2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Dynamic Control Allocation with Asymptotic Tracking of Time-Varying Control Input Commands

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Abstract— **In this paper, the problem of control allocation – distribution of control power among redundant control effectors, under a set of constraints – for the inner loop of a re-entry vehicle guidance and control system is studied. Our control allocation scheme extends a previously developed model-predictive algorithm by providing asymptotic tracking of time-varying control input commands. The approach accounts for non-negligible dynamics of the actuators with hard constraints, setting it apart from most existing control allocation schemes, where a static relationship between control surface deflections (actuator outputs) and moments about a three-body axis (plant inputs) is assumed. The approach is readily extended to encompass a variety of linear actuator dynamics without the need for redesign of the overall control allocation scheme, allowing for increased effectiveness of the inner loop in terms of speed of maneuverability. Simulation results, with consideration given toward implementation, are provided for an experimental reusable launch vehicle, and are compared to those of static control allocation schemes.**

I. INTRODUCTION

Modern advanced aircraft, such as hypersonic re-entry vehicles, are generally characterized by the presence of more control effectors than controlled variables. The implication of this characteristic is that the control system possesses a certain degree of redundancy, and can in principle achieve multiple control objectives and maintain certain performance when control authority is limited by failures. With this increased capability comes the requirement for (inner loop) control allocation schemes, which usually must meet some criteria for optimality, to select in real time the control configuration for the available actuators. However, it is typical for inner loop control allocation modules and outer loop closed-loop control laws to undergo separate design in order to simplify the control design and ensure the stability and robustness of the overall system.

The basic control allocation objective is to generate appropriate commands to the actuators in order to produce the desired control at the plant input. In the presence of hard constraints, the control allocation scheme must distribute the available control authority among redundant control effectors to meet the control objectives, and simultaneously satisfy the constraints (see [1], and references therein). Most existing algorithms for control allocation neglect actuator dynamics, or deal with the actuator dynamics separately. However, the presence of actuator dynamics can decrease the overall effective bandwidth of the control system, and severely degrade the closed-loop performance.

This work was supported by the AFRL/AFOSR Collaborative Center of Control Sciences at The Ohio State University.

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In [5] we introduced preliminary results for a dynamic control allocation scheme (one that included effects of linear actuator dynamics) where the effectiveness of a simple Model Predictive Control Allocation (MPCA) architecture was demonstrated. Tracking of asymptotically constant input commands was guaranteed, and simulation results were given for a high-fidelity model of a re-entry vehicle which had four control effectors (two wing flaps and two tail rudders). Building on the results of [5], this paper provides results for a new MPC-based dynamic control allocation algorithm for asymptotically stable tracking of non-constant input commands. The approach directly accounts for non-negligible dynamics of the actuators with hard constraints, setting it apart from most existing control allocation schemes where a static relationship between control surface deflections (actuator outputs) and moments about a three-body axis (plant inputs) is assumed. The simulation results of this paper also extend those of [5] in that two additional control surfaces are used in the simulation (flaps on the aircraft body itself), thereby increasing the importance of accounting for actuator dynamics, and highlighting the requirement for sophisticated control allocation. The dynamic control allocation problem for this paper is posed as a two-step mixed-optimization problem with constraints. The first step aims at finding a reference input/state trajectory for the actuator dynamics that satisfies the given constraints, while the second stage poses the problem of tracking the reference trajectory computed at the previous step as a receding-horizon quadratic optimization.

The paper is organized as follows: In Section II, a specific control architecture for RLVs is introduced, and the corresponding dynamic control allocation problem is formalized. The proposed MPCA algorithm is presented in Section III. In Section IV comparative simulation studies with a static control allocation scheme are presented.

II. CONTROL ARCHITECTURE FOR A RLV MODEL

The guidance and control architecture for reentry vehicles considered in this paper is based on the one proposed in Doman and Ngo [3], and adopted in [5]. According to [3], the control input required to let the vehicle track a given angular velocity command $\omega_{\rm cmd} = [p_{\rm cmd}\ q_{\rm cmd}\ r_{\rm cmd}]^{\rm T}$ is computed in the 3-dimensional angular acceleration space by means of a dynamic inversion-based controller. The resulting desired angular acceleration profile, $\dot{\omega}_{\rm des}$ must then be allocated within a set of redundant actuators, given in this case by six aerodynamic control surfaces. To be specific, let the aircraft rotational dynamics be given as

