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The effects of state feedback and input transformation with application to Lur'e systems

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***Abstract*— Given a linear system, necessary and sufficient conditions are established for the existence of a state feedback and input transformation such that the closed-loop system becomes extended strictly positive real(ESPR) or SPR. The application to Lur'e systems is also given. The results show that a multivariable Lur'e system with bounded sector constraints can always be stabilized by a state feedback and input/output coupling under a stabilizability condition. An extended Chua's circuit with two nonlinear functions is given as an example to illustrate the results.**

I. INTRODUCTION

Modern control theory has got a great success in theory and applications. The idea of feedback control has been used in linear and nonlinear systems to solve practical engineering problems. Various feedback controller design methods have been established, see [14], [9], [28] and references therein. The effects of state feedback in the closed-loop pole assignment and optimal control were studied very early[11], [1]. Compared with state feedback or output feedback, the input transformation has been paid much less attention in linear system theory by now except that it played an important role in input and output decoupling control[26]. The effects of input transformation are always ignored from a mathematical viewpoint, since it is only a variable transformation. In fact, input transformation can also be used as a design degree of freedom to improve system properties.

The strict positive realness(SPR) and ESPR of transfer functions are important properties of dynamic systems. They play crucial roles in absolute stability and hyperstability, passivity analysis, quadratic optimal control, and adaptive system theory[1], [19], [4], [16]. Along with the development of the study on SPR and ESPR of transfer functions, the corresponding algebraic conditions for SPR have been established, namely, the Kalman-Yakubovich-Popov(KYP) lemma[27], [20]. It has been recognized as one of the most basic tools in systems theory. By combining frequency-domain and algebraic criteria, analysis and synthesis for absolute stability of Lur'e systems have been studied broadly[3], [25], [18], [2]. Special Lur'e systems, Chua's circuits, have also attracted a lot of interest[12], [15], [17], [23], [24]. This paper is devoted to studying the effects of state feedback and input transformation for SPR, ESPR and absolute stability of Lur'e systems.

The rest of this paper is organized as follows: In section 2, necessary and sufficient conditions are established for the existence of a state feedback and input transformation such that the closed-loop system is ESPR or SPR for given linear systems. In section 3, the idea of input transformation is applied to Lur'e systems. Here we also call it input and output coupling. It is proved that a multivariable Lur'e system with bounded sector constraints can always be absolute stable by choosing a state feedback and input transformation under a stabilizability condition. Lur'e systems with unbounded sector constraints can be considered similarly. In section 4, an extended Chua's circuit with two nonlinear functions is presented as an example to illustrate the results.

II. STATE FEEDBACK AND INPUT TRANSFORMATION FOR ESPR AND SPR

The standard definitions of ESPR and SPR are as follows[13].

Definition 1 A transfer function $G(s)$ is said to be ESPR if $G(s)$ is analytic in $\mathbf{Re}(s) \geq 0$ and satisfies $G(jw) + G^H(jw) > 0$ for $w \in \mathbf{R} \cup \{\infty\}$.

Definition 2 A transfer function $G(s)$ is said to be PR if $G(s)$ is analytic in $\mathbf{Re}(s) > 0$ and $G(\lambda) + G^H(\lambda) \geq 0$ for all $\mathbf{Re}(\lambda) > 0$. $G(s)$ is said to be SPR if $G(s - \epsilon)$ is PR for some $\epsilon > 0$.

Before giving the main results in this section, the following important KYP lemma is needed.

