2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

On the Dynamics of the Pitch Control Loop in Horizontal-Axis Large Wind Turbines

Shashikanth Suryanarayanan and Amit Dixit

Abstract— **Most commercial large wind turbines use blade pitch action to mitigate structural loads in high wind velocity conditions. In this paper, we study the linearized dynamics of the map from blade pitch to tower top fore-aft deflection in horizontal-axis wind turbines. We show that the mass and stiffness distribution of the blades at certain operating conditions determine the presence (or absence) of a right half-plane zero in the transfer function of interest. We conclude that blade design can impose constraints on the achievable tower fore-aft oscillation mitigation through collective blade pitch control. Consequences of this result to wind turbine design are discussed.**

I. INTRODUCTION

Wind power is the fastest growing electric power industry in the world [1]. Current global installed capacity exceeds 32000 MW with a projected growth rate of 10000 MW/year for the next five years. The phenomenal growth of this industry can be attributed, primarily, to the rapid progress made in wind turbine technology. Over the last couple of decades, technological progress has led to the development of turbines with high power capture efficiency. Today, many wind farms produce electrical power at a Cost-of-Energy ($\approx$ 4-6 c/kWHr) comparable to that of coal and natural gas based power plants.

For economic reasons, much of the work on wind turbine development has focused on large wind turbines. Typical large wind turbines of today are massive structures (Fig. 1) with enormous blade spans (70-100m in diameter), tall towers (60-100m in height) with power ratings in the 1-5 MW range. Amongst the technologies that enable realization of these machines, advanced control plays a pivotal role. Modern large wind turbines are endowed with sophisticated control systems which are organized to support several modes of operation such as start-up. shut-down, power production etc.

For the purposes of this paper, we look at a component of the control routine used in the power production mode. The control objectives for this mode differ vastly depending on the prevailing wind conditions. During low wind operation, the goal is to maximize energy capture. However, in conditions where wind speeds are greater than the turbine's "rated" wind speed (usually around 10-12 m/s) the primary objective is to minimize fatigue loading of the turbine structure. At higher wind speeds, most large wind turbines use blade pitch action to keep structural loads within design limits at high wind speeds.

S. Suryanarayanan and A. Dixit are with the Department of Mechanical Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India. Email: : adixit@me.iitb.ac.in, shashisn@iitb.ac.in

Fig. 1. Horizontal-Axis Large Wind Turbine (Courtesy: GE Energy)

In this paper, we study the input-output dynamics of the map from blade pitch on the fore-aft (swaying into and away from wind) motion of the tower in large horizontal-axis wind turbines. More specifically, our interest will be in investigating the linearized behaviour of this map at higher wind speeds. The study of this map gains significance owing to the flexible nature of large wind turbine towers. Tower top deflections as much as 15 cm are not uncommon in multi-megawatt turbines. Since tower oscillations are correlated strongly with fatigue loading of the tower, the interest is in mitigating oscillations in the tower.

The paper is organized as follows. Section II discusses wind turbine control preliminaries. This is followed, in Section III by an analysis of behaviour of the linearized dynamics of interest, namely the transfer function from blade pitch to tower top fore-aft deflection. This section also discusses the implications of the results of the analysis on control performance as well as turbine component design. The paper concludes with a summary and a discussion on future problems of interest.

The primary contribution of the paper is the result that blade design significantly impacts achievable load mitigation performance through pitch control action. It is shown that for certain turbine configurations and operating conditions, the transfer function of interest consists of a right half-plane zero - a situation which imposes a constraint on achievable control performance. In addition, the result discussed in the paper exposes the need for an integrated "systems" approach to wind turbine design.

II. PRELIMINARIES

In this section we focus on the background required for the reader to appreciate the discussion in Section III. We begin with a description of the architecture of the class of turbines studied in this paper. This will be followed by a discussion on blade aerodynamics. This, in turn, will be followed by a description of the control problems of interest in the paper. The section will conclude with a note on the modal approach to simulating the structural dynamics of wind turbines.