$$\dot{\omega} = f(\omega, \theta) + g(\delta, \theta)\,, \tag{1}$$

where $\omega = [p\ q\ r]^{\rm T}$ is the vector of angular body rates and the vector $\theta \in \mathbb{R}^p$ contains measurable or estimable time-varying parameters associated with the relevant operating point (i.e., Mach number, angle of attack, etc.) The components of the vector $\delta \in \mathbb{R}^6$ are the deflection of the control surfaces, playing the role of the control input to (1). The term $f(\omega, \theta)$ includes accelerations due to the aircraft body and engine, while $g(\delta, \theta)$ represents the control dependent accelerations. Following [4], the control dependent portion is modeled as a time-varying affine mapping of the form

$$g(\delta, \theta) = G(\theta)\delta + \epsilon(\theta) \tag{2}$$

which, with intercept correction, results in an improved approximation of the relationship between surface deflections and angular accelerations with respect to the standard linear formulation commonly adopted in the literature (see, for instance, [1]). A stable second-order linear system of the form

$$\begin{array}{rcl} \dot{x}_{\rm act} &=& A_{\rm act}x_{\rm act} + B_{\rm act}\delta_{\rm cmd} \\ \delta &=& C_{\rm act}x_{\rm act} \end{array} \tag{3}$$

is assumed for the non negligible actuator dynamics governing the control surface deflections. In (3), $x_{\rm act} = [\delta^{\rm T}\ \dot{\delta}^{\rm T}]^{\rm T}$ is the state and $\delta_{\rm cmd} \in \mathbb{R}^6$ is the commanded deflection input. Letting

$$y_{\rm des} = \dot{\omega}_{\rm des} - f(\omega, \theta) - \epsilon(\theta)\,, \tag{4}$$

the specific control allocation problem considered in this paper is that of finding the input command $\delta_{\rm cmd}$ in (3), such that the control effective mapping

$$y = G(\theta)\delta$$

satisfies $y = y_{\rm des}$, which in turn yields $\dot{\omega} = \dot{\omega}_{\rm des}$ for (1). The control effective mapping is assumed to be unto, that is, the matrix $G(\theta)$ has full row rank. Usually, hard constraints on the amplitude and rate of the commanded deflection $\delta_{\rm cmd}$, as well as on the magnitude of the angular acceleration y that can be effectively produced by the control surfaces, must be explicitly taken into account in the formulation of the problem. The redundancy provided by having a greater number of control effectors than controlled variables must then be exploited by the control allocation algorithm to obtain a solution that satisfies the given constraints whenever possible, or yield a suitable approximation when a feasible solution does not exist. Assuming that the amplitude and rate limits are given respectively as

$$\begin{array}{c} \delta_{\min} \leq \delta_{\rm cmd}(t) \leq \delta_{\max}\,, \quad \dot{\delta}_{\min} \leq \dot{\delta}_{\rm cmd}(t) \leq \dot{\delta}_{\max} \\ y_{\min} \leq y(t) \leq y_{\max}\,,\ t \geq 0\,, \end{array} \tag{5}$$

where the inequalities must be intended componentwise, the *dynamic control allocation problem* is posed as follows: Given $y_{\rm des}$ as in (4), find $\delta_{\rm cmd}$ such that the output of the system

$$\begin{array}{rcl} \dot{x}_{\rm act} &=& A_{\rm act}x_{\rm act} + B_{\rm act}\delta_{\rm cmd} \\ y &=& G(\theta)C_{\rm act}x_{\rm act}\,, \end{array} \tag{6}$$

subject to the constraints (5), tracks $y_{\rm des}$ as closely as possible.

While in most cases it is reasonable to assume that the dynamics of the actuators are known and time-invariant, that is, that the pair $(A_{\rm act}, B_{\rm act})$ is fixed over a given time interval, the matrix $G(\theta)$ should be considered time-varying to allow a more accurate representation of the control effective mapping over the envelope of flight conditions. Since the control allocation algorithm must be performed by a digital computer, we derive a sampled data equivalent of (6) in the form

$$\begin{array}{rcl} x_{k+1} &=& Ax_k + Bu_k \\ y_k &=& C_kx_k\,, \end{array} \tag{7}$$