Lemma 1[20],[22] Given $G(s) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$, $G(s)$ is ESPR, if and only if there exists $P > 0$ such that

$$\begin{pmatrix} PA + A^TP & PB - C^T \\ B^TP - C & -D - D^T \end{pmatrix} < 0.$$

In addition, if (A, B) is controllable, (A, C) is observable, C is with full row rank and $D = 0$, $G(s)$ is SPR if and only if there exists $P > 0$ such that

$$PA + A^TP < 0 \quad PB = C^T.$$

Given a linear system

$$\begin{cases} \dot{x} &= Ax + Bu, \\ y &= Cx + Du, \end{cases} \tag{1}$$

where $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}, C \in \mathbf{R}^{m\times n}, D \in \mathbf{R}^{m\times m}$. The transfer function from u to y is $G(s) = C(sI -$

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$A)^{-1}B + D$. We propose the following state feedback and input transformation,

$$u = Kx + Lv, \tag{2}$$

then the closed-loop system becomes

$$\left\{ \begin{array}{rcl} \dot{x} & = & (A + BK)x + BLv, \\ y & = & (C + DK)x + DLv. \end{array} \right. \tag{3}$$

In what follows, we see if there exists a state feedback and input transformation as in (2) such that system (3) is ESPR. For this problem, we can get the following result easily.

Theorem 1 There exists a state feedback and input transformation as in (2) such that system (3) is ESPR, if and only if $G^{-1}(s) \in \mathbf{RH}_\infty$, i.e., $A - BD^{-1}C$ is Hurwitz stable.

Proof By Lemma 1, system (3) is ESPR, if and only if there exists $Q > 0$ and any matrix $Y(Y = KQ)$ such that

$$\left(\begin{array}{cc} AQ + QA^T + BY + Y^T B^T & BL - QC^T - Y^T D^T \\ L^T B^T - CQ - DY & -DL - L^T D^T \end{array} \right) < 0. \tag{4}$$

Rewrite (4) as

$$\left(\begin{array}{cc} AQ + QA^T & -QC^T \\ -CQ & 0 \end{array} \right) + \left(\begin{array}{c} B \\ -D \end{array} \right) (Y \quad L) + \left(\left(\begin{array}{c} B \\ -D \end{array} \right) (Y \quad L) \right)^T < 0,$$

by the LMI method [13], [10], there exists Y, L such that the inequality above holds, if and only if there exists $\lambda > 0$ such that

$$\left(\begin{array}{cc} AQ + QA^T & -QC^T \\ -CQ & 0 \end{array} \right) - \lambda \left(\begin{array}{c} B \\ -D \end{array} \right) (B^T \quad -D^T) < 0.$$

By Schur complement, the inequality above holds if and only if D is nonsingular and the following two inequalities hold

$$AQ + QA^T - BD^{-1}CQ - QC^T D^{-T} B^T + \frac{1}{\lambda} QC^T D^{-T} D^{-1} CQ < 0, \tag{5}$$

$$AQ + QA^T - \lambda BB^T < 0. \tag{6}$$

Take λ sufficiently large, the inequality (5) holds if and only if

$$AQ + QA^T - QC^T D^{-T} B^T - BD^{-1} CQ < 0. \tag{7}$$

Using LMI method in [13] again, there exists λ such that (6) holds, if and only if there exists $Y(Y = KQ)$ such that

$$AQ + QA^T + BY + Y^T B^T < 0.$$

The inequality above holds naturally when (7) holds. Therefore, (5) and (6) hold simultaneously, if and only if (7) holds. That is, $A - BD^{-1}C$ is Hurwitz stable, i.e., $G^{-1}(s) \in \mathbf{RH}_\infty$. This completes the proof.

Corollary 1 There exists a state feedback matrix K and input transformation $L = I$ in (2) such that system (3) is ESPR, if and only if $A - BD^{-1}C$ is Hurwitz stable and $-D - D^T < 0$.

Corollary 2 If we do not consider the state feedback, then the problem above becomes to find a matrix L such that $G(s)L$ is ESPR. This is equivalent to the existence of a common Q such that $AQ + QA^T < 0$ and $(A - BD^{-1}C)Q + Q(A - BD^{-1}C)^T < 0$ hold simultaneously.

