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Competitive Team Strategies for Routing Control in Parallel-Link Communication Networks

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Abstract- In this paper we consider a two-node parallel-link communication network shared by competitive teams of users. Each team has several users with various types of traffics or jobs to be routed on the network and must compete with the other teams for the network resources. The users in each team cooperate for the benefit of their team so as to achieve optimal routing over network. For each team, there is a centralized decision-maker, called the team manager, or leader, who coordinates the routing strategies among for all users in his team. A game theoretic approach to deal with both cooperation within each team and competition among the teams, called the Noninferior Nash Strategy with a team Manager (NNSM) is introduced. This multi-team solution provides a new framework for analyzing hierarchically controlled systems so as to address complicated coordination problems among the various users. This strategy is applied to derive the optimal routing policies for all users in the network. It is shown that the Noninferior Nash Strategy with a team Manager is effective in improving the overall network performance. Several examples are presented.

1. INTRODUCTION

The problem of routing is encountered in all and every communication network shared by a large number of users. Conventional networks are traditionally designed as a single entity with a single performance objective under the assumption that users are passive and would cooperate in optimizing the overall performance of routing in the entire network. In modern communication networks, however, this assumption may no longer be valid since the users typically have various, and sometime even contradictory, performance measures and demands. One possible way to manage such a network is to allow each user to change its routing strategy based on the state of the network so as to optimize its own performance criterion. However, the change of strategy taken by one user is likely to cause changes in other users' strategies, resulting in a continuously changing network. The final outcome in such a network is therefore heavily dependent on the competitive actions by all the users and hence its overall optimization is best analyzed within the framework of game theory. The analysis of competitive routing problems using game theory has recently received considerable attention in the network communication and control literatures [1-6]. The Nash equilibrium, a main concern in [1-6], when reached, ensures that no user finds it advantageous to change his behavior unilaterally in an attempt to further improve his own performance.

In this paper we consider a network structure where several competitive teams of users (rather than single users) share the network resources. We assume that each team has a manager (or team leader) who centralizes all decisions for that team. A practical example of such a structure is a set of organizations (or companies) each with different classes of data traffics requirements, such as email, audio, image, video, etc. and all sharing the same internet resources to send their data. However, rather than allow each user in an organization to unilaterally decide how to compete with users from other organizations, our model assumes that each organization has a manager whose job is to optimize the needs of users within his own organization. At the same time, the managers of all the organizations have to compete with each other so as to best serve the performance objectives of their own organizations over the network.

One natural way of managing such a network is for users belonging to the same team (organization) to cooperate with each other and to let the team managers compete with each other and settle to an equilibrium in which each of them reaches its optimum operating point. We note that [7] considers a similar organizational structure except that the objective of each team manager is to optimize the average of the objectives of all users in his team. In this paper, we assume that each team manager has his own objective function which is, in general, completely independent from those of the users in his team. This problem can be modeled as a multi-team game and solved using the concept of Noninferior Nash Strategy with a Manager (NNSM) discussed in [8].

In this paper, we will apply the NNSM to a simple two node network interconnected by a number of parallel links. Our main goal is to derive a routing control scheme for the network and investigate the effectiveness of the NNSM strategies. This simple network structure could form the basis for a more complex structure with several nodes and serial as well as parallel links.

2. MODEL AND PROBLEM FORMULATION

We consider a set $N=\{1,\cdots,\mathrm{N}\}$ of teams each with several types of users that share a set $L=\{1,\cdots,\mathrm{L}\}$ of parallel communication links interconnecting a common source node and a common destination node. Let c_j be the capacity of link j, $c=(c_1,\cdots,c_{\mathrm{L}})$ be the capacity configuration, and $C=\sum_{j=1}^{\mathrm{L}} c_j$ be the total capacity of the entire network. Without loss of generality, let $c_1<c_2<\cdots<c_L$. Let TM_X denote the manager of team X $(X=1,\cdots,\mathrm{N})$ who serves n_X users in team X. Assume that the i^{th} $(i=1,\cdots,n_X)$ user in team X has a throughput demand that is a Poisson process with average rate $\lambda_i^X>0$. Let $\lambda=\sum_{X=1}^{\mathrm{N}}\sum_{i=1}^{n_X}\lambda_i^X$ be the total throughput demand of all users in the networks. Furthermore, for stability reasons assume that the total throughput demand is less than the total capacity of the parallel links, i.e., $\lambda<C$. The i^{th} user managed by TM_X sends its flow by splitting its demand λ_i^X over the parallel links. Let $f_i^X(j)$ denote the (expected) fraction of flow that user i from team X sends on link j. The user flow fraction configuration