where, to streamline the notation, we have denoted by $x_k = x(t_k) \in \mathbb{R}^{12}$ the state, by $u_k = \delta_{\rm cmd}(t_k) \in \mathbb{R}^6$ the commanded input, by $y_k = y(t_k) \in \mathbb{R}^3$ the acceleration produced by the actuators, and by $r_k = y_{\rm des}(t_k)$ the reference to be tracked. The output matrix of (7) is given by $C_k = G(\theta_k)C_{\rm act}$. Since rate limits on continuous-time signals can be expressed as equivalent amplitude limits for their sampled-data version (see [3]), there is no loss of generality in limiting ourselves to consider in the sequel constraints on the amplitude of u_k and y_k in the form $u_{\min} \leq u_k \leq u_{\max}$ and $y_{\min} \leq y_k \leq y_{\max}$. Equivalently, for the sake of brevity, we will refer to the above constraints as $u_k \in \mathcal{U} \subset \mathbb{R}^6$ and $y_k \in \mathcal{Y} \subset \mathbb{R}^3$, where $\mathcal{U}$ and $\mathcal{Y}$ are polytopes. The model (7) is assumed to satisfy the well-posedness condition:

Assumption 1: (A, B) is controllable, and (A, C_k) is observable for all $k \geq 0$.

III. Model Predictive Control Allocation

A. Target calculation

Since the reference trajectory r_k is time-varying, the first step in solving the dynamic control allocation problem is to find a suitable inverse of the actuator dynamics, in such a way that a feasible target control input u_k^r and a corresponding target state trajectory x_k^r is made available to the control allocator. We assume that the reference trajectory is available over a given interval:

Assumption 2: At each time $k \geq 1$ the sequence $\{r_{k+i}\}_{i=0}^{N-1}$ is known, where $N > 1$ is the length of the prediction horizon.

The *target calculation problem* is then stated as follows: For any $k \geq 1$, given the sequence $\{r_{k+i}\}_{i=0}^{N-1}$ and the initial state x_{k-1}^r, find $\{u_{k+i-1}^r\}_{k=0}^{N-1}$ such that the corresponding solution of

$$\begin{array}{rcl} x_{k+1}^r &=& Ax_k^r + Bu_k^r \\ y_k &=& C_kx_k^r\,, \end{array} \tag{8}$$

yields $y^r_{k+i} = r_{k+i}$, and satisfies $u^r_{k+i-1} \in \mathcal{U}$, $y^r_{k+i} \in \mathcal{Y}$, for all $i = 0, \ldots, N-1$. Once a solution of the target calculation problem is available, then by means of the simple coordinate transformation $\tilde{x}_k = x_k - x^r_k$, $\tilde{u}_k = u_k - u^r_k$, the tracking problem for (7) is cast as the problem of stabilizing the equilibrium $\tilde{x} = 0$ of the *error system*

$$\begin{aligned} \tilde{x}_{k+1} &= A\tilde{x}_k + B\tilde{u}_k \\ \tilde{y}_k &= C_k \tilde{x}_k \,, \end{aligned} \tag{9}$$

where $\tilde{y}_k = y_k - r_k$, subject to the constraints

$$\begin{aligned} u_{\min} - u^r_k \le \tilde{u}_k \le u_{\max} - u^r_k \\ y_{\min} - y^r_k \le \tilde{y}_k \le y_{\max} - y^r_k \,. \end{aligned} \tag{10}$$

There are several issues that need to be addressed regarding the feasibility of both the target calculation problem and the stabilization of the constrained system (9). In order to guarantee that the origin of (9) is in the interior of the feasible set of the time-varying constraints (10) for all feasible sequences $\{u^r_k, y^r_k\}$, the constraints of the target calculation problem are tighten by means of positive offsets, that is, the original set of constraints are replaced by

$$\begin{aligned} u^r_k \in \bar{\mathcal{U}} = \{u : u_{\min} + \Delta u_{\min} \le u \le u_{\max} - \Delta u_{\max}\} \\ y^r_k \in \bar{\mathcal{Y}} = \{y : y_{\min} + \Delta y_{\min} \le y \le y_{\max} - \Delta y_{\max}\} \end{aligned}$$

where the components of the vectors $\Delta u_{\min}, \ldots, \Delta y_{\max}$ are strictly positive. Secondly, the target calculation problem may not have a solution (u^r_{k+i-1}, x^r_{k+i}) that does not temporarily violate the given constraints, even if the reference satisfies $r_{k+i} \in \bar{\mathcal{Y}}$ for all $i \in [0, N-1]$. As a matter of fact, the reference is not even guaranteed to satisfy $r_k \in \bar{\mathcal{Y}}$ for all $k \ge 1$, as r_k is the output of the dynamic inversion module, which may at times require unattainable control efforts. It is precisely the role of the target calculation algorithm to compute a "steady-state solution" of the actuator dynamics that achieves a compromise between fidelity of response and exact fulfillment of the constraints.

