2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Stochastic Multiple Model Adaptive Control with Hypothesis Testing

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***Abstract*— A hypothesis test switching method is proposed for multiple model adaptive control of a stochastic system. The method is compared to a heuristic performance index switching method. Benefits are shown when simulating the control of plant without *a priori* knowledge of parameter jumps and model placement.**

I. INTRODUCTION

Adaptive control of stochastic discrete-time linear systems using multiple models is covered in [1] and [2]. Two popular sum-of-error model performance indices suited for stochastic environments use a fading memory [2], [3] or finite window size [1]. These are both heuristic switching methods that require user observation and adjustment for optimal performance in a given situation.

A hypothesis test is useful for detecting parameter jumps when coupled with a Kalman filter for system identification [4], [5]. This paper demonstrates the fusion of two concepts in switching systems, namely, hypothesis testing and multiple model adaptive control. An n-sample hypothesis test method is proposed to provide optimal model switching without need for heuristic tuning. The benefits of hypothesis test switching are shown using simulations in Section III.

II. MATHEMATICAL PRELIMINARIES

A. Multiple Model Adaptive Control with Performance Index Switching

The objective is to have the plant output y track a reference output y^*. Two adaptive models (one free-running and one resetting) and $(N-2)$ fixed models are used. This combination is a good compromise between computational complexity and performance [1], [2]. The resetting model takes initial parameter estimates from any newly selected fixed model. For model i, the identification error $e_i = \hat{y}_i - y$ is used in the adjustable performance index [2],

$$J_i(k) = d_1 e_i^2(k) + d_2 \sum_{\tau=0}^{k} d_3^{k-\tau} e_i^2(\tau), \quad d_3 \in [0,1], \quad (1)$$

where the weights d_1, d_2, and d_3, are heuristically adjusted to achieve the desired switching behavior. This performance index is well suited for the stochastic and/or time varying case, as it incorporates a sum-of-error with a fading memory. Proof of stability using a performance index is provided in [1]. This switching method is compared to hypothesis test switching in Section III.

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B. Hypothesis Test Switching

The null hypothesis H_0 is declared as: no plant parameter jump has occurred. In the event of a parameter jump, the resulting change in statistics can be detected using the t-test:

$$t_r = \frac{\sum_{k=0}^{n-1} |e_c(k)|}{S/\sqrt{n}}. \quad (2)$$

The user-defined value, *n*, is the sample size of control error $e_c = y - y^*$. The running standard deviation S is calculated,

$$S^2 = \frac{\sum_{k=0}^{r-1} (e_c(k) - \bar{e}_c)^2}{r-1}, \quad (3)$$

where the running sample size, r, is reset to 2 whenever H_0 is rejected. Should H_0 never be rejected, r will saturate at the user-defined value r_{sat}. A user-defined significance level α corresponds to a threshold T_r, using statistical tables. Threshold T_r is used in the following binary decision rule:

1) if $t_r \le T_r$, accept H_0 (no parameter jump)
2) if $t_r > T_r$, reject H_0 (a parameter jump)

When H_0 is accepted, switching is not permitted. When H_0 is rejected, the model with the smallest performance index,

$$J_i(k) = \frac{1}{n} \sum_{\tau=0}^{n} |e_i(k-\tau)|, \quad (4)$$

becomes selected. The following statistical terms are defined: α, the probability of falsely detecting a parameter jump (a Type I error), and β, the probability of missing a parameter jump (a Type II error). Reducing α will cause β to increase. It is worth emphasizing that false detections may not impact performance because only index (4) determines if a switch is needed. Implementing a larger samples size n will increase the accuracy of the hypothesis test, at the cost of reducing the speed in which a jump can be detected. Due to space limitation, the proof of stable control using hypothesis test switching is not included.

III. SIMULATIONS

Hypothesis test switching is implemented with single-sample testing $(n = 1)$ for fastest detection, and standard deviation $r_{sat} = 40$. The performance index (1) switching method uses $d_1 = 1$ and $d_2 = 2$, as suggested in [3]. A 2^{nd} order plant has the parameter vector defined as,

$$\theta^{\mathrm{T}}(k) = [a_1(k) \quad a_2(k) \quad b_1(k) \quad b_2(k) \quad c_1(k) \quad c_2(k)],$$

where parameters a_i, b_i, and c_i, are associated with the output, input, and noise, respectively. Each simulation contains 400 time steps and has three possible states for θ^{T}. Initially

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at state θ_1^{T}, θ^{T} jumps to θ_2^{T}, θ_3^{T}, then back to θ_1^{T}, at time steps 100, 200, and 300, respectively. In each simulation, three fixed models are placed in the neighborhood of the three plant states. The resetting and free-running adaptive models employ the extended least squares algorithm, with forgetting factor $\lambda = 0.97$. The objective is to have the plant output track the reference signal, $y^*(k+1) = \sin(2\pi k/50) + \sin(2\pi k/150)$, with zero control error e_c.

