

MULTISENSOR TRACKING OF MULTIPLE MANEUVERING TARGETS USING MULTISCAN JPDA AND IMM FILTERING

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ABSTRACT

We present a novel suboptimal filtering algorithm developed by applying a multiscan joint probabilistic data association (Mscan-JPDA) technique and interacting multiple model (IMM) filtering approach. In the past this algorithm has been restricted to single sensor tracking which we now extend to a multiple sensor tracking scenario. Multiscan JPDA addresses the problem of measurement-to-track data association for multiple targets and clutter. An IMM approach is exploited to consider the tracking of highly maneuvering targets. The proposed algorithm is illustrated via a simulation example involving tracking of two maneuvering targets by two sensors (radar and infrared) and a multiscan data window of length two.

I INTRODUCTION

We consider the problem of multisensor tracking of multiple maneuvering targets in presence of clutter using switching multiple target motion models. In the Bayesian framework the standard JPDA algorithm uses only a single (latest) scan data available at the sensors. To use more information to solve the data association problem, multiple scans of data (current and past scans) have been used in [3] where only non-maneuvering targets (i.e. one model per target) have been considered. In [4], Mscan-JPDA technique was extended for multiple maneuvering targets where a filtering algorithm was developed by applying the basic interacting multiple model (IMM) approach and multiple scan joint probabilistic data association (Mscan-JPDA) technique. But the algorithm in [4] is applicable only to single sensor tracking. In this paper, we extend the algorithm in [4] to multisensor scenario.

In a single scan JPDA we define single scan joint events. Similarly in the multiscan scenario we define multiple-scan joint events as follows. A marginal association event $\theta_{ir}(k)$ is said to be effective at time scan k when the validated measurement $y_k^{(i)}$ is associated with (i.e. originates from) target r ($r = 0, 1, \dots, N$ where $r = 0$ means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event Θ_k is said to be effective when a set of marginal events $\{\theta_{ir}(k)\}$ holds true simultaneously. That is, $\Theta_k = \bigcap_{i=1}^m \theta_{ir_i}(k)$ where r_i is the index of the target to which measurement $y_k^{(i)}$ is associated in the event under consideration, ($i = 1, 2, \dots, m$). In the multiscan case with a scan window size L (L -scan-back) and $k_s = k - L + s$, we define multiscan joint events

$$\Theta_{kL} = \bigcap_{s=1}^L \bigcap_{i=1}^m \theta_{ir_{is}}(k_s)$$

where $\theta_{ir_{is}}(k_s)$ is the marginal association event that at time scan k_s , i th the validated measurement $y_{k_s}^{(i)}$ is associated with target r_{is} . The number of feasible multiscan joint events grow exponentially with multiscan window length, and even for a multiscan window of length two or three, this number can be large.

II PROBLEM FORMULATION

Assume that there are total N targets with the target set denoted as $\mathcal{T}_N := \{1, 2, \dots, N\}$. Assume that the dynamics of each target can be modeled as one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \dots, n\}$. For target r ($r \in \mathcal{T}_N$), the event that model i is in effect during the sampling period

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$(t_{k-1}, t_k]$ will be denoted by $M_k^i(r)$. For the j -th hypothesized model (mode), the state dynamics and measurements of target r ($r \in \mathcal{T}_N$) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r), \quad (1)$$

$$z_k^l(r) = h^{j,l}(x_k(r)) + w_k^{j,l}(r) \quad \text{for } l = 1, \dots, q, \quad (2)$$

where $x_k(r)$ is the system state of target r at t_k and of dimension n_x (assuming all targets share a common state space), $z_k(r)$ is the (true) measurement vector (i.e. due to target r) from sensor l at t_k and of dimension $n_{z,l}$, $F_{k-1}^j(r)$ and $G_{k-1}^j(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r and $h^{j,l}$ is the nonlinear transformation of $x_k(r)$ to $z_k^l(r)$ ($l = 1, 2, \dots, q$) for model j . A first-order linearized version of (2) is given by

$$z_k^l(r) = H_k^{j,l}(r)x_k(r) + w_k^{j,l}(r) \quad (3)$$

where $H_k^{j,l}(r)$ is the Jacobian matrix of $h^{j,l}$ evaluated at some value of the estimate of state $x_k(r)$ (see Sec. III). The process noise $v_{k-1}^j(r)$ and the measurement noise $w_k^{j,l}(r)$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1}^j (same for all targets) and $R_k^{j,l}$ (same for all targets), respectively. At the initial time t_0 , the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j(r)$ and the known covariance $P_0^j(r)$. The probability of target r in model j at t_0 , $\mu_0^j(r) = P\{M_0^j(r)\}$, is also assumed to be known. The switching from model $M_{k-1}^i(r)$ to model $M_k^j(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P\{M_k^j(r)|M_{k-1}^i(r)\}$. Henceforth, t_k will be denoted by k .

Note that, in general, at any time k , some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time k . The measurement set (not yet validated) generated by sensor l at time k is denoted as $Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \dots, z_k^{l(m_l)}\}$ where m_l is the number of measurements generated at time k . Variable $z_k^{l(i)}$ ($i = 1, \dots, m_l$) is the i th measurement within the set. The validated set of measurements from sensor l at time k will be denoted by Y_k^l , containing \bar{m}_l ($\leq m_l$) measurement vectors. The cumulative set of validated measurements from sensor l up to time k is denoted as $\mathcal{Z}^{k(l)} = \{Y_1^l, Y_2^l, \dots, Y_k^l\}$. The cumulative set of validated measurements from all sensors up to time k is denoted as $\mathcal{Z}^k = \{\mathcal{Z}^{k(1)}, \mathcal{Z}^{k(2)}, \dots, \mathcal{Z}^{k(q)}\}$ where q is the number of sensors.

We make the same standard assumptions as detailed in [4]. Multiscan window of length two will be used to compute multiscan joint probabilities. Extension to higher lengths is straightforward but tedious. The goal is to find the filtered state estimate for target r ($r \in \mathcal{T}_N$)

$$\hat{x}_{k|k}(r) = E\{x_k(r)|\mathcal{Z}^k\} \quad (4)$$

and the associated error covariance matrix

$$P_{k|k}(r) = E\{[x_k(r) - \hat{x}_{k|k}(r)][x_k(r) - \hat{x}_{k|k}(r)]'|\mathcal{Z}^k\} \quad (5)$$

where $x_k(r)'$ denotes the transpose of $x_k(r)$.