Assumption 3: Given a feasible reference sequence $\{r_k\}_{k=0}^{\infty} \subset \bar{\mathcal{Y}}$, there exists a number $\bar{k} \in \mathbb{N}$, an input sequence u^r_k and an initial condition x^r_0 such that the solution $x^r_k = x(k, x^r_0, u^r_{[0,k)})$ of (7) satisfies $r_k = C_k x^r_k$ and $u^r_k \in \mathcal{U}$ for all $k \ge \bar{k}$.

This assumption is needed to guarantee that the problem is indeed solvable, in the sense there exists an input/state trajectory for (8) that ultimately is feasible and reproduces the given reference. Since $\dim(u^r_k) > \dim(y^r_k)$, the required input/state reference trajectory may not be uniquely defined if a feasible solution exists. For this reason, it is convenient to cast the target calculation as a constrained optimization problem.

Assumption 4: The sampled-data equivalent of (3) has relative degree one, that is, the matrix $C_{\text{act}}B \in \mathbb{R}^6$ is nonsingular.

The above assumption is easily verified for the second-order actuator model considered in this paper, due to the zero-order hold introduced with the sampling of the continuous-time model. This in turn implies that the matrix $C_k B = G(\theta_k) C_{\text{act}} B$ is full rank for any k. To find the state/input target sequence in each moving interval $[k, k+N-1]$, we employ the following recursive algorithm. Assigning the initial state $x^r_0 = 0$, the solution of (8) satisfies at each step

$$y^r_k = C_k \sum_{i=0}^{k-1} A^{k-1-i} B u^r_i \,.$$

Since the matrix $C_k B$ has full rank, the target trajectory over the first moving interval $\mathcal{I}_0 = [0, N-1]$ is computed solving iteratively N systems of underdetermined equations of the form

$$\begin{aligned} &C_k B u^r_{k-1} = \mathbf{r}_k \\ &\text{subject to } \; u^r_{k-1} \in \bar{\mathcal{U}}, \; y^r_k \in \bar{\mathcal{Y}} \,, \end{aligned} \tag{11}$$

where $\mathbf{r}_k = r_k - C_k \sum_{i=0}^{k-2} A^{k-1-i} B u^r_i$. The state trajectory $\{x^r_k\}_{k=1}^{N}$ is then computed from the sequence $\{u^r_k\}_{k=0}^{N-1}$ by means of the state propagation equation. The reference trajectory over any subsequent moving interval $\mathcal{I}_k = [k, k+N]$, $k \ge 1$, is obtained simply adding the solution of

$$C_{k+N} B u^r_{k+N-1} = \mathbf{r}_{k+N} \,, \text{ subject to } u^r_{k-1} \in \bar{\mathcal{U}}, \; y^r_k \in \bar{\mathcal{Y}}$$

to the sequence $\{u^r_i\}_{i=k-1}^{k+N-2}$ already available at the previous step. To resolve possible infeasibility, some constraints must necessarily be relaxed through the introduction of an appropriate number of slack variables. Assuming that the preference is given to the input constraints, a convenient way of solving (11) is looking at the solution of the Linear Programming (LP) problem

$$\min_{u^r,\, y^s,\, r^s} J^{\text{tar}}_k = \begin{bmatrix} w_1^{\mathrm{T}} & w_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} r^s_k \\ y^s_k \end{bmatrix} \tag{12}$$

subject to

$$\begin{bmatrix} -r^s_k \\ -y^s_k \\ u^r_{k-1} \\ -u^r_{k-1} \\ C_k B u^r_{k-1} - r^s_k \\ -C_k B u^r_{k-1} - r^s_k \\ y^r_k - y^s_k \\ -y^r_k - y^s_k \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ u_{\max} - \Delta u_{\max} \\ -u_{\min} - \Delta u_{\min} \\ \mathbf{r}_k \\ -\mathbf{r}_k \\ y_{\max} - \Delta y_{\max} \\ -y_{\min} - \Delta y_{\min} \end{bmatrix},$$

where r^s, $y^s_k \in \mathbb{R}^3$ are vectors of nonnegative slack variables, and w_1, $w_2 \in \mathbb{R}^3$ are vectors of positive weights. The LP problem corresponds to the minimization of the functional

$$\min_{u^r} J^{\text{tar}}_k = \| w_1^{\mathrm{T}} (C_k B u^r_{k-1} - \mathbf{r}_k) \|_1$$

