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On Control Performance Assessment Based on Lag-Delay Models

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Abstract— **In the paper, suitability of control quality assessment based on delay approximation for delay-free plants is investigated. To this end, the LQG control benchmark, which can be seen as a MV benchmark with bounded control variance, is compared for both linear delay-free continuous-time plants with outputs corrupted by a stochastic disturbance and their lag-delay models. Using approximated plant models, the area of achievable accuracy is then defined for control performance assessment. Control systems with discrete-time PID controllers tuned both classically and optimally in such way that disturbance characteristics are taken into account are considered and compared with the benchmark**

I. INTRODUCTION

Complex systems are comprised of numerous loops which are controlled by local SISO controllers. The decision to retune or replace any of these controllers should be preceded by an investigation whether and to what extent this would improve performance. Such procedure is referred to as benchmarking or control performance assessment [7]-[10], [15]. Most related pieces of research done so far assume MV control as the performance lower bound. The well known performance measure is as follows

$$\eta = \frac{\sigma_{mv}^2}{\sigma_y^2}, \qquad \text{where} \quad 0 \le \eta \le 1 \tag{1}$$

Invariable part of output variance, equal to hypothetical minimum variance, is determined by

$$\sigma_{mv}^2 = \left(f_0^2 + f_1^2 + f_2^2 + \ldots + f_{d-1}^2\right)\sigma_a^2 \tag{2}$$

with impulse response coefficients $f_0, f_1, \ldots f_{d-1}$,variance of stochastic disturbance σ_a^2 , and discrete-time delay d.

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The main point stressed in the literature is that the system delay d is known to the process engineer. Unfortunately, since the true transport delay is rarely present in industrial plants, the requirement of a'priori knowledge of the plant delay seems to be too optimistic. In industrial practice the delay is usually meant as the delay part in the popular lag–delay approximation, see Fig. 1, of the plant dynamics. Therefore it is interesting to know whether such delay is authoritative enough for control performance assessment.

Another disadvantage of MV based benchmark is that it does not take the control effort into account. In order to remove this drawback a modified MV strategy is considered in this paper. This results in the LQG problem. Since most of control loops in the industry are equipped with PID type controllers, it is interesting to know the distance of their control performance from the best achievable one. Furthermore, it is interesting to know how the controller settings determined optimally on the basis of the disturbance characteristics would improve the performance.

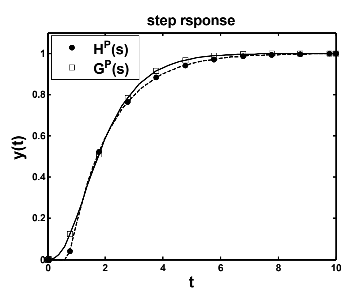

Fig. 1. Step responses of the plant $G^p(s)$, and its lag-delay approximation $H^p(s)$.

The paper is organized as follows. In Sections 2 we state the control problem for a continuous-time delayed SISO system with a stochastic disturbance. In Section 3 we present algorithms resulting from problem defined in Section 2. Next in Section 4 we derive formulas to calculate output and control variances for both LQG and classical controllers. In Section 5 an example is used to compare the control performance assessment based on lag approximation with that based on original non-delayed system, showing uncertainty of the benchmark lower bound.

II. PROBLEM STATEMENT

Our main goal is to check whether the delay obtained via lag-delay approximation, and the approximation itself, are reasonable for control performance assessment of original non-delayed plants.

The problem requires calculation of output and control variances for systems controlled by optimal LQG and PID type controllers.

Since the solution for delay-free systems has been already extensive presented [3], in this paper we focus on delayed model problem.