III IMM/MSCAN-JPDA FILTERING ALGORITHM

We now extend the multiscan IMM/JPDA filtering algorithm of [4] to apply to the multisensor case. As in [3] we will follow a sliding window multiscan approach. We will only briefly outline the basic steps in “one cycle” (i.e. processing needed to update for a new set of measurements and a new multiscan window) of the IMM/JPDA multiscan filter. We assume that the scan window size is two. Given state estimate at time $k-1$ based on data up to time $k-1$, in Sec. III.1 we provide first scan steps (using data up to time k) and in Sec. III.2 we provide the second scan steps (using data up to time $k+1$).

Assumed available: Given the state estimate $\hat{x}_{k-1|k-1}^j(r)$:= $E\{x_{k-1}(r)|M_{k-1}^j(r), \mathcal{Z}^{k-1}\}$, the associated covariance $P_{k-1|k-1}^j(r)$ and the conditional mode probability $\mu_{k-1}^j(r) = P[M_{k-1}^j(r)|\mathcal{Z}^{k-1}]$ at time $k-1$ for each mode $j \in \mathcal{M}_n$ and each target $r \in \mathcal{T}_N$.

III.1 First Scan Steps

These mimic [4] except that now we have 2 sensors.

Step 1.1. Interaction – mixing of the estimate from the previous time ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): The expressions for the predicted mode probability $\mu_k^{j-}(r) := P\{M_k^j(r)|\mathcal{Z}^{k-1}\}$ and the mixing probability $\mu^{ij}(r) := P\{M_{k-1}^i(r)|M_k^j(r), \mathcal{Z}^{k-1}\}$ are as in [2, Sec. 4.2]. Similarly, the expressions for the mixed estimate $\hat{x}_{k-1|k-1}^{0j}(r) := E\{x_{k-1}(r)|M_k^j(r), \mathcal{Z}^{k-1}\}$ and the associated covariance $P_{k-1|k-1}^{0j}(r) := E\{[x_{k-1}(r) - \hat{x}_{k-1|k-1}^{0j}(r)][x_{k-1}(r) - \hat{x}_{k-1|k-1}^{0j}(r)]'|M_k^j(r), \mathcal{Z}^{k-1}\}$ are as in [2, Sec. 4.2].

Step 1.2. Predicted state ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): State prediction:

$$\hat{x}_{k|k-1}^j(r) := E\{x_k(r)|M_k^j(r), \mathcal{Z}^{k-1}\} = F_{k-1}^j \hat{x}_{k-1|k-1}^{0j}(r). \quad (6)$$

State prediction error covariance:

$$P_{k|k-1}^j(r) = F_{k-1}^j P_{k-1|k-1}^{0j}(r) F_{k-1}^{j'} + G_{k-1}^j Q_{k-1}^j G_{k-1}^{j'}. \quad (7)$$

The mode-conditioned predicted measurement of target r , $\hat{z}_k^{j,1}(r)$, and the covariance $S_k^{j,1}(r)$ of the mode-conditioned residual $\nu_k^{j,1(i)}(r) := z_k^{1(i)} - \hat{z}_k^{j,1}(r)$ are as in [2].

Step 1.3. Measurement validation for sensor 1: This step is exactly the same as the Step 1.3 in [2]. The volume of validation region for the whole target set is approximated by $V_k^1 = \sum_{r=1}^N V_k^1(r)$ where $V_k^1(r)$ is the validation region for target r .

Step 1.4. State estimation with validated measurements from sensor 1 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): From among all the raw measurements at time k , i.e., $Z_k^1 := \{z_k^{1(1)}, z_k^{1(2)}, \dots, z_k^{1(m_1(k))}\}$, define the set of validated measurement for sensor 1 at time k as $Y_k^1 := \{y_k^{1(1)}, y_k^{1(2)}, \dots, y_k^{1(\bar{m}_1(k))}\}$ where $\bar{m}_1(k)$ is the total number of validated measurement at time k and $y_k^{1(i)} := z_k^{1(l_i)}$ where $1 \leq l_1 < l_2 < \dots < l_{\bar{m}_1(k)} \leq m_1(k)$ when $\bar{m}_1(k) \neq 0$. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \dots, \bar{m}_1(k), \quad r = 0, \dots, N \quad (8)$$

where $\omega_{ir} = 1$ if the measurement i lies in the validation gate of target r , else it is zero. A joint association event Θ_k^1 is represented by the event matrix

$$\hat{\Omega}(\Theta_k^1) = [\hat{\omega}_{ir}(\Theta_k^1)] \quad i = 1, \dots, \bar{m}_1(k), \quad r = 0, \dots, N \quad (9)$$

where

$$\hat{\omega}_{ir}(\Theta_k^1) = \begin{cases} 1 & \text{if } \theta_{ir}(k) \subset \Theta_k^1 \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

A feasible association event is one where a measurement can have only one source $\sum_{r=0}^N \hat{\omega}_{ir}(\Theta_k^1) = 1 \quad \forall i$, and where at

most one measurement can originate from a target $\delta_r(\Theta_k^1) := \sum_{i=0}^{\bar{m}_1(k)} \hat{\omega}_{ir}(\Theta_k^1) \leq 1$ for $r = 1, \dots, N$. The above joint events Θ_k^1 are mutually exclusive and exhaustive.