$$\mathrm{f}_i^X=\left(f_i^X(1),\cdots,f_i^X(\mathrm{L})\right) \tag{1}$$

is called a routing strategy of user i from team X and the set

$$\mathrm{F}_i^X=\left\{\begin{array}{l}\mathrm{f}_i^X\in R^{\mathrm{L}}:0\le\lambda_i^X f_i^X(j)\le c_j,\\ \sum_{j=1}^{\mathrm{L}} f_i^X(j)=1, 0\le f_i^X(j)\le 1, j\in L\end{array}\right\} \tag{2}$$

of strategies that satisfy the user's demand is called the strategy space of user i from team X. The routing control profile for the users from team X is denoted by

$$\mathrm{f}^X=\left(\mathrm{f}_1^X,\cdots,\mathrm{f}_{n_X}^X\right) \tag{3}$$

and takes values in the product strategy space

$$\mathrm{F}^X=\otimes_{i=1}^{n_X}\mathrm{F}_i^X. \tag{4}$$

The system routing control profile is given by

$$\mathrm{f}=\left(\mathrm{f}^1,\cdots,\mathrm{f}^{\mathrm{N}}\right) \tag{5}$$

and takes values in the overall product strategy space

$$\mathrm{F}=\otimes_{X=1}^{\mathrm{N}}\mathrm{F}^X. \tag{6}$$

The details of such a system are shown in Figure 1.

The users from team X have certain routing decisions $\mathrm{f}_i^X=\left(f_i^X(1),\cdots,f_i^X(\mathrm{L})\right)$ to make for the purpose of, for example, minimizing their average delay time. Consider the average delay for each user as a cost function. Without loss of generality, we let the service requirement of each user be exponentially distributed with mean equal to 1. We concentrate on the $M/M/1$ delay function [9] $d(j)$ on link l ($j\in\mathrm{L}$):

$$d(j)=\begin{cases}\dfrac{1}{c_j-\sum_{X=1}^{\mathrm{N}}\sum_{i=1}^{n_X}\lambda_i^X f_i^X(j)} & \sum_{X=1}^{\mathrm{N}}\sum_{i=1}^{n_X}\lambda_i^X f_i^X(j)<c_j\\ \infty & \sum_{X=1}^{\mathrm{N}}\sum_{i=1}^{n_X}\lambda_i^X f_i^X(j)\ge c_j\end{cases} \tag{7}$$

Thus, the total delay for user i from team X is:

$$d_i^X=\sum_{j=1}^{\mathrm{L}}\lambda_i^X f_i^X(j)d(j). \tag{8}$$

and the cost function (i.e., average delay) for user i from team X under control strategy profile f_j^i to be minimized is given by

$$J_i^X(\mathrm{f})=\frac{d_i^X}{\lambda_i^X}=\sum_{j=1}^{\mathrm{L}} f_i^X(j)d(j) \tag{9}$$

where $J_i^X:\mathrm{F}\to R$. Obviously, this cost function depends on the control strategies of other users as well.