Remark 1 Comparing Theorem 1 with Corollary 1, one can see that the effect of L is to adjust D such that $-DL - L^T D^T < 0$. By the canonical LMI method, all solutions K, L can be parameterized under the condition in Theorem 1.

Remark 2 State feedback can change the closed-loop eigenvalues of systems, and input transformation can change zeros of systems. We can improve system properties by combining state feedback with input transformation. The effects of input transformation are always ignored from a mathematical viewpoint. In this paper, we take input transformation as a design index to improve system properties.

In system (3), state feedback and input transformation are implemented simultaneously. If we implement them separately, we have the following system,

$$\left\{ \begin{array}{rcl} \dot{x} & = & Ax + Bu_1 + Bu_2, \\ y & = & Cx + Du_2. \end{array} \right. \tag{8}$$

Let $u_1 = Kx, u_2 = Lv$, then we get a new closed-loop system as follows,

$$\left\{ \begin{array}{rcl} \dot{x} & = & (A + BK)x + BLv, \\ y & = & Cx + DLv. \end{array} \right. \tag{9}$$

Compared with system (3), there is no state feedback in system output in the system above. By the method in Theorem 1, one can get

Theorem 2 If (A, B) is stabilizable and D is nonsingular, then there exists (K, L) such that system (9) is ESPR.

Proof By Lemma 1, system (9) is ESPR, if and only if there exists $Q > 0$ and any matrix $Y(Y = KQ)$ such that

$$\left(\begin{array}{cc} AQ + QA^T + BY + Y^T B^T & BL - QC^T \\ L^T B^T - CQ & -DL - L^T D^T \end{array} \right) < 0. \tag{10}$$

Rewrite (10) as

$$\left(\begin{array}{cc} AQ + QA^T + BY + Y^T B^T & -QC^T \\ -CQ & 0 \end{array} \right) + \left(\begin{array}{c} B \\ -D \end{array} \right) L(0 \quad I) + \left(\left(\begin{array}{c} B \\ -D \end{array} \right) L(0 \quad I) \right)^T < 0.$$

By the method in [10], [13], there exists L such that the inequality above holds, if and only if there exists $\lambda > 0$ such that

$$\left(\begin{array}{cc} AQ + QA^T + R & -QC^T \\ -CQ & 0 \end{array} \right) - \lambda \left(\begin{array}{cc} BB^T & -BD^T \\ -DB^T & DD^T \end{array} \right) < 0, \tag{11}$$

where $R = BY + Y^T B^T$ and

$$AQ + QA^T + BY + Y^T B^T < 0. \tag{12}$$

By Schur complement as in the proof of Theorem 1, the inequality (11) holds if and only if D is nonsingular and

$$AQ + QA^T + BY + Y^T B^T - BD^{-1} CQ - QC^T D^{-T} B^T < 0. \tag{13}$$

(13) and (12) hold simultaneously if

$$AQ + QA^T + BY + Y^T B^T + BB^T + QC^T D^{-T} D^{-1} CQ < 0.$$

Obviously, BB^T can be cancelled by taking Y, so there exist $Q > 0$ and any matrix Y such that the inequality above holds if and only if there exist $Q > 0$ and Y such that

$$AQ + QA^T + BY + Y^T B^T + QC^T D^{-T} D^{-1} CQ < 0.$$

Because of the stabilizability of (A, B), the inequality above holds naturally. This completes the proof.

Remark 3 It is well known that the existence of a common Lyapunov matrix for two Hurwitz matrices is very difficult to solve, see [21] and references therein. From (12) and (13), one can see that a special kind of problem of the existence of a common Lyapunov matrix under state feedback appears here, that is, if (A, B) is stabilizable, there exist $Q > 0$ and a state feedback matrix K such that $(A + BK)Q + Q(A + BK)^T < 0$ and $(A+BK+BC)Q+Q(A+BK+BC)^T < 0$. Obviously, this problem can be generalized to the existence of $Q > 0$ and K such that

$$(A + BK + BC_1)Q + Q(A + BK + BC_1)^T < 0, \cdots,$$

$$(A + BK + BC_r)Q + Q(A + BK + BC_r)^T < 0$$

for given r matrices $C_1, \cdots, C_r$ with compatible dimensions, if (A, B) is stabilizable.