Following [1], define the binary measurement association indicator $\tau_i(\Theta_k^1) := \sum_{r=1}^N \hat{\omega}_{ir}(\Theta_k^1)$, $i = 1, \dots, \bar{m}_1(k)$, to indicate whether the validated measurement $y_k^{1(i)}$ is associated with a target in event Θ_k^1 . Furthermore, the number of false (unassociated) measurements in event Θ_k^1 is $\phi(\Theta_k^1) = \sum_{i=1}^{\bar{m}_1(k)} [1 - \tau_i(\Theta_k^1)]$. We will limit our discussion to nonparametric JPDA [1]. One can evaluate the likelihood that the target r is in model j_r as

$$\begin{aligned} \Lambda_k^{j_r,1}(r) &:= p[Y_k^1 | M_k^{j_r}(r), \mathcal{Z}^{k-1}] \\ &= \sum_{\Theta_k^1} p[Y_k^1 | \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}] P\{\Theta_k^1\}. \end{aligned} \quad (11)$$

The first term in the last line of (11) can be written as

$$\begin{aligned} p[Y_k^1 | \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}] &= \sum_{j_1=1}^n \dots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \dots \sum_{j_N=1}^n \\ & p[Y_k^1 | \Theta_k^1, M_k^{j_1}(1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}] \times P\{M_k^{j_1}(1), \dots, \\ & M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \dots, M_k^{j_N}(N) | \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}\}. \end{aligned} \quad (12)$$

The second term in the last line of (11) turns out to be ([1, Sec. 6.2])

$$P\{\Theta_k^1\} = \frac{\phi(\Theta_k^1)! \epsilon^N}{\bar{m}_1(k)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_k^1)} (1 - P_D)^{1 - \delta_s(\Theta_k^1)} \quad (13)$$

where P_D is the detection probability (assumed to be the same for all targets) and $\epsilon > 0$ is a “diffuse” prior (for nonparametric modeling of clutter) whose exact value is irrelevant. We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term on the right-side of (12) is

$$p[Y_k^1 | \Theta_k^1, M_k^{j_1}(1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}]$$

$$\begin{aligned} &\approx \prod_{i=1}^{\bar{m}_1(k)} p[y_k^{1(i)} | \theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}], \quad \theta_{ir_i}(k) \subset \Theta_k^1, \end{aligned} \quad (14)$$

where the conditional pdf (probability density function) of the validated measurement $y_k^{1(i)}$ given its origin and target mode, is given by

$$\begin{aligned} &p[y_k^{1(i)} | \theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}] = \\ &\begin{cases} \mathcal{N}(y_k^{1(i)}; \hat{z}_k^{j_{r_i},1}(r_i), S_k^{j_{r_i},1}(r_i)) & \text{if } \tau_i(\Theta_k^1) = 1, \\ 1/V_k^1 & \text{if } \tau_i(\Theta_k^1) = 0 \end{cases} \end{aligned} \quad (15)$$

where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp[-\frac{1}{2}(x - y)' P^{-1}(x - y)]. \quad (16)$$

The second term on the right-side of (12) is given by

$$\prod_{s=1, s \neq r}^N \mu_k^{j_s^-}(s). \quad (17)$$

The probability of the joint association event Θ_k^1 given that model j is effective for target r from time $k-1$ through k is

$$P\{\Theta_k^1 | M_k^j(r), \mathcal{Z}^{k-1}, Y_k^1\}$$

$$= \frac{1}{c} p[Y_k^1 | \Theta_k^1, M_k^j(r), \mathcal{Z}^{k-1}] P\{\Theta_k^1\} =: \beta_k^j(r, \Theta_k^1) \quad (18)$$

where c is such that $\sum_{\Theta_k^1} \beta_k^j(r, \Theta_k^1) = 1$.

The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_k^{j,r-1}(r)$ (needed in Step 1.5 later) via (11)-(17). Define the target and mode-conditioned innovations

$$\nu_k^{j,1}(r, \Theta_k^1) :=$$

$$\begin{cases} y_k^{1(i)} - \widehat{z}_k^{j,1}(r) & \text{for } i = 1, \dots, \overline{m}_1(k) \text{ if } \theta_{ir}(k) \subset \Theta_k^1 \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

Using $\widehat{x}_{k|k-1}^j(r)$ (from (6)) and its covariance $P_{k|k-1}^j(r)$ (from (7)), one computes the partial state update $\widehat{x}_{k|k}^{j,1}(r)$ and its covariance $P_{k|k}^{j,1}(r)$ according to the standard PDAF [1],[2].

Step 1.5. Mode and joint event conditioned predicted measurements for sensor 2 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): The mode and joint event conditioned measurement for sensor 2 is given by

$$\widehat{z}_k^{j,2}(r, \Theta_k^1) := h^{j,2}(\widehat{x}_{k|k}^{j,1}(r, \Theta_k^1)). \quad (20)$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu_k^{j,2(i)}(r, \Theta_k^1) := z_k^{2(i)} - \widehat{z}_k^{j,2}(r, \Theta_k^1)$ is given by

$$\begin{aligned} S_k^{j,2}(r, \Theta_k^1) &:= E\{\nu_k^{j,2(i)}(r, \Theta_k^1) \nu_k^{j,2(i)'}(r, \Theta_k^1) | M_k^j(r), \mathcal{Z}^{k-1}, Y_k^1\} \\ &= H_k^{j,2}(r, \Theta_k^1) P_{k|k}^{j,1}(r, \Theta_k^1) H_k^{j,2'}(r, \Theta_k^1) + R_k^{j,2} \end{aligned} \quad (21)$$

where $H_k^{j,2}(r, \Theta_k^1)$ is the first order derivative (Jacobian matrix) of $h^{j,2}(\cdot)$ at $\widehat{x}_{k|k}^{j,1}(r, \Theta_k^1)$. Note that (21) assumes that $z_k^{2(i)}$ originates from the target r .

Step 1.6. Measurement validation for sensor 2: This is similar to Step 1.3. For target r , the validation region is taken to be the same for all models, i.e., as the largest of them. Let $|A| = \det(A)$

$$(j_r, \Theta_k^1) := \arg \left\{ \max_{j \in \mathcal{M}_n, \Theta_k^1} |S_k^{j,2}(r, \Theta_k^1)| \right\}. \quad (22)$$

Then measurement $z_k^{2(i)}$ ($i = 1, 2, \dots, m(k)$) is validated if and only if

$$|z_k^{2(i)} - \widehat{z}_k^{j_r,2}(r, \Theta_k^1)|' [S_k^{j_r,2}(r, \Theta_k^1)]^{-1} [z_k^{2(i)} - \widehat{z}_k^{j_r,2}(r, \Theta_k^1)] < \gamma \quad (23)$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is

$$V_k^2(r) := c_{n_{z2}} \gamma^{n_{z2}/2} |S_k^{j_r,2}(r, \Theta_k^1)|^{1/2} \quad (24)$$

where n_{z2} is the dimension of the measurement and $c_{n_{z2}}$ is the volume of the unit hypersphere of this dimension. Choice of γ is discussed in more detail in [1, Sec. 2.3.2]. The volume of validation region for the whole target set is approximated by $V_k^2 = \sum_{r=1}^N V_k^2(r)$.