A. Mathematical Problem Statement

Consider the linear SISO plant modeled by the following stochastic, continuous-time system

$$\frac{dx(t)}{dt} = Ax(t) + bu(t-\tau) + c\dot{\xi}(t) \tag{3}$$

$$y(t) = d'x(t), \tag{4}$$

where *x(t)* is *p* – dimensional state vector, *A* is $p \times p$ – dimensional matrix, *b, c* and *d* are *p* – dimensional vectors. The initial condition x_0 is assumed to be a normal random vector, $x_0 \sim N(0,Q_0)$. $\xi(t)$ is a Wiener process, and $var\ \xi(t)=\delta(t)$.

The time delay is defined as follows

$$\tau = lh - h + \theta \tag{5}$$

where $l \geq 0$ and $0 < \theta \leq h$. The plant is controlled by the output *u(t)* of a ZOH device with period *h*

$$u(t) = u_k, \quad \text{for } t \in (kh, kh+h], \quad k = 0,1,\ldots, \tag{6}$$

driven by the digital controller output *u(k)*, which changes its value in discrete time instants $t_k=kh$, k=0,1,... . The output of the system is assumed to be measured synchronically at instants t_k as:

$$z_k = d' x_k + n_k, \tag{7}$$

where n_k is the measurement error modeled as white noise with zero mean $E[n_k]=0$, and variance $E[n_i^2]=\nu^2$.

The aim of the system is to minimize the average value of the system error variance with a limit imposed on control variance. The considered problem is equivalent to minimization of the following performance index

$$I^c = \lim_{N\to\infty} E\frac{1}{Nh}\int_0^{Nh} \left\{ e^2(t+\tau) + \lambda u^2(t) \right\} dt \tag{8}$$

III. ALGORITHMS

A. LQG Benchmark

Introduce the predictable state

$$x^p(t) = x(t+\tau) \tag{9}$$

The system defined by state equation (3), measurement equation (7), modulation equation (6) and performance index (8) can be described at sampling instants as

$$x_{k+1}^p = Fx_k^p + gu_k + w_k^p \tag{10}$$

$$z_k = d' x_k^p (kh - \tau) + n_k \tag{11}$$

$$I_k = \lim_{N\to\infty} E\frac{1}{N}\sum_0^{N-1} \left\{ x_j^{p}{}' Q_1 x_j^p + 2x_j^p q_{12} u_j + q_2 u_j^2 + q_w \right\} \tag{12}$$

where $w_k^p = w(kh+\tau)$ is zero mean Gaussian white noise vector with covariance $E\{w_k^p, w_k^{p}{}'\} = W$. Vectors x_0 and $[w_k^{p}{}', n_k]$ are uncorrelated for all $k \geq 0$. The relationship [1,13,14,15,16] between matrices and vectors defining system (10)-(11) with the index (12) and the system (3)-(4) with (8) are presented in Appendix.

The optimal control law minimizing the performance index (12) for the system (10)-(11) is

$$u_k = -k_c' \hat{x}_{k|k}^p = -k_c' F_p \underbrace{\begin{bmatrix} \hat{x}_{k|k} \\ u_{k-d} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}}_{\hat{x}_{k|k}^p} \tag{13}$$

where

$$F_p = [F(\tau) \quad F(\tau-\theta)g(\theta) \quad F(dh-2h)g \quad \cdots \quad Fg \quad g] \tag{14}$$

$\hat{x}_{k|k}^p$ is an estimate of the state x_k^p using measurements z_k up to and including k. The feedback gain vector depends on the positive solution *S* of the following algebraic Riccati equation

$$S = Q_1 + F'SF - \frac{(q_{12} + F'Sg)(q_{12} + F'Sg)'}{q_2 + g'Sg} \tag{15}$$

$$k_c = \frac{q_{12} + F'Sg}{q_2 + g'Sg} \tag{16}$$

Stationary Kalman filter [1,2,4,6,13,14] for the system (10)-(11) takes the following form

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + k^f\left(z_k - d'\hat{x}_{k|k-1}\right) \tag{17}$$

$$\hat{x}_{k+1|k} = F\hat{x}_{k|k} + g(h-\theta)u_{k-d+1} + F(h-\theta)g(\theta)u_{k-d} \tag{18}$$

where

$$k^f = \frac{\Sigma d}{\nu^2 + d'\Sigma d} \tag{19}$$

$$\Sigma = W + F\left(\Sigma - \frac{\Sigma d d' \Sigma'}{\nu^2 + d'\Sigma d}\right)F' \tag{20}$$

B. PID type controllers

The system in (3)-(4) controlled by means of discrete time classical PID, PI, PD and P controllers is considered. Controller settings are supplied in two ways: as minimization result of the performance index (12), and by means of one of the classical methods [3] called QDR (Quarter Decay Ratio).