In what follows, we discuss the effects of state feedback and input transformation for SPR in strictly proper systems. Given a linear system

$$\left\{ \begin{array}{lcl} \dot{x} & = & Ax + Bu, \\ y & = & Cx, \end{array} \right. \quad (14)$$

where A, B and C are as given in (1). Under the state feedback and input transformation as in (2), the closed-loop system becomes

$$\left\{ \begin{array}{lcl} \dot{x} & = & (A + BK)x + BLv, \\ y & = & Cx. \end{array} \right. \quad (15)$$

Then by Lemma 1, if C is with full row rank and (A, B) is controllable, (A, C) is observable, system (15) is SPR if and only if there exists $Q > 0$ such that

$$(A + BK)Q + Q(A + BK)^T < 0, \quad QC^T = BL. \quad (16)$$

For simplicity, in what follows we suppose $C = (I_m \quad 0)$ without lost of generality. Obviously, there exist $Q > 0$ and any matrix L such that $QC^T = BL$ if and only if CB is nonsingular. Write Q and B as the following blocked matrices

$$Q = \begin{pmatrix} Q_1 & Q_{12} \\ Q_{12}^T & Q_2 \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},$$

where Q_1 and B_1 are $m \times m$ nonsingular matrices, by $C = (I_m \quad 0)$ and $QC^T = BL$, one knows that

$$Q_1 = B_1 L, \quad Q_{12}^T = B_2 L.$$

Then take

$$S = \begin{pmatrix} I & 0 \\ -Q_{12}^T Q_1^{-1} & I \end{pmatrix} = \begin{pmatrix} I_1 & 0 \\ -B_2 B_1^{-1} & I_2 \end{pmatrix},$$

one gets

$$SQS^T = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 - Q_{12}^T Q_1^{-1} Q_{12} \end{pmatrix}, \quad SB = \begin{pmatrix} B_1 \\ 0 \end{pmatrix}.$$

Let

$$SAS^{-1} = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix},$$

then one can get the following result by taking congruent transformation on (16).

Theorem 3 If CB is nonsingular and (A, B) is stabilizable, let $C = (I_m \quad 0)$, then there exists a state feedback and input transformation (K, L) such that (15) is SPR if and only if A_2 as given above is Hurwitz stable.

Proof Taking congruent transformation by S on (16), one gets

$$S(A + BK)S^{-1}SQS^T + SQS^T S^{-T}(A + BK)^T S < 0,$$

$$SQS^T S^{-T} C^T = SBL.$$

Noticing the structures of SQS^T and SB discussed above, the inequality above implies that A_2 is stable. When A_2 is stable, one can find K and Q such that the inequality above holds by stabilizability of (A, B).

Remark 4 From Theorem 3, one can see that the existence of (K, L) such that (15) is SPR is equivalent to the existence of a stable submatrix in A after a similarity transformation. Because of the equality $QC^T = BL$, Q and B have a special structure. It results in a special structure of A. In addition, the effect of L here is actually to adjust B such that CBL is positive definite.

Remark 5 If state feedback disappears in (15), the problem becomes to find a matrix L such that $C(sI - A)^{-1}BL$ is SPR. From the proof of Theorem 3, this is equivalent to the existence of a diagonal blocked matrix $Q = diag(Q_1, Q_2) > 0$ such that $A_0 Q + QA_0^T < 0$, where $A_0 = SAS^{-1}$, S is the matrix defined above by the structure of B.