The team managers TM_X for $X=1,2,....N$ may have different objective functions $P^X:\mathrm{F}^X\to R$. In this paper, we consider two types of objective functions for team managers: efficiency objective functions (Type 1) and flow cost objective functions (Type 2). A team manager with an objective function of Type 1 wants to maximize the efficient utilization of the highest capacity link, which is given by

$$\text{Type 1:}\quad P^X(\mathrm{f}^X)=\sum_{i=1}^{n_X}\lambda_i^X f_i^X(L) \tag{10a}$$

A team manager with an objective function of Type 2 wants to minimize the total cost of flow for his users. Let $p^X(j)$ be the cost paid by users from team X for their flow on link j, and the manager of that team wishes to minimize the total cost of the flow for team X given by

Type 2: $P^X(\mathrm{f}^X) = \sum_{j=1}^{\mathrm{L}} \left(p^X(j) \sum_{i=1}^{n_X} \lambda_i^X f_i^X(j) \right)$ (10b)

The optimal routing problem is therefore formulated as

$$\max_{\mathrm{f}^X} P^X(\mathrm{f}^X) \text{ for Type 1} \quad \mathrm{f}^X \in \mathrm{F}^X, X \in N$$
or (11a)
$$\min_{\mathrm{f}^X} P^X(\mathrm{f}^X) \text{ for Type 2} \quad \mathrm{f}^X \in \mathrm{F}^X, X \in N$$

for each Team Manager TM_X in the network;

$$s.t. \quad \min_{\mathrm{f}_i^X} J_i^X(\mathrm{f}) \quad \mathrm{f} \in \mathrm{F}, \mathrm{f}_i^X \in \mathrm{F}_i^X, X \in N, i = 1, \cdots, n_X \quad (11b)$$

for each user in team *X*. Note that when N=1, the above problem reduces to a team optimization problem.

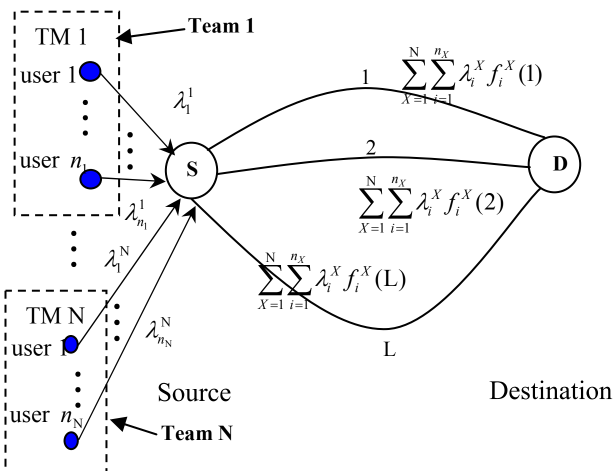

Figure 1. Two-Node Parallel-Link Communication Network with Multiple Teams of Users

3. NONINFERIOR NASH STRATEGIES FOR ROUTING PROBLEMS

The two-node parallel-link communication network discussed in the previous section is a typical example of multi-team systems [8] that are controlled by several competing teams of decision-makers, with each team consisting of several cooperating decision-makers. The optimization of a multi-team system must be done within a framework that combines team theory with game theory. Focusing on routing problem, we refer to this framework as nonzero-sum multi-team routing games. A set of strategies called Noninferior Nash Strategies (NNS) [8] has been developed which represents cooperation among all members within each team but insures a non-cooperative Nash equilibrium among all teams. We will assume that the specific choice of a Nash equilibrium within this set is done by the team managers based on their own criteria. The resulting choice is referred to as a Noninferior Nash Solution with a Manager (NNSM). Before applying NNSM, let's consider the routing problem with only one team (i.e., N=1).

3 a. Team Optimization for Single-Team Routing Control Problems.

For simplicity, we consider a network with two parallel links and two users as illustrated in Figure 2. Let the users' throughput demands be λ_1 and λ_2, and the link capacities be c_1 and c_2 respectively. Let x and y denote the fraction of flow demand of user 1 and user 2 will be assigned to link 1, respectively. According to the constraints in (1), 1-*x* (or 1-*y*) is the fraction of flow demand of the user 1 (or user 2) will be assigned to link 2.