A. Turbine Architecture

The wind turbine modeled in this study is a horizontal-axis (axis of rotation of rotor is horizontal), three-bladed, upwind (rotor points into wind), large wind turbine. This is the most common architecture in current commercial wind turbines. Most of today's turbines support variable speed operation. In other words, the rotor and generator speed is not rigidly coupled to that of the grid frequency. The most popular generator architecture that supports variable speed action is the doubly-fed induction generator. Such a machine allows for the generator torque demand to be controlled independent of generator speed/slip. Each of the blades of the turbine can be pitched about its longitudinal axes. The common actuators used for large wind turbine control are the generator and the three blade pitch actuators. Generator speed is the primary sensed variable that is used for real-time control.

B. Blade Aerodynamics

In order to appreciate the forces and moments acting on the turbine structure, it is useful to understand the elements of blade aerodynamics. Here, we present the gist of the so-called Blade Element Momentum (BEM) theory [2].

Fig. 2 shows a typical blade section (note the aerofoil shape). We assume that wind is incident normal to the plane of rotation with velocity V. Due to the rotation of the blade, the section sees a relative wind velocity W. The angle α made by the relative wind velocity with blade chord line is called the Angle-of-Attack (AoA). The angle made by the chord line with the plane of rotation is the sum of the twist angle ϕ of the blade element and the pitch angle β of the blade. The incident wind produces a "lift" force F_L and a "drag" force F_D which are, respectively, normal to and parallel to W. The resultant of these forces may be resolved into an axial component F_T and a tangential component F_r. If the blades are rigid, the sum of axial forces from all blade elements is termed as the thrust T on the rotor. The tangential component of the force on the blade element contributes to aerodynamic torque.

The lift and the drag forces on the blade section can be written as:$F_L = (\frac{1}{2}\rho A W^2)C_L, F_D = (\frac{1}{2}\rho A W^2)C_D$ where C_L and C_D are the lift and drag coefficients associated with the aerofoil. It is well known that C_L and C_D depend primarily on the AoA. The typical variation of these coefficients with AoA is shown in Fig. 3.

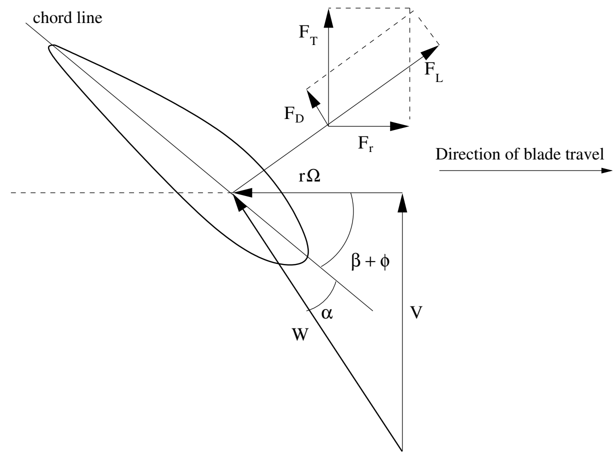

Fig. 2. Aerodynamics of Blade Section

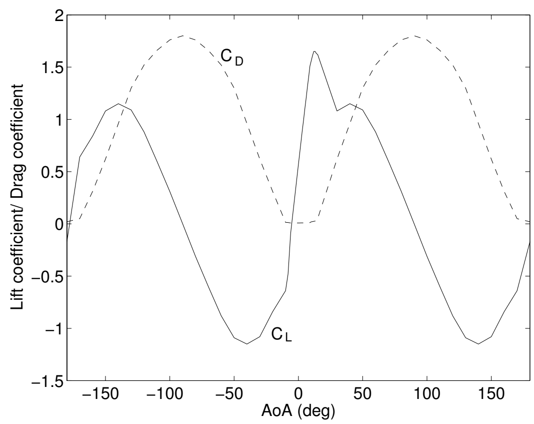

Fig. 3. Variation of C_L, C_D with Angle-of-Attack

C. Pitch Control of Large Wind Turbines

From the above expressions for lift and drag forces, it can be seen that the thrust force and rotor aerodynamic torque may be controlled by controlling the angles of attack of the wind's interaction with the blade elements. The angles of attack associated with different blade elements can be controlled, to a significant extent, by appropriate choices for the pitch angle and rotor rotational speed. The rotor rotational speed is often controlled by imposing an appropriate generator torque. In wind turbine control parlance, control of the AoA using pitching action and generator torque demand variation are called 'pitch control' and 'torque control' respectively.