III. Applications to multivariable Lur'e systems

Consider the following multivariable Lur'e system

$$\left\{ \begin{array}{lcl} \dot{x} & = & Ax + Bu - B\varphi(y), \\ y & = & Cx, \end{array} \right. \quad (17)$$

where A, B and C are as given in (1), $\varphi(y) = (\varphi_1(y_1), \cdots, \varphi_m(y_m))^T$. In the Lur'e system above, every y_i is only input to φ_i, i.e., it has a diagonal nonlinearity. Combining with the idea of input transformation given above, we consider the following input and output coupling Lur'e system under state feedback. Take $u = Kx$ and $z = Ly$, consider system

$$\left\{ \begin{array}{lcl} \dot{x} & = & (A + BK)x - B\varphi(z), \\ y & = & Cx, \end{array} \right. \quad (18)$$

where $\varphi(z) = (\varphi_1(z_1), \cdots, \varphi_m(z_m))^T$. Obviously, system (18) is generated by (17) under state feedback and a special kind of output transformation. In order to study the absolute stability of (18), we consider the following system,

$$\left\{ \begin{array}{lcl} \dot{x} & = & (A+BK)x - B\varphi(z), \\ z & = & LCx. \end{array} \right. \quad (19)$$

One can get the following simple conclusion for the absolute stability of (18).

Conclusion 1 system (18) is absolute stable if and only if (19) is absolute stable.

Remark 6 In the viewpoint of mathematics, system (18) is very simple, its absolute stability can be studied by system (19). But it is interesting in the viewpoint of input and output information coupling of systems, see [6], [7], [8] for the effects of input and output coupling in nonlinear systems, and [5] for the multiplier design for ESPR.

In what follows, we discuss the absolute stability of (18) with nonlinear functions satisfying sector constraints

$$0 \le \varphi_i(\sigma)\sigma \le \mu_i \sigma^2, \quad i = 1, \ldots, m. \quad (20)$$

Suppose $\mu = diag(\mu_1, \cdots, \mu_m)$ and the transfer function from $-\varphi(z)$ to z in (19) is $G_{KL}(s) = LC(sI - A - BK)^{-1}B$. Combining Conclusion 1 with the Popov criterion for Lur'e systems[18], one can get

Conclusion 2 If there exist K and L such that $\mu^{-1} + G_{KL}(s)$ is ESPR, then system (18) is absolute stable with sector nonlinearities (20).

By Conclusion 2 and the method in the section above, one can get

Theorem 4 If (A, B) is stabilizable and μ_i are positive bounded numbers, then there always exists (K, L) such that (18) is absolute stable with sector nonlinearities (20).

Proof By Conclusion 2, we only prove that there always exist K and L such that $\mu^{-1} + G_{KL}(s)$ is ESPR, that is, there exists $P > 0$ such that

$$\left(\begin{array}{cc} P(A+BK) + (A+BK)^T P & PB - C^T L^T \\ B^T P - LC & -2\mu^{-1} \end{array} \right) < 0.$$

By the method in [22-24], there exists L such that the inequality above holds, if and only if there exists $\lambda > 0$ such that

$$\left(\begin{array}{cc} P(A+BK) + (A+BK)^T P - \lambda C^T C & PB \\ B^T P & -2\mu^{-1} \end{array} \right) < 0 \quad (21)$$

and

$$\left(\begin{array}{cc} P(A+BK) + (A+BK)^T P & PB \\ B^T P & -2\mu^{-1} - \lambda I \end{array} \right) < 0 \quad (22)$$

hold. Obviously, if (A, B) is stabilizable and μ is positive bounded, then there always exist K and $P > 0$ such that (21) and (22) hold by taking λ large enough. This completes the proof.

Theorem 5 If A is Hurwitz stable and μ is positive bounded, then there always exists L such that such that (18)(without state feedback) is absolute stable with sector nonlinearities (20).

Proof Obviously, if A is Hurwitz stable, (21) and (22) always hold.