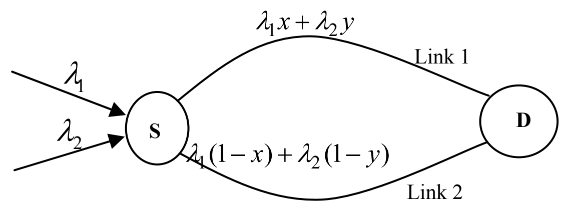

Figure 2. Single-Team Routing Problem

As expressed in (9), the cost functions J_1 and J_2 for users 1 and 2 are given by

$$J_1(x,y) = \frac{x}{c_1 - \lambda_1 x - \lambda_2 y} + \frac{1-x}{c_2 - \lambda_1(1-x) - \lambda_2(1-y)} \quad (12)$$

and

$$J_2(x,y) = \frac{y}{c_1 - \lambda_1 x - \lambda_2 y} + \frac{1-y}{c_2 - \lambda_1(1-x) - \lambda_2(1-y)} \quad (13)$$

In the single team optimization problem, both users cooperate with each other and the team manager' objective is to maximize the efficient usage of the link with high capacity (objective function of Type 1). The objective function for the team manager is given by

$$J_M(x,y) = \lambda_1(1-x) + \lambda_2(1-y) \quad (14)$$

since $c_1 < c_2$. The team optimization problem can be therefore be formulated as:

$$\max_{x,y} J_M(x,y) \quad (15)$$

$$\text{s.t. } \min_{x} J_1(x,y) \text{ and } \min_{y} J_2(x,y) \quad (16)$$

$$c_1 - \lambda_1 x - \lambda_2 y > 0 \text{ and } c_2 - \lambda_1(1-x) - \lambda_2(1-y) > 0 \quad (17)$$

$$0 \le x, y \le 1 \quad (18)$$

The cost functions $J_1(x,y)$ and $J_2(x,y)$ are convex with respect to x and y over the convex space given by (17) and (18). Thus, the optimal solution for (14) can be determined by minimizing a weighted scalar-valued cost function $J(x,y;\alpha)$ [10] as follows:

$$\min_{x,y} J(x,y;\alpha) = \alpha J_1(x,y) + (1-\alpha) J_2(x,y) \quad (19)$$

where α is a weight factor satisfying $0 \le \alpha \le 1$. For each α, there exists an optimal solution ($x^*(\alpha), y^*(\alpha)$). Since the cost function of manager on the higher level is also determined by the optimal controls x by user 1 and y by user 2, $J_M(x,y)$ becomes a function of the weight factor α. In other words, the objective of the manager is to decide on an optimal choice of α so as to minimize $J(x,y;\alpha)$.

As a numerical example, let p_1=400, p_2=100, $\lambda_1 = 1$, $\lambda_2 = 3, c_1 = 3$ and $c_2 = 6$, where p_i is the cost paid by using link *i* (*i*=1,2). The convex set given by (17) and (18) is expressed as the blue-shaded area shown in Figure 3. The cost function of user 1, $J_1(x,y)$, is given in Figure 4. We observe that $J_1(x,y)$ is convex with respect convex set given by (17). However, the objective function for user 1, the average delay, is extremely large with respect to the decisions around the boundaries $c_1 - \lambda_1 x - \lambda_2 y$=0 and $c_2 - \lambda_1(1-x) - \lambda_2(1-y)$=0. Therefore, in practice, user 1 has to avoid the use of those decision choices. The objective functions $J_1(x,y)$ and $J_2(x,y)$ in reasonable areas are given in Figure 5. After figuring out all the possible cooperative controls for both users, i.e., ($x^*(\alpha), y^*(\alpha)$) for all α's, we substitute these solutions to (14) to calculate the optimal value of J_M. A plot of J_M versus α is shown in Figure 6 from which we see that: $\alpha^* = 0.25$, $x^*(\alpha^*) = 0.03$, $y^*(\alpha^*) = 0.3$, $J_1^* = 0.3456$, $J_2^* = 0.3838$ and $J_M^* = 3.07$.