Step 1.7. State update with validated measurements for sensor 2 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): From among all the raw measurements at time k , i.e., $Z_k^2 := \{z_k^{2(1)}, z_k^{2(2)}, \dots, z_k^{2(m_2(k))}\}$, define the set of validated measurement for sensor 2 at time k as $Y_k^2 := \{y_k^{2(1)}, y_k^{2(2)}, \dots, y_k^{2(\overline{m}_2(k))}\}$ where $\overline{m}_2(k)$ is the total number of validated measurement at time k and $y_k^{2(i)} := z_k^{2(i)}$ where $1 \leq l_1 < l_2 < \dots < l_{\overline{m}_2(k)} \leq m_2(k)$ when $\overline{m}_2(k) \neq 0$. Define the variables similar to (8)-(10) but for sensor 2 with \overline{m}_1 and Θ_k^1 replaced with \overline{m}_2 and Θ_k^2 , respectively. One can evaluate the likelihood that the target r is in model j_r as

$$\Lambda_k^{j_r,2}(r) := p[Y_k^2 | M_k^{j_r}(r), \mathcal{Z}^{k-1}, Y_k^1]$$

$$= \sum_{\Theta_k^2} \sum_{\Theta_k^1} p[Y_k^2 | \Theta_k^2, \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}, Y_k^1] P\{\Theta_k^2\} \beta_k^j(r, \Theta_k^1). \quad (25)$$

The first term in the last line of (25) can be written as

$$\begin{aligned} p[Y_k^2 | \Theta_k^2, \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}, Y_k^1] &= \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_N=1}^n \\ & p[Y_k^2 | \Theta_k^2, \Theta_k^1, M_k^{j_1}(1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}, Y_k^1] \\ & \times P\{M_k^{j_1}(1), \dots, M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \dots, \\ & M_k^{j_N}(N) | \Theta_k^2, \Theta_k^1, M_k^{j_r}(r), \mathcal{Z}^{k-1}, Y_k^1\}. \end{aligned} \quad (26)$$

while the second term is

$$P\{\Theta_k^2\} = \frac{\phi(\Theta_k^2)! \epsilon}{\overline{m}_2(k)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_k^2)} (1 - P_D)^{1 - \delta_s(\Theta_k^2)}. \quad (27)$$

Then the first term on the right-side of (26) is

$$\begin{aligned} & p[Y_k^2 | \Theta_k^2, \Theta_k^1, M_k^{j_1}(1), \dots, M_k^{j_N}(N), \mathcal{Z}^{k-1}, Y_k^1] \\ & \approx \prod_{i=1}^{\overline{m}_2(k)} p[y_k^{2(i)} | \theta_{ir_i}(k), \Theta_k^1, M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}, Y_k^1], \quad \theta_{ir_i}(k) \subset \Theta_k^2, \end{aligned} \quad (28)$$

where the conditional pdf of the validated measurement $y_k^{2(i)}$ given its origin and target mode, is given by

$$\begin{aligned} & p[y_k^{2(i)} | \theta_{ir_i}(k), \Theta_k^1, M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}, Y_k^1] \\ & = \begin{cases} \mathcal{N}(y_k^{2(i)}; \widehat{z}_k^{j_{r_i},2}(r_i, \Theta_k^1), S_k^{j_{r_i},2}(r_i, \Theta_k^1)) & \text{if } \tau_i(\Theta_k^2) = 1, \\ 1/V_k^{j_{r_i}} & \text{if } \tau_i(\Theta_k^2) = 0 \end{cases} \end{aligned} \quad (29)$$

The second term on the right-side of (26) is given by

$$\prod_{s=1, s \neq r}^N \mu_k^{j_s}(s, \Theta_k^1). \quad (30)$$

The probability of the joint association event (Θ_k^2, Θ_k^1) given that model j is effective for target r from time $k-1$ through k is

$$\begin{aligned} \beta_k^j(r, \Theta_k^1, \Theta_k^2) &:= P\{\Theta_k^2, \Theta_k^1 | M_k^j(r), \mathcal{Z}^{k-1}, Y_k^1, Y_k^2\} \\ &= \frac{1}{c} p[Y_k^2 | \Theta_k^2, \Theta_k^1, M_k^j(r), \mathcal{Z}^{k-1}, Y_k^1] P\{\Theta_k^2\} \beta_k^j(r, \Theta_k^1) \end{aligned} \quad (31)$$

where c is such that $\sum_{\Theta_k^2} \sum_{\Theta_k^1} \beta_k^j(r, \Theta_k^1, \Theta_k^2) = 1$.

The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_k^{j,r,2}(r)$ (needed in Step 1.8 later) via (25)-(30). Define the target and mode-conditioned innovations

$$\nu_k^{j,2}(r, \Theta_k^1, \Theta_k^2) := \begin{cases} y_k^{2(i)} - \widehat{z}_k^{j,2}(r, \Theta_k^1) & \text{for } i = 1, \dots, \overline{m}_2(k) \\ & \text{if } \theta_{ir}(k) \subset \Theta_k^2 \\ 0 & \text{otherwise.} \end{cases} \quad (32)$$

Using $\widehat{x}_{k|k}^j(r, \Theta_k^1)$ and its covariance $P_{k|k}^j(r, \Theta_k^1)$, one computes the final state update $\widehat{x}_{k|k}^j(r)$ and its covariance $P_{k|k}^j(r)$ according to the standard PDAF [1]; details are omitted.

Step 1.8. Update of mode probabilities ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$):

$$\mu_k^j(r) := P[M_k^j(r) | \mathcal{Z}^k] = \frac{1}{c} \mu_k^{j-}(r) \Lambda_k^{j,1}(r) \Lambda_k^{j,2}(r) \quad (33)$$

where c is such that $\sum_{j=1}^n \mu_k^j(r) = 1$.