The control law for classical controllers is defined by the following equations

- state equation

$$x_{k+1}^c = A^c x_k^c - B^c z_k \tag{21}$$

- and output equation

$$u_k = C^c x_k^c - D^c z_k, \tag{22}$$

where for discrete time PID controller matrices and vectors defining the control law are the following form

$$\left.\begin{aligned} A^c &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C^c = \begin{bmatrix} K_p \dfrac{h}{T_i} & -Kp\dfrac{T_d}{h} \end{bmatrix}, \\ B^c &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad D^c = \begin{bmatrix} K_p\left(1 + \dfrac{h}{T_i} + \dfrac{T_d}{h}\right) \end{bmatrix}, \end{aligned}\right\} \tag{23}$$

Since the predictable description of the system (10)-(11) is not useful for the PID control algorithm, the system defined by state equation (3), measurement equation (7), modulation equation (6) and performance index (8) is described at sampling instants in alternative form

$$x_{k+1} = Fx_k + \Gamma_0 u_{i-d+1} + \Gamma_1 u_{k-d} + w_k \tag{24}$$

$$z_k = d' x_k + n_k \tag{25}$$

where

$$\Gamma_1 = e^{A(h-\theta)}\int_0^{\theta} e^{Av} b\,dv, \quad \Gamma_0 = \int_0^{h-\theta} e^{Av} b\,dv \tag{26}$$

The corresponding state-space description is as follows

$$\left.\begin{aligned} \begin{bmatrix} x_{k+1} \\ u_{k-d+1} \\ \vdots \\ u_{k-1} \\ u_k \end{bmatrix} &= \underbrace{\begin{bmatrix} F & \Gamma_1 & \Gamma_0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{\underline{F}} \underbrace{\begin{bmatrix} x_k \\ u_{k-d} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}}_{\underline{x}_k} \\ &+ \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\underline{\Gamma}} u_k + \underbrace{\begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\underline{\Omega}} w_k \end{aligned}\right\} \tag{27}$$

or in aggregated form

$$\underline{x}_{k+1} = \underline{F}\,\underline{x}_k + \underline{\Gamma} u_k + \underline{\Omega} w_k \tag{28}$$

Introduce the notation

$$\left.\begin{aligned} \overline{G} &= E\{\bar{x}_k, \bar{x}_k'\} = \begin{bmatrix} \overline{G}_{11} & \overline{G}_{12} \\ \overline{G}_{21} & \overline{G}_{22} \end{bmatrix}, \quad \bar{x}_k = \begin{bmatrix} \bar{x}_k \\ x_k^c \end{bmatrix}, \\ \underline{d} &= \begin{bmatrix} d' & 0 & \cdots & 0 & 0 \end{bmatrix} \end{aligned}\right\} \tag{29}$$