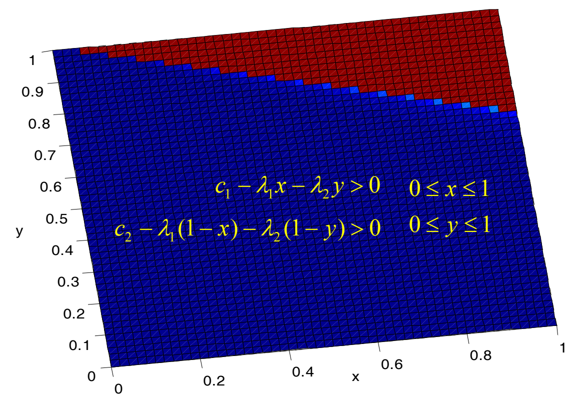

Figure 3. Convex set of the given example

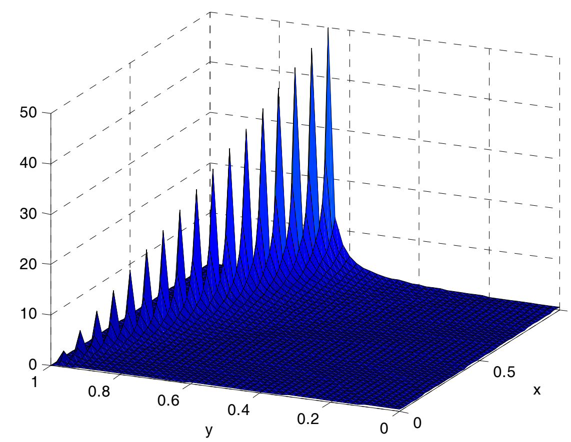

Figure 4. Convex cost function $J_1(x,y)$

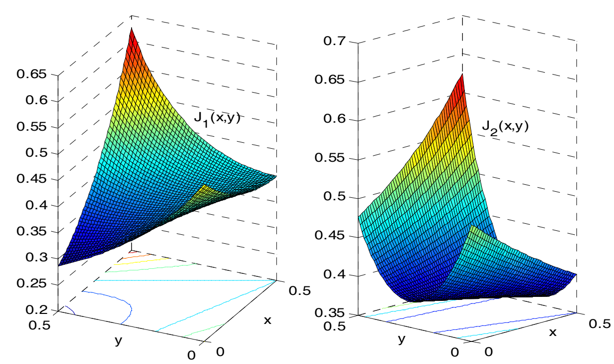

Figure 5. Cost functions $J_1(x,y)$ and $J_2(x,y)$ in reasonable areas

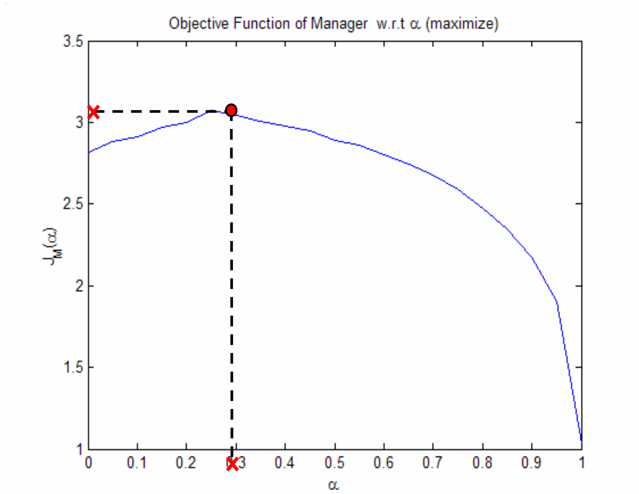

Figure 6. Objective function for the manager w.r.t. different values of weight factor

3.b. Noninferior Nash Strategies for Two-Team Routing Control Problems.