As seen from the expressions for lift and drag, high wind speeds result in large forces on blade elements and consequently on the turbine structure. In the interest of preserving the structural integrity of the turbine, it is important that the fatigue damage due to fluctuating structural loads is minimized. Blade pitch action is used for this purpose. Pitching results (almost always) in reduced angles of attack and consequently reduced loads.

In the past, pitch control strategies have been synthesized based on design principles for single-input, single-output feedback loops [3]. A popular strategy, that has been implemented on a large number of turbines, is to maintain a constant generator torque demand and to vary the blade pitch angles so as to regulate generator speed. A by-product of this strategy is the limiting of power and structural loads to within design limits. More recently, the wind turbine control community has begun to appreciate the importance of design of pitch controllers to account for the multivariable nature of the problem as well as explicitly address the problem of mitigation of structural loads. One of the primary loads of interest are the forces and moments at the tower top/base. In this context, we study the the input-output map from collective blade pitch (all three blades pitched by the same amount) to the tower top fore-aft displacements.

D. Modal Simulation

In this subsection, we discuss the "modal approach" to simulate the structural dynamics of wind turbines.

The structural components of wind turbines can be viewed as distributed parameter systems. The response of such a system to an external excitation may be written as weighted sum of an infinite number of basis functions (called "modes"). Associated with each mode is mode shape - a non-dimensional function which describes the spatial variation of the amplitude of vibration of the mode. The response of distributed parameter systems can often be approximated by superimposing responses of a finite number of 'dominant' modes of the system. The number of modes to be considered for a good approximation of the response is found by ensuring that the norm of the difference between the exact response and the approximated response for a predefined input is less than a critical value. This is the principle behind modal simulation and the approximation method is known as the "Assumed Modes" method [4].

III. Analysis of Blade-Tower Dynamics

As mentioned before, in this study we are interested in studying the linearized input-output map from blade pitch to tower top fore-aft displacement in large wind turbines. A linear model to describe this map may be obtained numerically from non-linear simulation design code such as

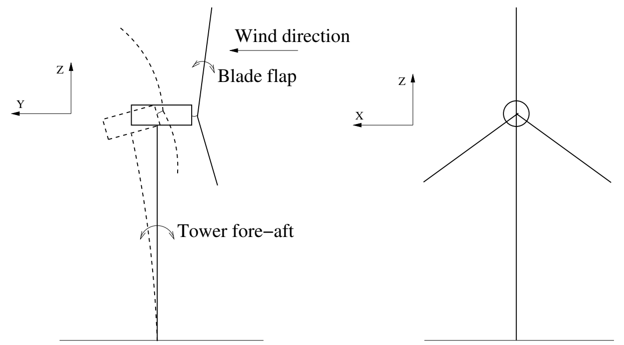

Fig. 4. Schematic Diagram showing tower fore-aft and blade flap degrees of freedom

BLADED, FAST etc. In this study we look to analytically generate the desired relationship.

We model the blade-tower system as a coupled two degree of freedom system. The degrees of freedom correspond to the flapping motion of the blade and the fore-aft motion of the tower. Fig. 4 motivates the choice of such a model structure. Owing to the flexible nature of large wind turbine components, blade flap motion (operating pitch angles are assumed to be small) is often coupled with tower fore-aft motion. About a suitable operating point, the linearized model describing the blade-tower dynamics may be written as:

$$[M]\ddot{q} + [C]\dot{q} + [K]q \quad = \quad [F]\beta$$

where, $[M]$, $[C]$ and $[K]$ denote the Mass, Damping and the Stiffness matrices respectively. The matrix $[F]$ denotes the "forcing" matrix, $q = [x_t \, x_b]^T$ where x_b and x_t respectively correspond to the blade flap and tower fore-aft modal deflections and β corresponds to the deviation of the pitch angle from its value at the operating point.