Employing (29) and (22), the performance index (12) can be rewritten as

$$\left.\begin{aligned} I_k = {} & tr\{Q_1 F_p \overline{G}_{11} F_p' + Q_1 W(\tau)\} - tr\{F_p' q_{12} D^c \underline{d}' \overline{G}_{11}\} \\ & - tr\{\underline{d} D^{c\,\prime} q_{12}' F_p \overline{G}_{11}\} + tr\{q_2 \underline{d} D^{c\,\prime} D^c \underline{d}' \overline{G}_{11}\} \\ & + tr\{C^{c\,\prime} q_{12}' F_p \overline{G}_{12}\} - tr\{q_2 C^{c\,\prime} D^c \underline{d}' \overline{G}_{12}\} \\ & + tr\{F_p q_{12} C^c \overline{G}_{21}\} - tr\{q_2 \underline{d} D^c C^c \overline{G}_{21}\} \\ & + tr\{q_2 C^{c\,\prime} C^c \overline{G}_{22}\} + tr\{q_2 D^{c\,\prime} D^c \nu^2 + q_w\} \end{aligned}\right\} \tag{30}$$

State covariance matrix $\overline{G}$ is the solution of the following discrete-time Lyapunov equation

$$\overline{G} = \overline{F}\,\overline{G}\,\overline{F}' + \overline{R}\,W\overline{R}' + \overline{N}\,\nu^2\,\overline{N}', \tag{31}$$

where the corresponding matrices for the system with PID type controller are as follows

$$\left.\begin{aligned} \overline{F} &= \begin{bmatrix} \underline{F} - \underline{\Gamma} D^c \underline{d}' & \underline{\Gamma} C^c \\ -B^c \underline{d}' & A^c \end{bmatrix}, \\ \overline{R} &= \begin{bmatrix} \underline{\Omega} \\ 0 \end{bmatrix}, \quad \overline{N} = \begin{bmatrix} -\underline{\Gamma} D^c \\ -B^c \end{bmatrix}, \end{aligned}\right\} \tag{32}$$

IV. Output and Control Variances

The output and control variances at sampling instants for the LQG controller can be calculated from the following expressions

$$\sigma^2_{y_k LQG} = \mathrm{var}\{ y_k \} = \underline{d}'\bar{V}(11)\underline{d} \quad (33)$$

$$\sigma^2_{u_k LQG} = \mathrm{var}\{ u_k \} = k^{c}{}' F_p \bar{V}(22) F_p{}' k^c \quad (34)$$

where

$$\left.\begin{aligned} &\bar{V} = E\{ \tilde{x}_k, \tilde{x}_k{}' \} = \begin{bmatrix} \bar{V}(11) & \bar{V}(12) \\ \bar{V}(21) & \bar{V}(22) \end{bmatrix}, \\ &\tilde{x}_k = \begin{bmatrix} \underline{x}_k \\ \hat{\underline{x}}_{k|k} \end{bmatrix}, \quad \hat{\underline{x}}_{k|k} = \begin{bmatrix} \hat{x}_{k|k} \\ u_{k-d} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}, \quad \underline{x}_k = \begin{bmatrix} x_k \\ u_{k-d} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix} \end{aligned}\right\} \quad (35)$$

and formulas in case of PID type controllers are defined as follows

$$\sigma^2_{y_k PID} = \mathrm{var}\{ y_k \} = \underline{d}'\bar{G}_{11}\underline{d} \quad (36)$$

$$\left.\begin{aligned} \sigma^2_{u_k PID} = \mathrm{var}\{ u_k \} = & C^c \bar{G}_{22} C^{c}{}' - D^c \underline{d}' \bar{G}_{12} C^{c}{}' \\ & - C^c \bar{G}_{21} \underline{d} D^{c}{}' + D^c \underline{d}' \bar{G}_{11} \underline{d} D^{c}{}' + D^c v^2 D^{c}{}' \end{aligned}\right\} \quad (37)$$

V. SUITABILITY OF DELAY APPROXIMATION FOR CONTROL PERFORMANCE ASSESSMENT

Exemplary plant is described by the following transfer function

$$G^p(s) = \frac{1}{(s+1)(0.5s+1)^2}\ , \quad (39)$$

and the disturbance generator with transfer function

$$G^d(s) = \frac{b_0}{T_0^2 s^2 + 2T_0 \xi s + 1}\ , \quad (40)$$

excited by white noise. The b_0 value is selected such that the noise variance equals to 1. Outputs of both paths are added together and they give system output *y(t)*. The variance of measurement noise characterizes accuracy of the sensor, transmitter and A/D converter.