Step 1.9. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final state estimate update at time k is given by

$$\widehat{x}_{k|k}(r) = \sum_{j=1}^n \widehat{x}_{k|k}^j(r) \mu_k^j(r) \quad (34)$$

and its covariance $P_{k|k}(r)$ is given by

$$\sum_{j=1}^n \left\{ P_{k|k}^j(r) + [\widehat{x}_{k|k}^j(r) - \widehat{x}_{k|k}(r)][\widehat{x}_{k|k}^j(r) - \widehat{x}_{k|k}(r)]' \right\} \mu_k^j(r). \quad (35)$$

III.2 Second Scan Steps

Here we update to scan $k+1$, given data up to time $k+1$, with a sliding scan window of size two, scans $\{k, k+1\}$.

Step 2.1. Interaction – mixing of the estimate from the previous time ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$):

$$\begin{aligned} \mu_k^j(r, \Theta_k^1, \Theta_k^2) &:= P\{M_k^j(r) | \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\} \\ &= \frac{1}{c} p[Y_k^2 | M_k^j(r), \mathcal{Z}^{k-1}, \Theta_k^1, \Theta_k^2, Y_k^1] \mu_k^j(r, \Theta_k^1). \end{aligned} \quad (36)$$

The first part of (36) is evaluated by using (26).

predicted mode probability: $\mu_{k+1}^{j-}(r, \Theta_k^1, \Theta_k^2) :=$

$$P\{M_{k+1}^j(r) | \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\} = \sum_{i=1}^n p_{ij} \mu_k^i(r, \Theta_k^1, \Theta_k^2). \quad (37)$$

mixing probability: $\mu^{ij}(r, \Theta_k^1, \Theta_k^2) :=$

$$P\{M_k^i(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\} = p_{ij} \mu_k^i(r, \Theta_k^1, \Theta_k^2) / \mu_{k+1}^{j-}(r, \Theta_k^1, \Theta_k^2). \quad (38)$$

mixed estimate: $\widehat{x}_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2) := E\{x_k(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\}$

$$= \sum_{i=1}^n \widehat{x}_{k|k}^i(r, \Theta_k^1, \Theta_k^2) \mu^{ij}(r, \Theta_k^1, \Theta_k^2). \quad (39)$$

covariance of the mixed estimate: $P_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2)$

$$\begin{aligned} &= \sum_{i=1}^n \{ P_{k|k}^i(r, \Theta_k^1, \Theta_k^2) + [\widehat{x}_{k|k}^i(r, \Theta_k^1, \Theta_k^2) - \widehat{x}_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2)] \\ &\quad \times [\widehat{x}_{k|k}^i(r, \Theta_k^1, \Theta_k^2) - \widehat{x}_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2)]' \} \mu^{ij}(r, \Theta_k^1, \Theta_k^2). \end{aligned} \quad (40)$$

Step 2.2. Predicted state ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$):

State prediction: $\widehat{x}_{k+1|k}^j(r, \Theta_k^1, \Theta_k^2) :=$

$$E\{x_{k+1}(r) | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\} = F_k^j \widehat{x}_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2). \quad (41)$$

State prediction error covariance: $P_{k+1|k}^j(r, \Theta_k^1, \Theta_k^2)$

$$= F_k^j P_{k|k}^{0j}(r, \Theta_k^1, \Theta_k^2) F_k^{j'} + G_k^j Q_k^j G_k^{j'}. \quad (42)$$

Using (2) and (41), the mode-conditioned predicted measurement of target r is

$$\widehat{z}_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2) := h^{j,1}(\widehat{x}_{k+1|k}^j(r, \Theta_k^1, \Theta_k^2)). \quad (43)$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu_{k+1}^{j,1(i)}(r, \Theta_k^1, \Theta_k^2) := z_{k+1}^{1(i)} - \widehat{z}_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2)$

is given by $S_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2)$

$$:= E\{\nu_{k+1}^{j,1(i)}(r, \Theta_k^1, \Theta_k^2) \nu_{k+1}^{j,1(i)}(r, \Theta_k^1, \Theta_k^2)' | M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k^1, \Theta_k^2\}$$

$$= H_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2) P_{k+1|k}^j(r, \Theta_k^1, \Theta_k^2) H_{k+1}^{j,1'}(r, \Theta_k^1, \Theta_k^2) + R_{k+1}^{j,1} \quad (44)$$

where $H_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2)$ is the first order derivative (Jacobian matrix) of $h^{j,1}(\cdot)$ at $\widehat{x}_{k+1|k}^j(r, \Theta_k^1, \Theta_k^2)$.

Step 2.3. Measurement validation for sensor 1: For target r , the validation region is taken to be the same for all models and Θ_k^1, Θ_k^2 's, i.e., as the largest of them. Let

$$(j_r, \bar{\Theta}_k^1, \bar{\Theta}_k^2) := \arg \left\{ \max_{j \in \mathcal{M}_n, \Theta_k^1, \Theta_k^2} |S_{k+1}^{j,1}(r, \Theta_k^1, \Theta_k^2)| \right\}. \quad (45)$$

Then measurement $z_{k+1}^{1(i)}$ ($i = 1, 2, \dots, m_1(k+1)$) is validated if and only if $[z_{k+1}^{1(i)} - \widehat{z}_{k+1}^{j_r,1}(r, \bar{\Theta}_k^1, \bar{\Theta}_k^2)]' [S_{k+1}^{j_r,1}(r, \bar{\Theta}_k^1, \bar{\Theta}_k^2)]^{-1} [z_{k+1}^{1(i)} - \widehat{z}_{k+1}^{j_r,1}(r, \bar{\Theta}_k^1, \bar{\Theta}_k^2)] < \gamma$ where γ is an appropriate threshold. The volume of the validation region with the threshold γ is $V_{k+1}^1(r) = c_{n_{z1}} \gamma^{n_{z1}/2} |S_{k+1}^{j_r,1}(r, \bar{\Theta}_k^1, \bar{\Theta}_k^2)|^{1/2}$ where n_{z1} is the dimension of the measurement and $c_{n_{z1}}$ is the volume of the unit hypersphere of this dimension. The volume of validation region for the whole target set is approximated by $V_{k+1}^1 = \sum_{r=1}^N V_{k+1}^1(r)$.

