text{BND}(\tilde{\theta}) |\bar{w}| + \frac{1}{L(s)} [\rho(y, t)], \end{aligned} \quad (4.11)$$

$$\begin{aligned} g_2 = |\dot{V}_1| = \left| \frac{\partial V_1}{\partial e} \dot{e} + \frac{\partial V_1}{\partial g_1} \dot{g}_1 \right| \\ \leq \frac{2|e|}{e^2 g_1^2 + \varsigma_1^2} \text{BND}(|\dot{e}|) + \frac{5}{2} \dot{g}_1, \end{aligned} \quad (4.12)$$

$$\mu_2 = (z_1 - V_1) g_2, \quad (4.13)$$

and

$$\begin{aligned} |\dot{e}| \leq \text{BND}(k_p) \left[ \left( 1 + \frac{|(a_{m1} - \alpha_0)s + a_{m2}|}{s^2 + a_{m1}s + a_{m2}} \right) (|z_1| + |g_1|) \right] \\ := \text{BND}(|\dot{e}|). \end{aligned} \quad (4.14)$$

The simulation results of zero-pole placement scheme are shown in Fig.3, where 3-1-*b* shows the system output without dynamic compensator and with  $d = 0$ , from which significant steady-state error can be observed. 3-1-*a* shows the output when a dynamic compensator is applied with  $d = 0$ . We can see that the latter method can keep the steady-state error no more than 4 percent of the final value. However, from Fig 5-1, we notice that once the dynamic compensator is used, the amplitude of the control signal  $\delta_e$  (5-1-*b*) would increase about 2.5 times as that without the compensator (5-1-*a*). Furthermore, according to Fig. 3-2, for both cases *with and without* the compensator (see (3-2-*a*) and (3-2-*b*), respectively), the disturbance  $d$  significantly degrades the performance.

The simulation results of the new MRRC scheme are shown in Fig.4, where 4-1-*a* and 4-2-*a* show the outputs of the reference model while 4-1-*b* and 4-2-*b* show the outputs of the aircraft plant, from which, we

can see that even with the existence of external disturbance, the steady-state error can be kept no more than 0.3 percent of the final value, and the control signal (5-2-*b*) is nearly the same as that of  $d = 0$  (5-2-*a*). It is shown that whether the disturbance exists or not, by using the new modified MRRC, a perfect tracking performance can be achieved. Furthermore, comparing 5-1-*b* to 5-2-*b*, the amplitude of the control signal is only about 35 percent of that by using the traditional zero-pole placement control scheme.

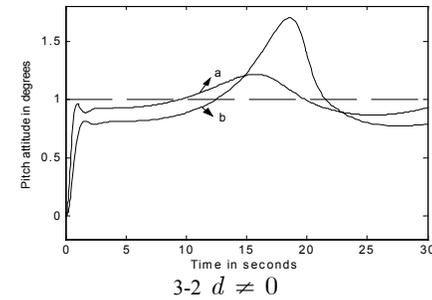
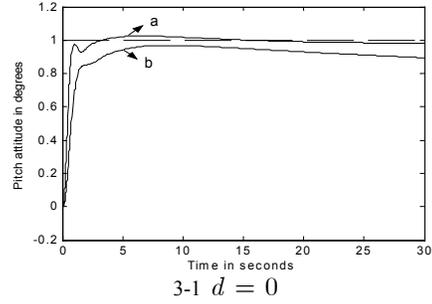


Fig.3 Simulation results when traditional zero-pole placement scheme is applied

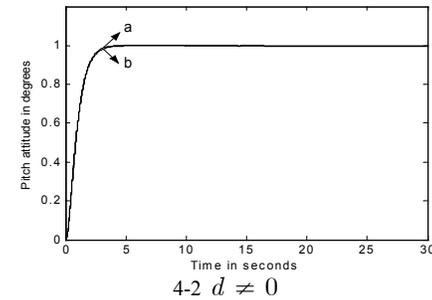
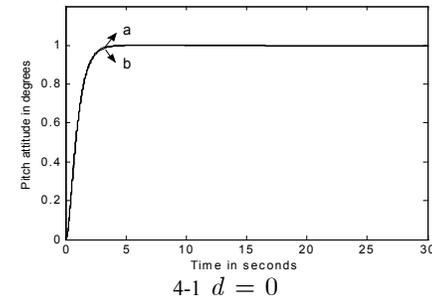
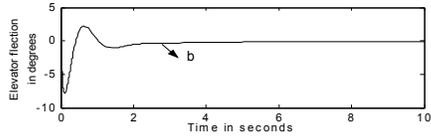
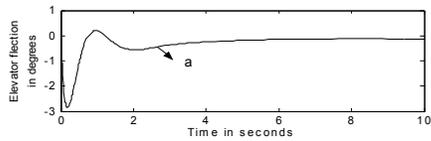
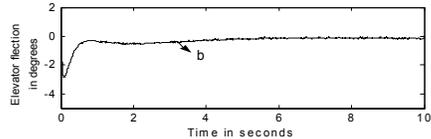
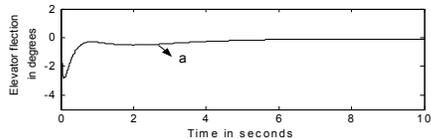


Fig.4 Simulation results when new MRRC scheme is applied



5-1 Using zero-pole placement scheme



5-2 Using new MRRC scheme

Fig.5 Control signal  $\delta_e$

## 5 Conclusion

In this paper, a modified model reference robust control (MRRC) scheme has been proposed and applied to the control law design of a pitch-attitude-hold autopilot. The simulation results have shown that by using the modified scheme, both the tracking performance and the control effort can be improved significantly.

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