The transfer function $G^p(s)$ is then approximated by the following lag-delay transfer function

$$H^p(s) = \frac{1}{Ts+1} e^{-s\tau} \quad (41)$$

Comparison of step responses of $G^p(s)$ and $H^p(s)$ is given in Fig. 1.
To assess the control performance and the control effort, standard deviation of the output and control signals at sampling points will be used. It should be noted here that for a reasonable designed system there is almost no difference between sampled and averaged values.

Another important observation is that parametric optimization of PID controllers practically led to PD controllers without integrating action. This is justified by the lack of bias in disturbance.

In Fig. 2a and 2b standard deviations of output signal against standard deviations of control signal for both delayed and non-delayed systems controlled by, respectively, optimal LQG, PD, and P controllers are plotted. The plots are parametrized by the weighting factor λ.
Results of the MV strategy are plotted as horizontal dashed lines and define areas of uncertainty of the performance lower bound when using the lag-delay approximation of non-delayed systems. Results of the benchmark designed for original model with realistic control signal magnitudes (λ=0.001) and PID type control with controllers settings supplied by means of QDR methods are plotted as points.

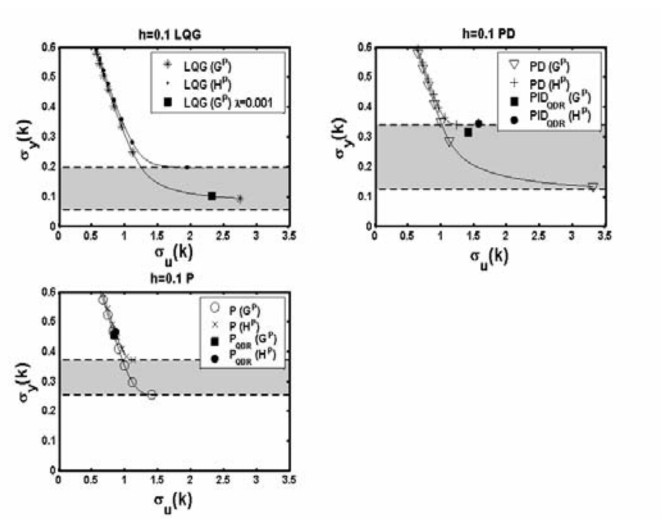

Fig. 2a. Standard deviation of output vs control; h=0.1.

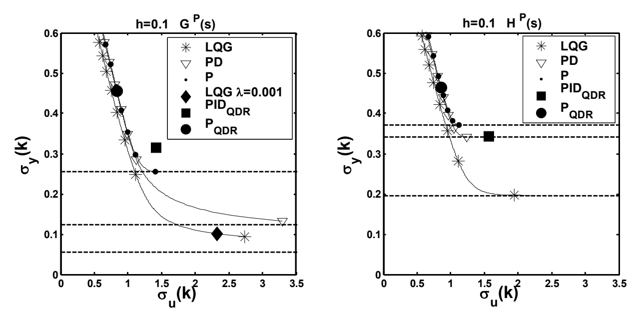

Fig. 2b. Standard deviation of output vs control; h=0.1.

Figure 2a shows that the pure MV control strategy used for the delayed model gives lower control quality than the MV algorithm for the delay-free model. Furthermore there is no significant improvement of control quality when optimal settings for classical controllers are used for the delayed model as compared to those obtained by means of QDR method.

In Fig. 3-6 exemplary realizations of output and control signals are presented illustrating the impact of lag approximation on disturbance attenuation.