Remark 7 From Theorem 4, one can see that Lur'e system (18) with bounded sectors can always be absolute stable by choosing an input/output coupling matrix L under a stabilizability condition. The extreme case here is to choose L as a zero matrix. Of course this case has no sense. All solutions L can be parameterized by the solutions to the matrix inequalities by the method in [22-24], one can choose some suitable L.

Similarly, one can also consider unbounded sector constraints as follows

$$0 \le \varphi_i(\sigma)\sigma, \quad i = 1, \ldots, m. \quad (23)$$

By the traditional absolute stability of Lur'e systems, one knows that if $G_{KL}(s)$ is SPR, then system (18) is absolute stable with nonlinear functions satisfying (23). Here the design problems for (K, L) can be studied by the method given in the section above.

IV. An Extended Chua's Circuit with Two Nonlinear Functions

By [23], [24], Chua's circuit can be considered as a single input and single output Lur'e system. Some coupled Chua's circuits were studied in [12]. In order to study the effects of input and output coupling here, we consider the following extended Chua's circuit with two nonlinear functions. Its differential equation is as follows

$$\dot{v} = Av + B(f_1(v_{11}),\, f_2(v_{12}))^*, \quad (24)$$

where

small $A = \left(\begin{array}{ccc} \frac{-1}{C_{11}R_{12}} & \frac{1}{C_{11}R_{12}} & 0 \\ \frac{1}{C_{12}R_{12}} & \frac{-1}{C_{12}R_{12}} & \frac{1}{C_{12}} \\ 0 & \frac{-1}{L_1} & \frac{-R_{11}}{L_1} \end{array} \right)$,

$B = \left(\begin{array}{cc} \frac{-1}{C_{11}} & 0 \\ 0 & \frac{-1}{C_{12}} \\ 0 & 0 \end{array} \right)$, $v = \left(\begin{array}{c} v_{11} \\ v_{12} \\ i_{13} \end{array} \right)$, f_1 and f_2 are two nonlinear functions defined as

$$f_1(x) = G_{12}x + 0.5(G_{11} - G_{12})(|x+1| - |x-1|),$$

$$f_2(x) = G_{22}x + 0.5(G_{21} - G_{22})(|x+1| - |x-1|).$$

Rewrite (24) as

$$\left\{ \begin{array}{lcl} \dot{v} & = & A_1 v - B_1(g_1(y_1),\, g_2(y_2))^*, \\ y & = & C_1 v, \end{array} \right. \quad (25)$$

where

$A_1 = \left(\begin{array}{ccc} \frac{-1}{C_{11}R_{12}} - \frac{G_{12}}{C_{11}} & \frac{1}{C_{11}R_{12}} & 0 \\ \frac{1}{C_{12}R_{12}} & \frac{-1}{C_{12}R_{12}} - \frac{G_{22}}{C_{12}} & \frac{1}{C_{12}} \\ 0 & \frac{-1}{L_1} & \frac{-R_{11}}{L_1} \end{array} \right)$, $B_1 = \left(\begin{array}{cc} \frac{G_{11}-G_{12}}{C_{11}} & 0 \\ 0 & \frac{G_{21}-G_{22}}{C_{12}} \\ 0 & 0 \end{array} \right)$, $C_1 = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right)$,

$y = \left(\begin{array}{c} y_1 \\ y_2 \end{array} \right)$, $g_1(x) = g_2(x) = 0.5(|x+1| - |x-1|)$, v is as given in (24). Obviously, system (25) is with Lur'e form. And g_i satisfy

$$0 \le x g_i(x) \le x^2, \quad i = 1, 2.$$

Taking parameters $C_{11} = 1, C_{12} = 10, R_{11} = 0, R_{12} = 1, L_1 = 1/1.487, G_{11} = -1.27, G_{12} = -0.68, G_{21} =$

$-0.1, G_{22} = 2$, system (25) has a canonical chaotic attractor with these parameters [17], see Fig. 1.