In this study $[M]$, $[C]$, $[K]$ and $[F]$ matrices were constructed based on the "Assumed Modes" method. The single most dominant tower fore-aft and blade flap modes are considered in the model. The structures of the matrices involved is as shown below.

$$M = \begin{bmatrix} m_t & m_c \\ m_c & m_b \end{bmatrix}$$

$$K = \begin{bmatrix} k_t & 0 \\ 0 & k_b \end{bmatrix}$$

$$C = \begin{bmatrix} c_t & c_c \\ c_c & c_b \end{bmatrix}$$

$$F = \begin{bmatrix} f_t \\ f_b \end{bmatrix}$$

The elements of $[M]$ and $[K]$ matrices were obtained by computing the modal mass and modal stiffness of the blade and the tower as follows: $m_b = \int_0^R \mu_b^2(r) m_{b,r}(r) dr; m_c =$

$\int_0^R \mu_b(r)m_{b,r}(r)dr; m_t = \int_0^H \mu_t^2(z)m_{t,z}(z)dz; k_b = \int_0^R [\mu_b'']^2(r)k_{b,r}(r)dr$ and $k_t = \int_0^H [\mu_t'']^2(z)k_{t,z}(z)dz$. where, μ_b and μ_t are the mode shapes of the first blade flap and first tower fore-aft mode respectively. R is the rotor radius and H is the tower height. $m_{b,r}$ and $m_{t,z}$ respectively denote the blade and tower mass per unit length while $k_{b,r}$ and $k_{t,z}$ denote the blade and tower stiffness per unit length. The symbol $\mu^{''}$ denotes the second derivative of the mode shape (with respect to space).

The damping matrix is composed of structural damping and aerodynamic damping elements. Given the structural damping ratios, the structural damping element in the damping matrix may be computed as: $(c_b)_{struct} = 2\zeta_b\sqrt{k_b m_b}; (c_t)_{struct} = 2\zeta_t\sqrt{k_t m_t}$

The aerodynamic damping (reduction of lift force due to blade flapping) and the forcing vector are nonlinear in nature and have to be linearized about an operating point. In this study, the expressions for the linearized aerodynamic damping and the linearized forcing vector were developed symbolically as: $(c_b)_{aero} = \int_0^R d(r)\mu_b(r)^2 dr; (c_c) = (c_c)_{aero} = \int_0^R d(r)\mu_b(r)dr; (c_t)_{aero} = 3\int_0^R d(r)dr; f_b = \int_0^R f(r)\mu_b(r)dr$ and $(f_t) = 3\int_0^R f(r)dr$ where

$$
\begin{aligned}
d(r) &= \frac{1}{2}\Omega\rho[rc(r)[\frac{dC_l}{d\alpha}](r)]cos(\Phi_r) + [C_L](r)sin(\Phi_r) \\
f(r) &= -\frac{1}{2}\rho c(r)\sqrt{r^2\Omega^2 + v^2}\, r\Omega\frac{dC_L}{d\alpha}
\end{aligned}
$$

Here, Ω is turbine rotational speed, ρ is the air density and $\Phi_r = atan(\frac{v}{r\Omega})$. Finally, we have $c_b = (c_b)_{aero} + (c_b)_{struct}$ and $c_t = (c_t)_{aero} + (c_t)_{struct}$

Once the $[M]$, $[K]$, $[C]$ and $[F]$ matrices are evaluated at a chosen operating point, the required transfer function of interest may be easily computed. Symbolically, the transfer function from pitch to tower fore-aft deflection looks like

$$G_{\beta\to x_t}(s) = \frac{a_2 s^2 + a_1 s + a_0}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

where, $a_0 = k_b f_t; a_1 = f_t c_b - 3c_c f_b; a_2 = f_t m_b - 3m_c f_b; b_0 = k_t k_b; b_1 = c_b k_t + k_b c_t; b_2 = k_t m_b - 3c_c^2 + c_b c_t + m_t k_b; b_3 = m_b c_t - 6m_c c_c + c_b m_t$ and $b_4 = m_t m_b - 3m_c^2$

The above transfer function was evaluated for various turbine configurations and operating conditions. The behaviour of the transfer function for three sets of turbine configurations/operating conditions, named 'WT1', 'WT2' and 'WT3' is discussed below.