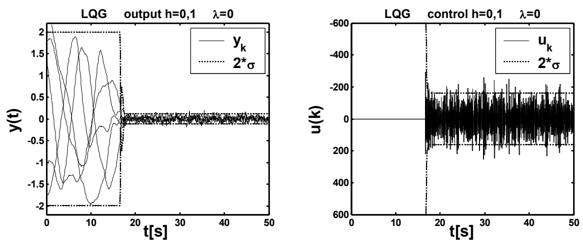

Fig. 3. Realizations of y(t) and u(t) for LQG controller; plant transfer function $G^p(s)$, λ=0.

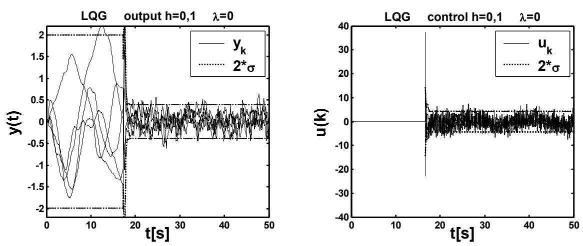

Fig. 4. Realizations of y(t) and u(t) for LQG controller; plant transfer function $H^p(s)$, λ=0.

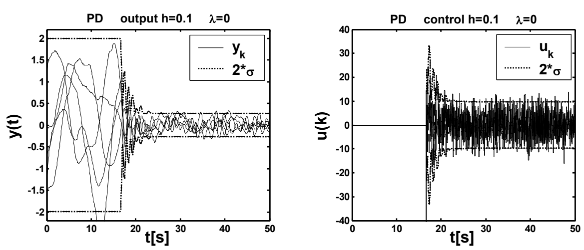

Fig. 5. Realizations of y(t) and u(t) for PD controller; plant transfer function $G^p(s)$, λ=0.

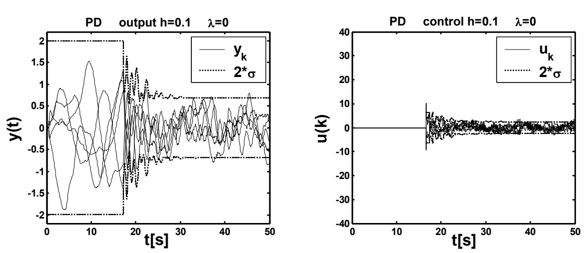

Fig. 6. Realizations of y(t) and u(t) for PD controller; plant transfer function $H^p(s)$, λ=0.

Table 1 presents the values of index η and standard deviation of output signal σ_y for the systems with the plant transfer function $G^P(s)$ and $H^P(s)$. Furthermore the value of sqrt(η)*σ_y is shown which is equivalent to standard deviation of the output signal obtained when pure MV Astrom's algorithm is used.

The bottom part of the table shows the values calculated for the delay-free system, $G^P(s)$, and calculated using the estimated substitute delay. The most important conclusion is that the use of the substitute delay to assess the system control performance often gives unreliable results. Another interesting observation is that the values from the last column belong to the uncertainty area of performance lower bound (see fig 2a). This can also be seen from Fig. 9 where the estimates of the performance lower bound are plotted when assuming different values of delay *l*.

TABLE I

VALUES OF PERFORMANCE MEASURE η AND ESTIMATED OUTPUT STANDARD DEVIATION UNDER PURE MV ASTROM'S CONTROL ALGORITHM

λ=0 h=0.1	η	σ_y	sqrt(η)	sqrt(η)*σ_y
LQG $[G^P(s)]$	0.8421	0.0751	0.9176	0.0689
PD $[G^P(s)]$	0.2668	0.1333	0.5166	0.0689
P $[G^P(s)]$	0.0701	0.2601	0.2648	0.0689
$PID_{QDR}[G^P(s)]$	0.0480	0.3145	0.2088	0.0689
P_{QDR} $[G^P(s)]$	0.0230	0.4545	0.1517	0.0689
LQG $[H^P(s)]$	0.9998	0.2026	0.9999	0.2026
PD $[H^P(s)]$	0.3465	0.3442	0.5886	0.2026
P $[H^P(s)]$	0.2938	0.3738	0.7362	0.2026
$PID_{QDR}[H^P(s)]$	0.3485	0.3432	0.5652	0.2026
P_{QDR} $[H^P(s)]$	0.1900	0.4649	0.4359	0.2026
substitute delay				
LQG $[G^P(s)]$	1.0000	0.0751	1.0000	0.0751
PD $[G^P(s)]$	0.4992	0.1333	0.7065	0.0942
P $[G^P(s)]$	0.4315	0.2601	0.6568	0.1709
$PID_{QDR}[G^P(s)]$	0.2799	0.3145	0.5094	0.1664
P_{QDR} $[G^P(s)]$	0.1773	0.4545	0.2791	0.1914