subject to a set of mixed constraints. In case $J^{\text{tar}}_k > 0$, the problem is not feasible, and the solution of (12) yields the best approximation in terms of the 1-norm of the error that does not violate the limits on the commanded input. If $J^{\text{tar}}_k = 0$, the problem is feasible and it may be possible to further exploit the non uniqueness of the solution to satisfy an additional performance requirement. This latter is posed as that of driving the commanded deflections u^r_{k-1} towards the "preferred" value u^p_{k-1} that solves the unconstrained weighted minimum energy problem

$$u^p_{k-1} = W^{-1}B^{\mathrm{T}}C_k^{\mathrm{T}}(C_kB\,W^{-1}B^{\mathrm{T}}C_k^{\mathrm{T}})^{-1}\mathbf{r}_k\,,$$

where $W > 0$ is a weighting matrix. The sub-objective function is chosen as the weighted 1-norm problem

$$\begin{aligned} &\min_{u^r_{k-1}} J^{\mathrm{p}}_k = \|w_3^{\mathrm{T}}(u^r_{k-1} - u^p_{k-1})\|_1 \\ &\text{subject to } C_kBu^r_{k-1} = \mathbf{r}_k\,,\ u^r_{k-1} \in \bar{\mathcal{U}}\,,\ y^r_k \in \bar{\mathcal{Y}} \end{aligned} \tag{13}$$

where $w_3 \in \mathbb{R}^{12}$ is a vector of positive weights to allow flexibility in the design. The sub-objective (13) is again formulated as the LP problem

$$\min_{u^r,\,u^s} J^{\mathrm{p}}_k = w_3^{\mathrm{T}}u^s_{k-1}$$

subject to

$$\begin{bmatrix} -u^s_{k-1} \\ u^r_{k-1} \\ -u^r_{k-1} \\ u^r_{k-1} - u^s_{k-1} \\ -u^r_{k-1} - u^s_{k-1} \end{bmatrix} \le \begin{bmatrix} 0 \\ u_{\max} \\ -u_{\min} \\ u^p_{k-1} \\ -u^p_{k-1} \end{bmatrix}, \quad C_kBu^r_{k-1} = \mathbf{r}_k$$

being $u^s_{k-1} \in \mathbb{R}^{12}$ a vector of slack variables. To reduce the computational demand, the two branches (11) and (13) of the target calculation may be combined into a single *mixed optimization problem*, involving the minimization of the functional

$$\min_{u^r_{k-1}} J^{\mathrm{m}}_k = \|w_1^{\mathrm{T}}(C_kBu^r_{k-1} - \mathbf{r}_k)\|_1 + \lambda\|w_3^{\mathrm{T}}(u^r_{k-1} - u^p_{k-1})\|_1\,,$$

where $\lambda > 0$ allows weighting between the main objective and the performance subobjective. The solution of the target optimization provides reference states, reference inputs, and reference output $y^r_k = C_kx^r_k$ for the actuator dynamics (7). The optimization problem guarantees that the reference output trajectory y^r_k remains close (in the 1-norm sense) to the actual reference r_k when the constraints are active. The reference trajectory (x^r_k, u^r_k) is then employed in the second stage of the dynamic control allocation algorithm, where a control that asymptotically tracks y^r_k is computed using a standard MPC algorithm.

B. Model-predictive tracking control

After the target input and state sequences have been computed, the control allocation problem at time k is posed as a standard sequential model-predictive tracking problem, equivalent to the stabilization of the origin of (9). Specifically, we consider the minimization of the cost function

$$\begin{aligned} J(x_k, k) = \sum_{i=0}^{N-1}\{&(y_{k+i|k} - y^r_{k+i})^{\mathrm{T}}Q(y_{k+i|k} - y^r_{k+i}) \\ &+ (u_{k+i} - u^r_{k+i})^{\mathrm{T}}R(u_{k+i} - u^r_{k+i})\} \\ &+ (x_{k+N|k} - x^r_{k+N})^{\mathrm{T}}\Psi_{k+N}(x_{k+N|k} - x^r_{k+N}) \end{aligned}$$

where $Q > 0$, $R > 0$, and $\Psi_i > 0$ for all $i \ge 0$. The variable $x_{k+i|k}$ is the predicted state of (7) at the step k, with $x_{k|k} = x_k$. Defining the error variables for the prediction model as

$$\bar{x}_{k+i} = x_{k+i|k} - x^r_{k+i}\,, \quad \bar{u}_{k+i} = u_{k+i} - u^r_{k+i}\,,$$