WT1: Rotor diameter = 70 m, Tower height = 90 m, Rated power = 1.5 MW operating conditions: Wind speed = 15 m/s, Pitch angle = 0^o and q_{op} =$[7.245\text{e-}2\ 3.207]^T$. The transfer function is:

$$G1_{\beta\to x_t}(s) = \frac{2.426s^2 - 4.6345s - 147.3}{s^4 + 4.857s^3 + 126.2s^2 + 266.4s + 3659}$$

WT2: Rotor diameter = 15 m, Tower height = 25 m, Rated power = 50 kW operating conditions: Wind speed = 15 m/s, Pitch angle = 0.75^o and q_{op} =$[1.97\text{e-}2\ 6.505\text{e-}2]^T$. The transfer function is:

$$G2_{\beta\to x_t}(s) = \frac{-0.2545s^2 - 0.0647s + 0.9384}{s^4 + 2.28s^3 + 878.5s^2 + 437.7s + 7.7e4}$$

WT3: Rotor diameter = 27 m, Tower height = 42m, Rated power = 275 kW operating conditions: Wind speed = 15 m/s, Pitch angle = 0^o and q_{op} =$[6.848\text{e-}3\ 8.915\text{e-}2]^T$. The transfer function is:

$$G3_{\beta\to x_t}(s) = \frac{-0.6219s^2 - 8.7165s - 2911}{s^4 + 5.02s^3 + 691s^2 + 1949s + 1.2e5}$$

The turbine parameters for these configurations were obtained from [5]. The transfer functions so obtained are found to be in close agreement with the the transfer functions obtained by numerical linearization carried out with FAST - an aforementioned aero-elastic wind turbine simulator. Also, the response predicted by these transfer functions near the operating point is found to be in agreement with that predicted by the nonlinear simulation routines in FAST.

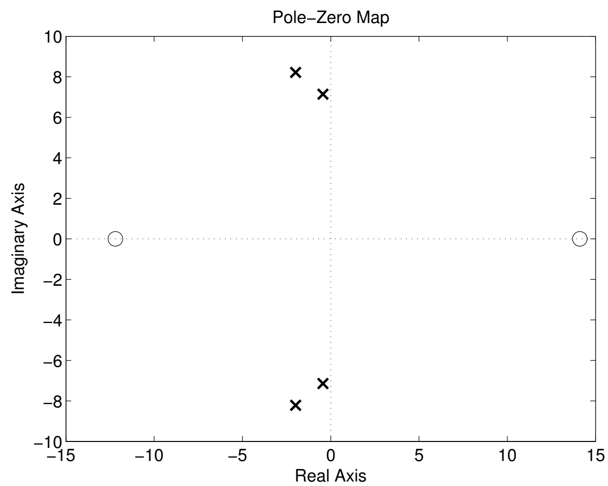

Fig. 5. Pole-zero map for WT1

Figs. 5, 6 and 7 show the pole-zero maps for WT1, WT2 and WT3 respectively. As can be seen from these plots, systems G_1 and G_2 have a right half-plane (RHP) zero while G_3 does not. Note that for system G_2, the RHP zero is "close" to the imaginary axis. We wish to mention that RHP zeros occur in several other turbine configurations for a variety of operating conditions. In the interests of brevity, we do not describe these configurations/operating conditions here.

It is well known that RHP zeros impose a limitation on achievable control performance (see, for example, [6]). In the context of the problem at hand, an RHP zero situation implies a limitation on achievable reduction of tower fore-aft oscillations using pitch control. The limitation can be especially serious in the case where the RHP zero is close to the imaginary axis (as in system G_2). Since the coefficients of the polynomials that define the transfer function from blade pitch to tower fore-aft deflection depend only on the