A. Demonstration of Hypothesis Test Switching

The applied significant level is $\alpha = 0.0125$ (corresponding to threshold $T_r = 2$). The three plant states are defined:

$$\begin{aligned} \theta_1^{\mathrm{T}} &= [\ \ 0.60 \quad 0.20 \quad 1.00 \quad\ \ 0.50 \quad -0.50 \quad 0.40]\ \textit{stable} \\ \theta_2^{\mathrm{T}} &= [\ \ 1.70 \quad 0.70 \quad 1.00 \quad -0.50 \quad -0.50 \quad 0.40]\ \textit{unstable} \\ \theta_3^{\mathrm{T}} &= [-0.50 \quad 0.20 \quad 1.00 \quad\ \ 0.20 \quad -0.50 \quad 0.40]\ \textit{stable} \end{aligned}$$

The resulting plant and switching behavior is shown in Fig. 1. Upon initialization, fixed model 1 was selected and the resulting control error was not zero mean Gaussian. At time step $k = 14$, the t-test value t_r became larger than the threshold $T_r = 2$, causing H_0 to be rejected. As a result, the free-running adaptive model was selected by index (4), yielding Gaussian control error. At each jump was the same behavior, where a fixed model was temporarily used before an adaptive model was selected.

B. Performance Index vs. Hypothesis Test Switching

This study compares performance index (1) and hypothesis test switching methods perform in 100 unique scenarios of parameter jumps and model placement. For each scenario, the plant states are pseudo-randomly generated as:

$$\begin{aligned} \theta_1^{\mathrm{T}} &= [p_{1,1} \quad p_{1,2} \quad 1 \quad p_{1,3} \quad -0.5 \quad 0.4] \\ \theta_2^{\mathrm{T}} &= [p_{2,1} \quad p_{2,2} \quad 2 \quad p_{2,3} \quad -0.5 \quad 0.4] \\ \theta_3^{\mathrm{T}} &= [p_{3,1} \quad p_{3,2} \quad 3 \quad p_{3,3} \quad -0.5 \quad 0.4] \end{aligned}$$

The parameters are uniformly distributed as follows: $p_{i,1} \sim [-1.0, 1.0]$, $p_{i,2} \sim [-0.5, 0.5]$, and $p_{i,3} \sim [-0.5, 0.5]$. To achieve the lowest possible MSE in one scenario, both switching methods are methodical tuned (100 tuning attempts each scenario). For attempt $j = 1, 2, \ldots, 100$, the hypothesis test uses threshold $T_r = 0.1j$, and index (1) uses $d_3 = 0.5 + 0.005j$. The resulting control MSE from each scenario and tuning attempt is plotted using two-dimensional contours, shown in Figs. 2 (a) and (b). A vertical dashed line is drawn in Fig. 2 (b) to illustrate how a low control MSE can be achieved for all scenarios without need for user adjustment of T_r. No such line can be drawn in Fig. 2 (a), as heuristic re-adjustment to weight d_3 would be necessary for reducing MSE in most new plant/model scenarios. In other comparisons (not included due to space), it was also found that hypothesis test switching performed much better model selection in the presence of noise.

IV. CONCLUSIONS

In this paper, a hypothesis test switching method was proposed to improve performance in multiple model adaptive control of stochastic systems. It was shown that hypothesis test switching could operate optimally, without user adjustment or *a priori* knowledge of plant parameter behavior and fixed model placement. This suggests that the switching method would be valuable where heuristic adjustments to switching would be too costly or challenging.

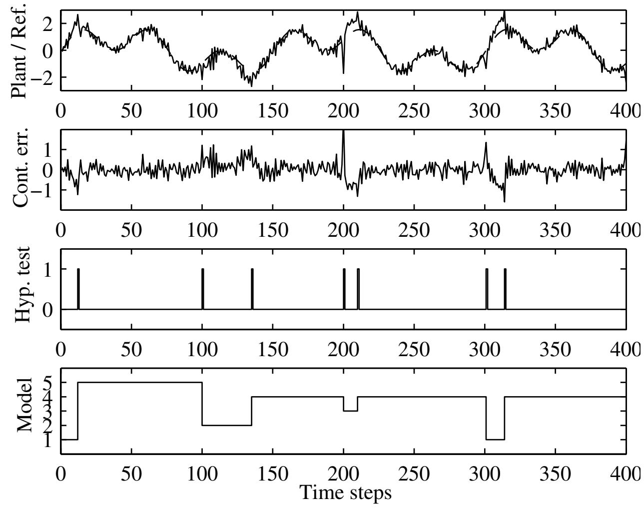

Fig. 1. Hypothesis test switching example with noise variance $\sigma^2 = 0.25$ (dashed line represents reference signal, models 1, 2, and 3 are fixed, model 4 is resetting, and model 5 is free-running).

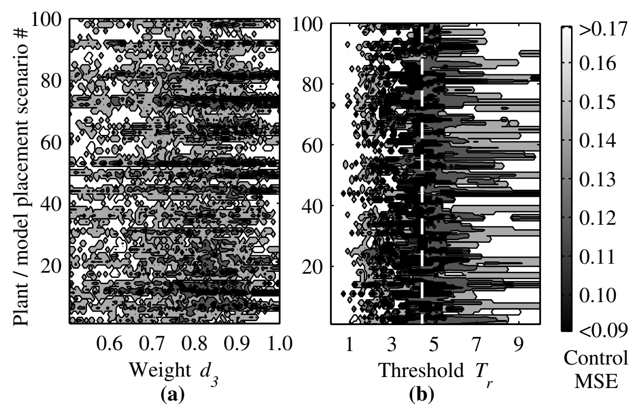

Fig. 2. Comparison of control error resulting from heuristic, (a), and hypothesis test, (b), switching in scenarios of different plant parameter behavior and model placement, using noise variance $\sigma^2 = 0.25$ (dashed line illustrates how low control MSE can be achieved using a 'fixed' T_r).

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