the cost function is rewritten as

$$\begin{aligned} J(\bar{x}_k, k) = \sum_{i=0}^{N-1}&(\bar{x}^{\mathrm{T}}_{k+i}Q_{k+i}\bar{x}_{k+i} + \bar{u}^{\mathrm{T}}_{k+i}R\bar{u}_{k+i}) \\ &+ \bar{x}^{\mathrm{T}}_{k+N}\Psi_{k+N}\bar{x}_{k+N} \end{aligned} \tag{14}$$

where $Q_i = C_i^{\mathrm{T}}QC_i$, and the problem is cast as the minimization of (14), subject to:

$$\begin{aligned} &\bar{x}_{k+i+1} = A\bar{x}_{k+i} + B\bar{u}_{k+i} \\ &D\bar{u}_{k+i} \le d_{k+i},\ E_{k+i}\bar{x}_{k+i} \le e_{k+i}, \end{aligned} \tag{15}$$

for all $i = 0, \cdots, N-1$, being

$$D = \begin{bmatrix} I \\ -I \end{bmatrix}, \quad E_{k+i} = \begin{bmatrix} C_{k+i} \\ -C_{k+i} \end{bmatrix},$$

$$d_{k+i} = \begin{bmatrix} u_{\max} - u^r_{k+i} \\ -u_{\min} + u^r_{k+i} \end{bmatrix},\ e_{k+i} = \begin{bmatrix} y_{\max} - y^r_{k+i} \\ -y_{\min} + y^r_{k+i} \end{bmatrix}.$$

The optimal control sequence $u^*_{k+i|k} = \bar{u}_{k+i} + u^r_{k+i}$ yields the implicit model predictive control allocation policy $\delta_{\mathrm{cmd}}(t_k) = u^*_{k|k}$. The optimization problem (14) may be infeasible or difficult to solve due to the given constraints on the input and output trajectories. The problem can be alleviated by relaxing the hard constraint for the state, and replacing (15) with the mixed constraints

$$\begin{aligned} &D\bar{u}_{k+i} \le d_{k+i} \\ &E_{k+i}\bar{x}_{k+i} \le e_{k+i} + \varepsilon_k, \quad i = 0, \cdots, N-1\,, \end{aligned} \tag{16}$$

where it is assumed that $\bar{u}_k = 0$ and $\bar{x}_k = 0$ is feasible. The cost function of the optimization problem is augmented with a penalty on the slack variables ε_k as

$$J_{\mathrm{constr}}(\bar{x}_k, \varepsilon_k, k) = J(\bar{x}_k, k) + \varepsilon_k^{\mathrm{T}}S\varepsilon_k\,, \tag{17}$$

where $S > 0$, and the mixed-constrained MPCA problem is posed as the minimization of (17), subject to the constraints given by (16), and the first equality in (15). Since the tracking problem in III-B is posed as a model predictive control with soft constraints on the state, and $(\bar{x}, \bar{u}) = (0, 0)$ lies in the interior of the feasible set, asymptotic stability of the the actuator dynamics suffices to guarantee feasibility of the optimization problem, and asymptotic convergence to the origin [7], [8]. In any case, a careful selection of the weights of the objective function is instrumental in improving the controller performance. Several methods exist to determine asymptotic convergence of model-predictive control algorithms through an appropriate selection of the terminal constraint (see the excellent survey [6], and references therein).

IV. Simulation Results

In this section we present simulation results for the MPC-based control allocation scheme. As a baseline for comparison, we also present results for an established *static* control allocation technique (that is, the design does not explicitly account for inclusion of actuator dynamics) developed by Doman et al. [4]. As a simulation testbed for these comparative studies, we adopt the six-degree of freedom reentry vehicle model in [3], which operates with a dynamic inversion based control architecture. The model has six control surfaces: right flap, left flap, right tail, left tail, body flap and speed brake, with upper deflection limit $30°$, lower deflection limit $-30°$ for the first four control surfaces, and rate limit $60°/s$ for all control surfaces under nominal conditions. The speed brake has an upper limit of $70°$ and a lower limit of $0°$, and the body flap has an upper limit of $25°$ and a lower limit of $-20°$.