Step 2.4. State estimation with validated measurements from sensor 1 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$):

From among all the raw measurements at time $k+1$, i.e., $\mathcal{Z}_{k+1}^1 := \{z_{k+1}^{1(1)}, z_{k+1}^{1(2)}, \dots, z_{k+1}^{1(m_1(k+1))}\}$, define the set of validated measurement for sensor 1 at time $k+1$ as $Y_{k+1}^1 := \{y_{k+1}^{1(1)}, y_{k+1}^{1(2)}, \dots, y_{k+1}^{1(\bar{m}_1(k+1))}\}$ where $\bar{m}_1(k+1)$ is the total number of validated measurement at time $k+1$. and $y_{k+1}^{1(i)} := z_{k+1}^{1(i)}$ where $1 \leq l_1 < l_2 < \dots < l_{\bar{m}_1(k+1)} \leq m_1(k+1)$ and $\bar{m}_1(k+1) \neq 0$. Note that all targets share a common validated measurement set Y_{k+1}^1 .

Define the variables similar to (8)-(10) with $\bar{m}_1(k)$ and Θ_k^1 replaced with $\bar{m}_1(k+1)$ and Θ_{k+1}^1 , respectively. One can evaluate the likelihood that the target r is in model j_r as

$$\Lambda_{k+1}^{j_r,1}(r) := p[Y_{k+1}^1 | M_{k+1}^{j_r}(r), \mathcal{Z}^k] = \sum_{\Theta_{k+1}^1} \sum_{\Theta_k^2} \sum_{\Theta_k^1}$$

$$p[Y_{k+1}^1 | \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1, M_{k+1}^{j_r}(r), \mathcal{Z}^k] P\{\Theta_{k+1}^1\} P\{\Theta_k^2, \Theta_k^1 | M_{k+1}^{j_r}(r), \mathcal{Z}^k\}. \quad (46)$$

The first term in the last line of (46) can be written as

$$\sum_{j_1=1}^n \dots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \dots \sum_{j_N=1}^n p[Y_{k+1}^1 | \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1, M_{k+1}^{j_1}(1),$$

$$\dots, M_{k+1}^{j_N}(N), \mathcal{Z}^k] \times P\{M_{k+1}^{j_1}(1), \dots, M_{k+1}^{j_{r-1}}(r-1),$$

$$M_{k+1}^{j_r+1}(r+1), \dots, M_{k+1}^{j_N}(N) | \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1, M_{k+1}^{j_r}(r), \mathcal{Z}^k\} \quad (47)$$

while the second term is

$$P\{\Theta_{k+1}^1\} = \frac{\phi(\Theta_{k+1}^1)! \epsilon}{\bar{m}_1(k+1)!} \prod_{s=1}^N (P_D)^{\delta_s(\Theta_{k+1}^1)} (1 - P_D)^{1 - \delta_s(\Theta_{k+1}^1)}. \quad (48)$$

The third term in the last line of (46) is given by

$$\begin{aligned} &P\{\Theta_k^2, \Theta_k^1 | M_{k+1}^{j_r}(r), \mathcal{Z}^k\} \\ &= \frac{1}{c^r} \mu_{k+1}^{j_r-}(r, \Theta_k^1, \Theta_k^2) P\{\Theta_k^2\} P\{\Theta_k^1\} p[Y_k^2, Y_k^1 | \Theta_k^2, \Theta_k^1, \mathcal{Z}^{k-1}] \end{aligned} \quad (49)$$

where

$$p[Y_k^2, Y_k^1 | \Theta_k^2, \Theta_k^1, \mathcal{Z}^{k-1}] = \sum_j p[Y_k^2, Y_k^1 | \Theta_k^2, \Theta_k^1, M_k^j(r), \mathcal{Z}^{k-1}] \mu_k^{j-}(r). \quad (50)$$

We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term on the right-side of (47) can be written as $(\theta_{ir_i}(k+1) \subset \Theta_{k+1}^1)$

$$\begin{aligned} & \bar{m}_1(k+1) \\ & \approx \prod_{i=1} p[y_{k+1}^{1(i)} | \theta_{ir_i}(k+1), \Theta_k^2, \Theta_k^1, M_{k+1}^{jr_i}(r_i), \mathcal{Z}^k] \end{aligned} \quad (51)$$

where the conditional pdf of the validated measurement $y_{k+1}^{1(i)}$ given its origin and target mode, is given by

$$\begin{aligned} & p[y_{k+1}^{1(i)} | \theta_{ir_i}(k+1), \Theta_k^2, \Theta_k^1, M_{k+1}^{jr_i}(r_i), \mathcal{Z}^k] = \\ & \begin{cases} \mathcal{N}(y_{k+1}^{1(i)}; \hat{z}_{k+1}^{jr_i,1}(r_i, \Theta_k^2, \Theta_k^1), S_{k+1}^{jr_i,1}(r_i, \Theta_k^2, \Theta_k^1)) & \text{if } \tau_i(\Theta_{k+1}^1) = 1, \\ 1/V_{k+1}^1 & \text{if } \tau_i(\Theta_{k+1}^1) = 0. \end{cases} \end{aligned} \quad (52)$$

The second term on the right-side of (47) is given by

$$\prod_{s=1, s \neq r}^N \mu_{k+1}^{js-}(s, \Theta_k^1, \Theta_k^2). \quad (53)$$

The probability of the joint association events $\Theta_{k+1}^1, \Theta_k^2$, and Θ_k^1 given that model j is effective for target r from time k through $k+1$ is

$$\begin{aligned} & \beta_{k+1}^j(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) := P\{\Theta_{k+1}^1, \Theta_k^2, \Theta_k^1 | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}^1\} \\ & = \frac{1}{c} p[Y_{k+1}^1 | \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1, M_{k+1}^j(r), \mathcal{Z}^k] P\{\Theta_{k+1}^1\} \\ & \quad \times P\{\Theta_k^2, \Theta_k^1 | M_{k+1}^j(r), \mathcal{Z}^k\} \end{aligned} \quad (54)$$

where the first term can be calculated from (47) and (51) - (53), the second term from (48), the third term from (49), and c is a normalization constant such that $\sum_{\Theta_{k+1}^1} \sum_{\Theta_k^2} \sum_{\Theta_k^1} \beta_{k+1}^j(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) = 1$.