(a) $v_{11}(t), v_{12}(t), i_{13}(t)$ (b) v_{11}, v_{12}, i_{23} space

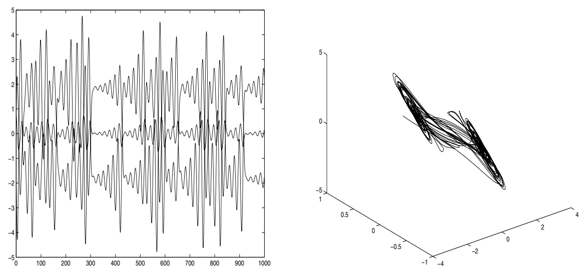

Fig. 1 The solution of (25) with initial value $v(0) = [0.1, 0.9, -1]^T$.

Let $K(s) = C_1(sI - A_1)^{-1}B_1$ be the transfer function from $(g_1(y_1, g_2(y_2))^*$ to $(y_1, y_2)^*$ in (25). For the parameters chosen above, A_1 is Hurwitz stable. By Theorem 5, there exists L such that $I + LK(s)$ is ESPR. Actually, by solving LMI, one can get such a L easily, $L = \begin{pmatrix} -1.279 & -2.7723 \\ -0.9959 & -5.3222 \end{pmatrix}$. For this L, of course the following system is absolute stable,

$$\left\{ \begin{array}{rcl} \dot{v} & = & A_1 v - B_1(g_1(z_1),\, g_2(z_2))^T, \\ y & = & C_1 v, \end{array} \right. \qquad (26)$$

where $z = (z_1,\, z_2)^T = Ly$, A_1, B_1 and C_1 are as given in (25). See Fig. 2 for the solution of (26) at the same initial value given above.

(a) $v_{11}(t), v_{12}(t), i_{13}(t)$ (b) v_{11}, v_{12}, i_{23} space

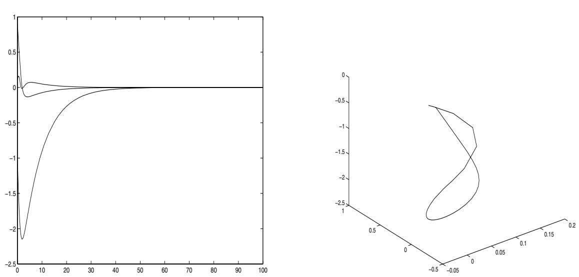

Fig. 2 The solution of (26) with initial value $v(0) = [0.1, 0.9, -1]^T$.

V. CONCLUSIONS

The effects of state feedback and input transformation are studied for ESPR, SPR and absolute stability of Lur'e systems. The results show that input transformation can play useful roles in these kinds of problems. Compared with state feedback, input transformation has been paid much less attention in linear system theory. In fact input transformation can change zeros of systems, and state feedback can change the eigenvalues of closed-loop systems. Therefore, by combining state feedback with input transformation, one can improve system performances effectively.

REFERENCES

[1] B.D.O. Anderson and J.B. Moore, *Linear Optimal Control.* New York: Prentice Hall, 1970.

[2] M. Arcak, M. Larsen and P. Kokotovic, Circle and Popov criteria as tools for nonlinear feedback design, *Automatica,* 39, 2003, pp 643-650.

[3] N.E. Barabanov, The state space extension method in the theory of absolute stability, *IEEE Trans. A. C.,* 45(12), 2000, pp 2335-2339.

[4] C.A. Desoer, and M. Vidyasagar, *Feedback systems: Input-Output Properties.* San Diego: Academic Press, 1975.

[5] Z. S. Duan, L. Huang, and L. Wang, Multiplier design for extended strict positive realness and its applications, *International Journal of Control,* 77(17), 2005, 1493-1502.

[6] Z. S. Duan, J.Z. Wang and L. Huang, Multi-input and multi-output nonlinear systems: Interconnected Chua's circuit, *International Journal of Bifurcation and Chaos,* 14(9), 2004, 3065-3081.