In this section, we consider a two-team routing problem (the results can be easily generalized to a multi-team problem). Assume the two teams are labeled H (Team 1) and T (Team 2), respectively, and that each team has two users as well. We still consider a two-node parallel-link network as before. The total system capacity is $C = c_1 + c_2$. Let the throughput demand of user *i* from Team H arrive at the network with rate λ_i^H (*i*=1, 2). The total throughput demand for the users from Team H is $\lambda^H = \lambda_1^H + \lambda_2^H$. The fractions of flow of user 1 and user 2 from Team H assigned to link 1 are x ($\in [0,1]$) and y ($\in [0,1]$), respectively. Let the throughput demand of user *j* in Team T arrive at the network with rate λ_j^T (*j*=1, 2). The total throughput demand for Team T users is $\lambda^T = \lambda_1^T + \lambda_2^T$. The fractions of flow of user 1 and user 2 from Team T assigned to link 1 are u ($\in [0,1]$) and v ($\in [0,1]$), respectively. Furthermore, we only consider the situation where the total capacity can accommodate the total user demand, that is, $\lambda^N + \lambda^T \le C$. The entire system is illustrated in Figure 7.

As before, each user wants to minimize its average delay in the system. It can be formulated as the following optimal problem: For the users from TEAM1,

$$\min_x J_1^H(x,y,u,v) = \frac{x}{g(x,y,u,v)} + \frac{1-x}{h(x,y,u,v)} \quad (20)$$

$$\min_y J_2^H(x,y,u,v) = \frac{y}{g(x,y,u,v)} + \frac{1-y}{h(x,y,u,v)} \quad (21)$$

and, for the users from TEAM2,

$$\min_u J_1^T(x,y,u,v) = \frac{u}{g(x,y,u,v)} + \frac{1-u}{h(x,y,u,v)} \quad (22)$$

$$\min_v J_2^T(x,y,u,v) = \frac{v}{g(x,y,u,v)} + \frac{1-v}{h(x,y,u,v)} \quad (23)$$

such that: $\quad g(x,y,u,v) > 0$ and $h(x,y,u,v) > 0 \quad (24)$

$$0 \le x,y,u,v \le 1 \quad (25)$$

where

$$g(x,y,u,v) = c_1 - \lambda_1^H x - \lambda_2^H y - \lambda_1^T u - \lambda_1^T v\,;$$

and

$$h(x,y,u,v) = c_2 - \lambda_1^H(1-x) - \lambda_2^H(1-y) - \lambda_1^T(1-u) - \lambda_1^T(1-v).$$

Clearly, this optimal problem can be formulated as a multi-team game with N=2 and $n_1 = n_2 = 2$. The solution to this problem is a noninferior Nash strategy. The average delay objective functions J_i^H and J_j^T (*i,j*=1,2) are strictly convex over the convex space given by (24) and (25).

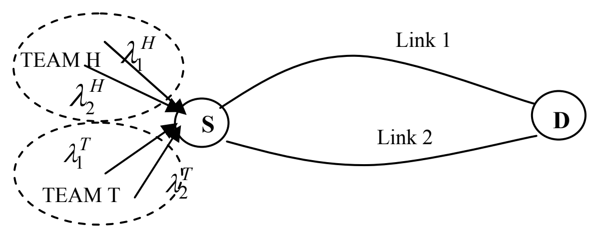

Figure 7. Two-Team Routing Problem

This problem admits a noninferior Nash strategy [8] with a weight vector $\xi = \left[\xi^H = (\alpha, 1-\alpha), \xi^T = (\beta, 1-\beta)\right]$ to the routing problem for the users served by two managers. The linear combination with these weights of the objective functions of both teams are given by

$$J^H(\alpha) = \alpha J_1^H + (1-\alpha) J_2^H \quad (26)$$

$$J^T(\beta) = \beta J_1^T + (1-\beta) J_2^T \quad (27)$$

Note that the noninferior Nash strategies for all four users will be functions of α and β, i.e., $x^* = x^*(\alpha,\beta)$, $y^* = y^*(\alpha,\beta)$, $u^* = u^*(\alpha,\beta)$ and $v^* = v^*(\alpha,\beta)$