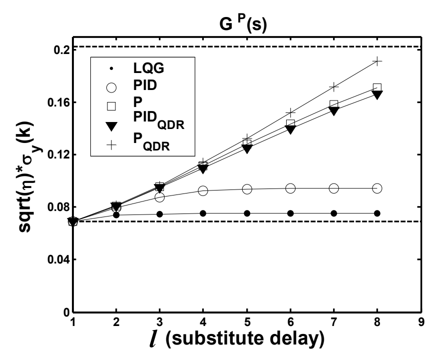

Fig. 9. Estimated control performance as a function of *l*.

VI. CONCLUSION

It has been shown in the paper that when the system to be controlled does not feature a known real delay then the substitute delay obtained from the popular lag-delay approximation can serve to determine the performance lower bound estimate. The estimate is located between two values: the smaller obtained assuming delay-free original system (*d=1*), and the greater valid for its lag-delay approximation. It is closer to the actual MV lower bound when calculated from the data coming from a system with high-quality control, and closer to the lower bound determined by the lag-delay approximate when the data come from a poorly controlled system.

It has also been shown that optimal tuning of classical PID controllers improves the disturbance attenuation bringing it closer to the lower bound when the original non-delayed model is used.

Large uncertainty concerning the estimate of achievable performance implies that identification of the continuous-time system is necessary both to asses the quality of the actual control system and to design high-quality controllers.

APPENDIX

Matrices and vectors for the system defined by equations (10)-(12) are as follows:

$$\left.\begin{aligned} F(h) &= e^{Ah}, & \mathrm{w}(h) &= \int_0^h e^{A(h-s)} c\xi(s)ds \\ g(h) &= \int_0^h e^{Av} b dv, & W(h) &= \int_0^h e^{As} cc' e^{A's} ds \end{aligned}\right\} \quad (42)$$

$$\left.\begin{aligned} Q_1 &= \frac{1}{h}\int_0^h F^{'}(\tau) M F(\tau) d\tau, \\ q_{12} &= \frac{1}{h}\int_0^h F^{'}(\tau) M g(\tau) d\tau, \\ q_2 &= \frac{1}{h}\int_0^h g'(\tau) M g(\tau) d\tau + \lambda, \\ q_w &= \frac{1}{h} d' \left\{ \int_0^h \int_0^\tau F(\tau - s) cc' F'(\tau - s) ds d\tau \right\} d \end{aligned}\right\} \quad (43)$$

and $M=dd'$ (44)

To obtain the values of index η we need to rewrite equations of the system (24)-(25) to the following state space representation [4]

$$x_{k+1} = F x_k + \Gamma_0 u_{i-d+1} + \Gamma_1 u_{k-d} + r a_k \quad (45)$$

$$z_k = d^{'} x_k + a_k \quad (46)$$

where

$$\left.\begin{aligned} & E\left[a_k^2\right] = \nu^2 + d'\Sigma d = \sigma^2, \quad r = \frac{F\Sigma d}{\sigma^2}, \\ & \Sigma = W + F\left(\Sigma - \frac{\Sigma d d' \Sigma'}{\nu^2 + d'\Sigma d}\right) F' \end{aligned}\right\} \quad (47)$$

In this paper a plant with delay and disturbance added to its output is considered. The disturbance dynamics differ from the plant dynamics.

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