[7] Z. S. Duan, J.Z. Wang and L. Huang, Frequency-domain method for the property of dichotomy of modified Chua's equations, *International Journal of Bifurcation and Chaos,* 15(8), 2005, In press.

[8] Z. S. Duan, J.Z. Wang and L. Huang, Input and output coupled nonlinear systems, *IEEE Trans. on Circuits and Systems-I: Regular Papers,* 52(3), 2005, In press.

[9] M. Green and D. J. N. Limebeer, *Linear Robust Control.* Englewood Cliffs, New Jersey: Prentice Hall, 1995.

[10] A. Helmersson, *Methods for robust gain scheduling.* Linkoping, Sweden, 1995.

[11] L. Huang, Y. P. Zheng and D. Zhang, The second method of Lyapunov and the analytical design of the optimum controller, *Acta Automatica Sinica*, Vol. 2, No. 4, 1964, pp 202-218.

[12] T. Imai, K. Konishi, H. Kokame and K. Hirata, An experimental suppression of spatial instability in one-way open coupled Chua's circuits,*International Journal of Bifurcation and Chaos,* 12, 2002, pp 2897-2905.

[13] T. Iwasaki,R.E. Skelton, All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas. *Automatica,* 30, 1994, 1307-1317.

[14] T. Kailath, *Linear Systems*, Englewood Cliffs, N. Y.: Prentice-Hall, 1980.

[15] T. Kapitaniak, L.O. Chua and G.Q. Zhong, Experimental hyperchaos in coupled Chua's circuits, *IEEE Trans. on Circuits and Systems-I,* 41, 1994, pp 499-503.

[16] Y.D. Landau, *Adaptive Control: The Model Reference Approach.* New York: Marcel Dekker, 1979.

[17] R. Madan, *Chua's Circuit: A Paradigm for Chaos,* Singapore, World Scientific, 1993.

[18] P. Park, A revisited Popov criterion for nonlinear Lur'e systems with sector restrictions, *International Journal of Control,* 68(3), 1997, pp 461-469.

[19] V.M. Popov, *Hyperstability of Automatic Control Systems.* New York: Springer-Verlag, 1973.

[20] A. Rantzer, On the Kalman-Yakubovich-Popov lemma. *Systems Control Letters,* 28, 1996, pp 7-10.

[21] R.N. Shorten and K.S. Narendra, On common quadratic Lyapunov functions for pairs of stable LTI systems whose system matrices are in companion form, *IEEE Trans. on Automatic Control,* 48(4), 2003, 618-621.

[22] W. Sun, P.P. Khargonekar and D. Shim, Solution to positive real control problem for linear time-invariant systems. *IEEE Trans. A. C.,* 39(10), 1994, pp 2034-2046.

[23] J. A. K. Suykens, P. F. Curran, J. Vandewalle and L.O. Chua, Robust nonlinear H_∞ synchronization of chaotic Lur'e systems, *IEEE Trans. on Circuits and Systems-I,* 44(10), 1997, pp 891-904.

[24] J. A. K. Suykens, T. Yang and L.O. Chua, Impulsive synchronization of chaotic Lur'e systems by measurement feedback, *International Journal of Bifurcation and Chaos,* 8(6), 1998, pp 1371-1381.

[25] R.D. Wang, Algebraic criteria for absolute stability, *Systems and Control Letters,* 47, 2002, pp 401-416.

[26] W. M. Wonham, *Linear Multivariable Control: A Geometric Approach*, New York: Springer-Verlag, 1979.

[27] V.A. Yakubovich, Solution of certain matrix inequalities in the stability theory of nonlinear control systems. *Soviet Math. Dokl.* 3, 1962, pp 620-623.

[28] K. Zhou, J.C. Doyle and K. Glover, *Robust and Optimal Control.* Englewood Cliffs, New Jersey: Prentice Hall, 1996 .