Since there are infinite combinations of α and β, we still need to decide on the optimal weight vector ξ^*. We introduce two different types of objective function for the two managers:

$$\max_{(x^*,y^*)} J_M^H(x^*,y^*) = \lambda_1^H(1-x^*) + \lambda_2^H(1-y^*) \quad (28)$$

for the manager of Team H, and

$$\min_{(u^*,v^*)} J_M^T(u^*,v^*) = p_1^T(\lambda_1^T u^* + \lambda_2^T v^*) + p_2^T(\lambda_1^T(1-u^*) + \lambda_2^T(1-v^*)) \quad (29)$$

for the manager of team T. The manager of Team H has an objective of Type 1 and wants to maximize the throughput on the link with higher capacity ($c_2 > c_1$), and the manager of Team T has an objective of Type 2 and wants to minimize the total cost of usage of different links. Let p_1^T and p_2^T be the price per unit flow for link 1 and link 2, respectively. It is clear that $J_M^H(\cdot)$ and $J_M^T(\cdot)$ will be functions of α and β as well. The optimal choices of α and β can be determined to satisfy a Nash equilibrium for a noncooperative game between the two managers with respective to the objective functions $J_M^H(\alpha,\beta)$ and $J_M^T(\alpha,\beta)$. Since it is not easy to obtain analytical expressions for $J_M^H(\alpha,\beta)$ and $J_M^T(\alpha,\beta)$, we will use a numerical example to illustrate the properties and effectiveness of the NNSM.

Let $c_1 = 3$, $c_2 = 6$, $\lambda_1^H = 1$, $\lambda_2^H = 3$, $\lambda_1^T = 0.5$, $\lambda_2^T = 1$, $p_1^T = 10$ and $p_2^T = 30$. The corresponding NNSM (optimal routing fractions) under the managers' objective functions are computed as follows: α^*=0.25, β^*=0.8, x^*=0.7, y^*=0, u^*=0, v^*=1, and the corresponding values of the objective functions are tabulated in Table 1.

Table 1. NNSM values of the objective functions

J_1^{H*}	J_2^{H*}	J_1^{T*}	J_2^{T*}	J_M^{H*}	J_M^{T*}
0.6748	0.4545	0.4545	0.7692	3.30	25

As we mentioned before, in our new game framework, we assume that users from the same team will cooperate with each other for the team benefit. For the purpose of comparison, let us consider the situation where all users ignore the fact that they belong to two teams and choose a Nash strategy irrespective of the managers' objective functions. The resulting strategies for four users in this example x^N=0.2, y^N=0.3, u^N=0.02 and v^N=0.2 and the corresponding values of the objective functions of all four users as well as the managers are shown in Table 2.

Table 2. Nash Strategy values of the objective functions

$J_1^{H^N}$	$J_2^{H^N}$	$J_1^{T^N}$	$J_2^{T^N}$	$J_M^{H^N}$	$J_M^{T^N}$
0.5424	0.5574	0.5194	0.5424	2.78	40

In comparison, we observe that, using a Nash strategy, some, but not all, users gain in reducing their average delay time. However, considering the team managers' objective functions and using the NNSM strategy, the total throughput on link 2 for Team H is 3.3 which is higher that the 2.78 resulting from the Nash strategy. (i.e., the NNSM results in a higher efficiency of using the highest capacity link). Furthermore, we note that the objectives functions of both managers are improved by using NNSM.

4. CONCLUSION

In this paper, we formulate the routing problem in a two node parallel-link network, shared by multiple teams of users as a multi-team game with multi team managers. We applied a game theoretic control strategy, called the Noninferior Nash Strategy with a Manager (NNSM) to this routing control problem. This strategy is chosen from the set of all Noninferior Nash Strategies (NNS) for the network in such a way as to satisfy a Nash equilibrium for separate managers criteria. Other types of solution concepts, such as the Stackelberg strategy, can also be easily implemented among the team managers. We use a simple example with two teams of two users each to illustrate the fact that NNSM is effective in improving the overall network performance. We also show that using Nash strategies among the four users only without the need of team managers is generally inefficient.

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