The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_{k+1}^{jr,1}(r)$ (needed in Step 2.5 later) via (46)-(53). Define the target and mode-conditioned innovations

$$\begin{aligned} & \nu_{k+1}^{j,1}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) \\ & := \begin{cases} y_{k+1}^{1(i)} - \hat{z}_{k+1}^{j,1}(r, \Theta_k^2, \Theta_k^1) & \text{for } i = 1, \dots, \bar{m}(k+1) \\ 0 & \text{if } \theta_{ir}(k+1) \subset \Theta_{k+1}^1 \\ & \text{otherwise.} \end{cases} \end{aligned} \quad (55)$$

Using $\hat{x}_{k+1|k}^j(r, \Theta_k^2, \Theta_k^1)$ (from (41)) and its covariance $P_{k+1|k}^j(r, \Theta_k^2, \Theta_k^1)$ (from (42)), one computes the state update $\hat{x}_{k+1|k+1}^j(r)$ and its covariance $P_{k+1|k+1}^j(r)$ in a manner similar to [4]; the details are omitted.

Step 2.5. Mode and joint event conditioned predicted measurements for sensor 2 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): The mode and joint event conditioned predicted measurement for sensor 2 is given by

$$\hat{z}_{k+1}^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) := h^{j,2}(\hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)). \quad (56)$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu_k^{j,2(i)}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) := z_{k+1}^{2(i)} - \hat{z}_{k+1}^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)$ is given by

$$\begin{aligned} & S_{k+1}^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) = H_{k+1}^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) \\ & \quad \times P_{k+1|k+1}^j(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) H_{k+1}^{j,2'}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1) + R_{k+1}^{j,2} \end{aligned} \quad (57)$$

where $H_{k+1}^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)$ is the first order derivative (Jacobian matrix) of $h^{j,2}(\cdot)$ at $\hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)$.

Step 2.6. Measurement validation for sensor 2: This is similar to Step 2.3. Let

$$\begin{aligned} & (j_r, \bar{\Theta}_{k+1}^1, \bar{\Theta}_k^2, \bar{\Theta}_k^1) := \\ & \arg \left\{ \max_{j \in \mathcal{M}_n, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1} |S_k^{j,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)| \right\}. \end{aligned} \quad (58)$$

Then measurement $z_{k+1}^{2(i)}$ ($i = 1, 2, \dots, m_2(k+1)$) is validated if and only if $[z_{k+1}^{2(i)} - \hat{z}_{k+1}^{j_r,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)]' [S_{k+1}^{j_r,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)]^{-1} [z_{k+1}^{2(i)} - \hat{z}_{k+1}^{j_r,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)] < \gamma$ where γ is an appropriate threshold. The volume of the validation region with the threshold γ is $V_{k+1}^2(r) := c_{n_2} \gamma^{n_2/2} |S_{k+1}^{j_r,2}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)|^{1/2}$. The volume of validation region for the whole target set is approximated by $V_{k+1}^2 = \sum_{r=1}^N V_{k+1}^2(r)$.

Step 2.7. State update with validated measurements from sensor 2 ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$): From among all the raw measurements at time $k+1$, i.e., $Z_{k+1}^2 := \{z_{k+1}^{2(1)}, z_{k+1}^{2(2)}, \dots, z_{k+1}^{2(m_2(k+1))}\}$, define the set of validated measurement for sensor 2 at time $k+1$ as $Y_{k+1}^2 := \{y_{k+1}^{2(1)}, y_{k+1}^{2(2)}, \dots, y_{k+1}^{2(\bar{m}_2(k+1))}\}$ where $\bar{m}_2(k+1)$ is the total number of validated measurement at time $k+1$ and $y_{k+1}^{2(i)} := z_{k+1}^{2(l_i)}$ where $1 \leq l_1 < l_2 < \dots < l_{\bar{m}_2(k+1)} \leq m_2(k+1)$ when $\bar{m}_2(k+1) \neq 0$. Define the variables similar to (8)-(10) One can evaluate the likelihood that the target r is in model j_r as

$$\begin{aligned} & \Lambda_{k+1}^{j_r,2}(r) := p[Y_{k+1}^2 | M_{k+1}^{j_r}(r), \mathcal{Z}^k, Y_{k+1}^1] = \sum_{\Theta_{k+1}^2} \sum_{\Theta_{k+1}^1} \sum_{\Theta_k^2} \sum_{\Theta_k^1} \\ & \quad p[Y_{k+1}^2 | \Theta_{k+1}^2, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1, M_{k+1}^{j_r}(r), \mathcal{Z}^k, Y_{k+1}^1] P\{\Theta_{k+1}^2\} \\ & \quad \times \beta_{k+1}^{j_r}(r, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1). \end{aligned} \quad (59)$$

Mimicking Secs. 1.7 and 2.4, one can calculate

$$\begin{aligned} & \hat{x}_{k+1|k+1}^j(r, \Theta_{k+1}^2, \Theta_{k+1}^1, \Theta_k^1, \Theta_k^2) \\ & := E\{x_{k+1}(r) | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}^1, \Theta_{k+1}^1, \Theta_k^1, Y_{k+1}^2, \Theta_{k+1}^2, \Theta_k^2\}, \\ & \hat{x}_{k+1|k+1}^j(r) := E\{x_{k+1}(r) | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}^1, Y_{k+1}^2\}, \end{aligned}$$

and their covariances matrices $P_{k+1|k+1}^j(r)$ and $P_{k+1|k+1}^j(r, \Theta_{k+1}^2, \Theta_{k+1}^1, \Theta_k^2, \Theta_k^1)$, respectively; details are omitted.

Step 2.8. Update of mode probabilities ($\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N$):

$$\mu_{k+1}^j(r) := P[M_{k+1}^j(r) | \mathcal{Z}^{k+1}] = \frac{1}{c} \mu_{k+1}^{j-}(r) \Lambda_{k+1}^{j,1}(r) \Lambda_{k+1}^{j,2}(r) \quad (60)$$

where c is such that $\sum_{j=1}^n \mu_{k+1}^j(r) = 1$.

Step 2.9. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): This is exactly as in [4, Step 2.6].

IV SIMULATION EXAMPLE

We now consider tracking two maneuvering targets by two sensors in clutter. We carry out sequential updating of state for each target using IMM multiscan JPDA with a scan window size of two and compare our results with single scan IMM/JPDA algorithm of [2].