A. Baseline Static Control Allocation

In most traditional control allocation schemes the actuator dynamics are neglected, since the dynamics of the actuators are assumed to be relatively fast compared with that of the aircraft. In this case, $\delta_{\rm cmd} = \delta$, and the *static control allocation problem* is that of finding δ such that

$$u_{\rm des} = G(t)\delta \,, \text{ subject to } \; \delta_{\min} \leq \delta \leq \delta_{\max} \,.$$

As a baseline static CA algorithm, we choose the Mixed Optimization with Intercept Correction (MOIC) scheme of [4], in which the control effective mapping is originally considered affine as in (2). Building upon the idea in [2], the MOIC problem is formulated as a 1-norm optimization problem of the form

$$\begin{aligned} &\min_{\delta} J = \|u_{\rm des} - G\delta\|_1 \\ &\text{subject to:} \quad \delta_{\min} \leq \delta \leq \delta_{\max} \,. \end{aligned} \tag{18}$$

If the optimal solution is such that $J_d = 0$, excess control authority can be utilized to optimize additional objectives, defined in terms of a the following 1-norm optimization

$$\begin{aligned} &\min_{\delta} J_S = \min_{\delta} \|W_p^{\rm T}(\delta - \delta_p)\|_1 \,, \\ &\text{subject to:} \quad u_{\rm des} = G\delta \,, \quad \underline{\delta} \leq \delta \leq \overline{\delta} \,, \end{aligned}$$

where $W_p \in \mathbb{R}^6$ is a vector of weights, and δ_p is a preferred value for the control surface deflection, determined on the basis of selected performance objectives such as drag minimization or minimum energy [2].

B. Comparative Simulation Studies

Typical tests for the control allocation schemes use a standard maneuver reference trajectory (variation in pitch) and include a wide variation on the dynamical characteristics of the six actuators in terms of damping ratio and natural frequency. Each actuator is assumed to behave as a second-order system with amplitude and rate limits, and damping ratio ranging from 0.5 to 0.7. Different natural frequencies, ranging from 20Hz and 5Hz, have been independently assigned and tested for each pair of tail and wing effectors, whereas a range from 10Hz and 5Hz is studied for the two body effectors. The feasible reference trajectory chosen for these tests is relatively benign, exhibiting change in only in the pitch motion (see Figure 1), corresponding to an approach and landing maneuver. The segment of command trajectory employed in the simulations has duration equal to 49 seconds. Table I displays results from several tests in which the damping and natural frequencies are varied for the tail effectors (right and left flaps), wing effectors (right and left flaps), and the body effectors (flap and speed brake). As performance metrics in comparing large numbers of tests, we typically calculate the mean square error (MSE) and the maximum error with respect to the reference trajectory over the entire test interval. The sample results given in Table I indicate that the error metrics for the MPC based control allocation scheme are approximately one order of magnitude smaller than those of the MOIC, depending on the test conditions. To illustrate performance for a typical test taken from the table, Figure 2 shows in the first plot the pitch tracking capabilities of the MPC and MOIC for the specific case in which the natural frequency of the tail effectors is 12 Hz, for the wing effectors is 8 Hz, whereas for the body effectors the natural frequency is 5 Hz. The damping ratio is $\zeta = 0.5$ for all effectors. The remaining plots in Figure 2 show the responses of the roll and yaw motions of the vehicle under these conditions. Recall that the reference trajectory specifies zero roll and yaw motions, but the MOIC scheme cannot track that request for the entire test. For this same test, Figure 3 shows the motion of the control surface deflections where distinctive differences in the action of the two schemes is

TABLE I

Performance comparison

ζ	ω_n			Maximum Error		MSE	
	W	T	B	MPC	MOIC	MPC	MOIC
0.7	20	12	10	1.81e-2	6.26e-2	3.07e-3	1.32e-2
0.5	20	12	10	1.80e-2	5.64e-2	3.20e-3	1.22e-2
0.7	12	8	5	2.51e-2	7.46e-2	5.03e-3	1.53e-2
0.5	12	8	5	3.43e-2	7.44e-2	5.66e-3	2.47e-2
0.7	10	8	5	3.41e-2	7.46e-2	5.63e-3	2.53e-2
0.5	10	8	5	3.42e-2	7.99e- 02	5.70e-3	2.92e-2

W: wing effectors, T: tail effectors, B: body effectors.
ζ: damping ratio, ω_n: natural frequency (Hz). MSE: mean-square error.