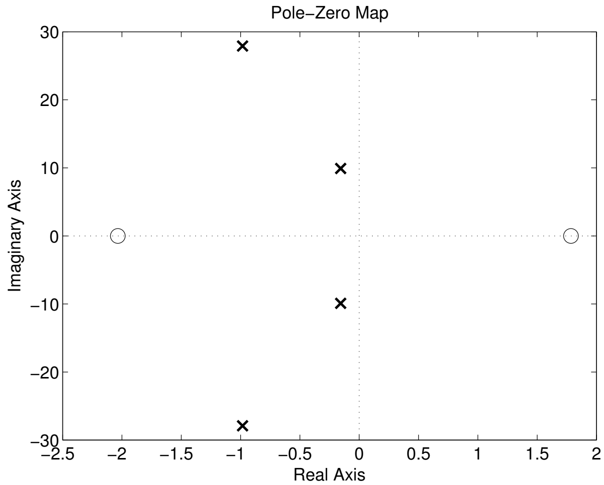

Fig. 6. Pole-zero map for WT2

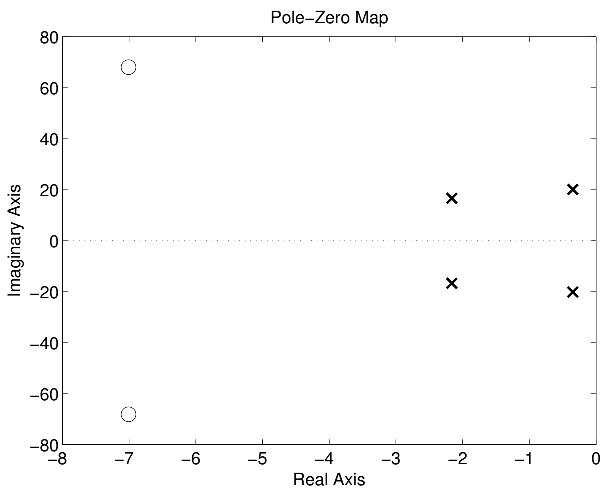

Fig. 7. Pole-zero map for WT3

blade mass and stiffness distributions (see expressions for a_2, a_1 and a_0), the above observations imply that blade design can significantly affect achievable tower load mitigation control performance through pitch control. Since towers are sized based on the expected fatigue/ultimate loads over the life of the turbine, this result indicates that if adequate care is taken in the design of the blades and the pitch control loop, towers may see smaller fatigue/ultimate loads. Consequently, they can be made lighter/cheaper.

We conclude our discussion with a possible explanation for the occurrence of the RHP zero for some turbine configurations/operating conditions. The existence of an RHP zero implies that when pitch angle is increased the tower experiences an increase in fore-aft deflection and tower top thrust force. This condition can occur if the decrease in lift forces on blade elements (due to increase in pitch angle) is offset by the inertial force associated with the flapping of the blade. We have noticed that such a situation occurs when the ratio of the coupled mass m_c to the blade mass m_b is large. A large $\frac{m_c}{m_b}$ ratio usually implies mass concentration nearer to the blade root. Further investigation is necessary to corroborate this postulate.

IV. SUMMARY AND FUTURE WORK

Blade pitch action is used in large wind turbines to mitigate loads on the turbine structure in high wind speed conditions. In this paper, the linearized dynamics of the effect of collective blade pitch action on tower top fore-aft deflection in horizontal axis wind turbines were discussed. A two degree of freedom linearized model was constructed to capture the coupled effects of blade flap and tower fore-aft motions. It was shown that for certain blade mass/stiffness distributions and operating conditions, the linearized dynamics of interest demonstrate non-minimum phase behaviour. We postulated that this behaviour could be due to mass concentration away from the blade root. The non-minimum phase condition imposes a constraint on achievable tower fore-aft oscillation mitigation through blade pitch action. The consequences of this result to wind turbine tower sizing were discussed.

The result discussed in this paper provides a platform for several interesting investigations. Our immediate interest is in performing limits of performance calculations on achievable tower load mitigation performance using collective blade pitch action given blade mass and stiffness distributions. Longer term problems of interest include mulitavariable design of pitch/torque control and the evolution of an integrated systems approach based paradigm for wind turbine component design.

REFERENCES

[1] Danish Wind Industry Association*http://www.windpower.org.*
[2] T. Burton, D. Sharpe, N. Jenkins and E. A. Bossanyi, *Wind Energy Handbook*, John Wiley & Sons, Ltd, NY; 2001.
[3] E. A. Bossanyi, *The Design of Closed-loop Controllers for Wind Turbines*, Wind Energy, vol. 3, 2000, pp. 149-163.
[4] L. Meirovich, *Elements of Vibration Analysis*, McGraw-Hill , NY; 1986.
[5] FAST: An aero-elastic design code for horizontal axis wind turbines *http://wind.nrel.gov/designcodes/simulators/fast*
[6] K. J. Astrom, *Limitations on Control System Performance*, European Journal of Control Energy, vol. 6(1), 2000, pp. 2-20.