The True Trajectories: Target 1 starts at location [10100 1740 40] in Cartesian coordinates in meters. The initial velocity

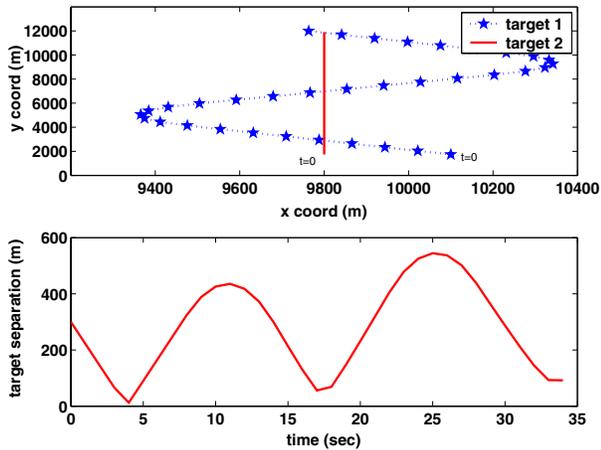


Figure 1. Trajectories of the two maneuvering targets (read top to bottom). (a) Position in xy plane. (b) distance between targets.

is $[-78 \ 299.9 \ 0]$ in m/s. Target stays at constant altitude with a constant speed of 309.87 m/s. Its trajectory is a straight line with constant velocity between 0 and 8 sec., a coordinated turn of -0.09 rad/s with a constant acceleration of 27.889 m/s² between 8 and 14 s, a straight line with a constant velocity between 14 and 22 s, a coordinated turn of 0.09 rad/s with a constant acceleration of 27.889 m/s² between 22 and 28 s, and a straight line with a constant velocity between 28 and 35 s. Target 2 starts at location $[9800 \ 1740 \ 40]$ in Cartesian coordinates in meters. The initial velocity is $[0 \ 299 \ 0]$ in m/s. The target stays at a constant altitude with a constant speed of 299 m/s. Its trajectory is a straight line with constant velocity between 0 and 35 sec.

The Target Motion Models: The motion models are identical for both targets. In each mode target dynamics are modeled in Cartesian coordinates as $x_k = Fx_{k-1} + Gv_{k-1}$ where state of the target is position, velocity and acceleration in each of the three Cartesian coordinates (x , y and z). Thus x_k is of dimension 9 ($n_x=9$). Two models are considered in the following discussion and they are the first two models of [2, Sec. 5]. Model 1 is a nearly constant velocity model with zero mean perturbation in acceleration with process noise standard deviation chosen as 5m/s^2 and Model 2 is a Wiener process acceleration (nearly constant acceleration motion) model with process noise standard deviation chosen as 30m/s^2 . The initial model probabilities for three targets are identical: $\mu_1^0 = 0.98$ and $\mu_2^0 = 0.02$. The mode switching probability matrix for the two targets is also identical and is given by

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}.$$

The Sensors: Two sensors, (radar and infrared) are used to obtain the measurements. The measurements are range and azimuth from radar. Azimuth and elevation angles from the infrared. The range, azimuth and elevation transformations, respectively, are given by $r = (x^2 + y^2 + z^2)^{1/2}$, $a = \tan^{-1}(y/x)$, $e = \tan^{-1}[z/(x^2 + y^2)^{1/2}]$. The measurement noise $w_k^{j,l}$ for sensor l ($l = 1, 2$) is assumed to be zero-mean white Gaussian with known covariance matrices $R^1 = \text{diag}[400\text{m}^2, 49\text{mrad}^2]$ and $R^2 = \text{diag}[4\text{mrad}^2, 4\text{mrad}^2]$. The sensors are assumed to be located at the origin of the coordinate system. The sampling interval was $T = 1\text{s}$ and it was assumed that the probability of detection $P_D = 0.997$.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 20 \times 10^{-6}/\text{m mrad}$ for sensor 1 and $\lambda_2 = 1.5 \times 10^{-4}/\text{mrad}^2$ for sensor 2. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for the sensor were based on the threshold $\gamma = 16$. With

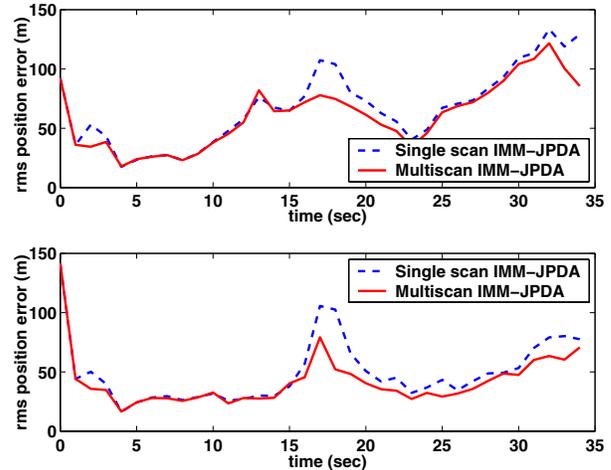


Figure 2. Root mean-square error (RMSE) in position using single scan IMM/JPDAF [2] and the proposed multiscan (window size 2 scans) IMM/Mscan-JPDAF algorithms.

the measurement vector of dimension 2, this leads to a gate probability $P_G = 0.9997$ (see p. 96 of [1]).

Simulation Results: The results were obtained from 100 Monte Carlo runs. Fig. 1 shows the true trajectories of the two targets and the distances among targets as a function of time. Fig. 2 shows the RMSE (root mean-square error) for the filtered position estimates for the two targets as a function of time. It is seen from Fig. 2 that the multiscan approach does provide a significant improvement over the single scan approach.

V CONCLUSIONS

We investigated the problem of multisensor tracking of multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal filtering algorithm was developed by applying the basic interacting multiple model (IMM) approach and multiscan joint probabilistic data association (JPDA) technique. We exploited sequential updating of measurements from multiple sensors. Past work (see [4]) on this problem is restricted to single sensor tracking.

The algorithm was illustrated via a simulation example involving tracking of two maneuvering targets and a multiscan data window of length two. The simulation example shows a significant improvement in target position estimate by the proposed IMM multiscan JPDA (with a scan window size of two) compared to the results of the single scan IMM/JPDA algorithm of [2].

VI REFERENCES

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