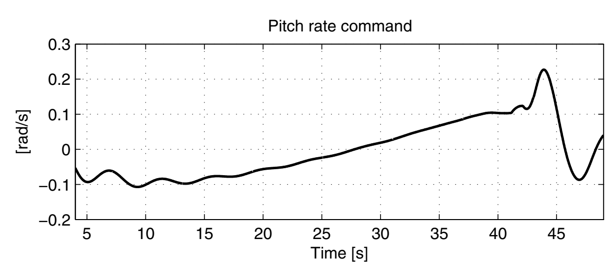

Fig. 1. Pitch rate commands

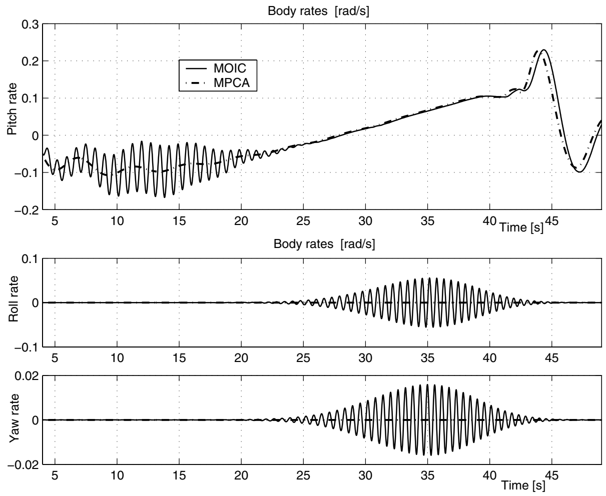

Fig. 2. Picth, roll and yaw rates

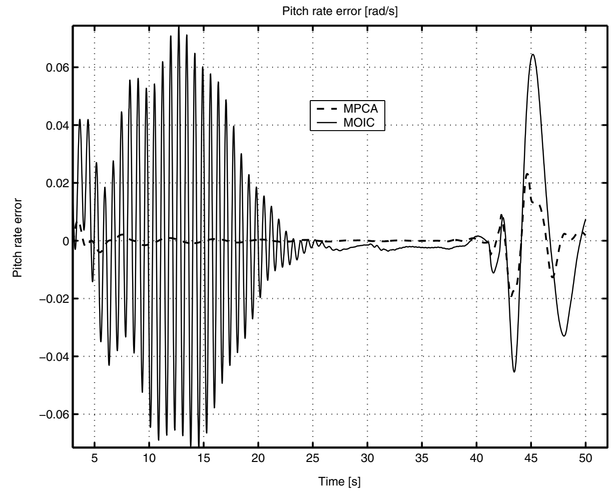

Fig. 4. Pitch rate error

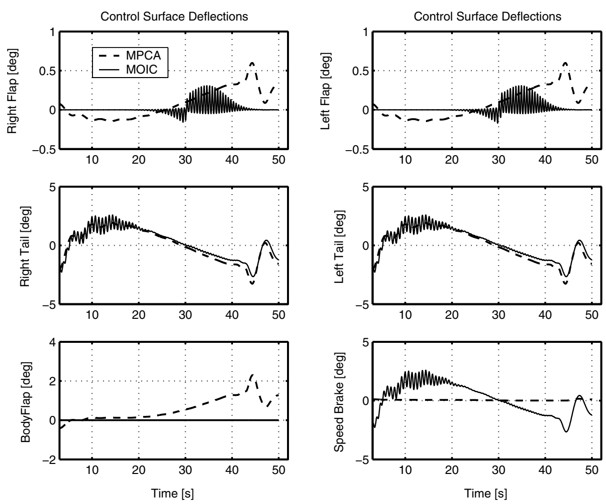

Fig. 3. Control surface deflections

apparent, whereas Figure 4 exhibits the relative pitch errors.

V. CONCLUSION

A dynamic control allocation scheme for asymptotically stable tracking of time-varying reference trajectories has been proposed to account for actuator dynamics and constraints in the inner loop of a re-entry vehicle guidance and control system. Building on the results of [5], this paper provides a formal proof of stability for the asymptotic tracking result, and offers control design guidelines for re-entry vehicle applications. A time-varying affine internal model, based on a high-fidelity simulation of an experimental re-entry vehicle, has been used in a model-predictive control design. The proposed scheme provides a generic approach to distribute control authority among different types of actuators. Extensive simulation studies indicate that the proposed approach shows significant performance improvement over traditional static control allocation algorithms in the presence of realistic actuator dynamics. Other advantages of the dynamic control allocation method proposed in this paper include the scheme's ability to deal with different types of actuators exhibiting a time-scale separation into fast and slow dynamics. Moreover, preliminary studies have shown that a large class of failure conditions can also be dealt with by the proposed algorithm, without the need to redesign the control allocation scheme. Such results, that will appear in an upcoming paper, render the proposed methodology extremely appealing for reconfigurable control. General issues regarding robust on-line identification of the control effecting mapping, and the application of recently developed robust MPC techniques to deal with uncertain actuator models are currently under investigation.

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