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Semi-active suspension control using "Fast" Model Predictive Control

M. Canale, M. Milanese, C. Novara and Z. Ahmad

Abstract— **The problem considered in this paper is the design and the analysis of a control strategy, for semi-active suspensions in road vehicles, based on Model Predictive Control (MPC) techniques. The computed control law, using predictive techniques aims to optimize, the suspension performance, by minimizing a quadratic cost function while ensuring that the magnitude of the forces generated by the control law satisfies the physical constraints of passive damping. The on line computation difficulties related to the predictive control law are overcome by means of a "Fast" implementation of the MPC algorithm (FMPC). The estimation of the system state variable needed for control law computation is provided by a suitable "robust" observer, whose accuracy is not affected by variation of the system parameters (i.e. masses, damping and stiffness coefficients, etc.). A performance comparison with the well established semi-active Sky-Hook strategy is presented. The achievable performance improvements of the proposed design procedure over Sky-Hook control law are showed by means of simulation results.**

I. Introduction

The design of controlled suspension systems for road vehicles aims to enhance the vehicle performances with regard to ride comfort and road handling. Such performance requirements have received, in the last two decades, a growing interest witnessed by an intense research activity developed from both industrial and academic sides (see e.g. [1], [2]). Vehicle suspensions serve several conflicting purposes: in addition to counteracting the body forces resulting from cornering, acceleration or braking and changes in payload, suspensions must isolate the passenger compartment from road irregularities. For driving safety, a permanent contact between the tires and the road should be assured. Passive suspension systems built of springs and dampers have serious limitations. Their parameters have to be chosen to achieve a certain level of compromise between road holding, load carrying and comfort, under wide variety of road conditions. This motivated extensive researches on active and semi-active suspension systems. Active suspension systems have the ability to store, dissipate and introduce energy to the system. As a result, the trade-offs among conflicting design goals can be better resolved. On the other hand, they require suitable actuator devices and high levels of energy-consumption leading to an increase of the system costs. To reduce such costs, semi-active suspensions were proposed, making use of dampers that can vary the damping coefficient.

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The goal of this paper is to introduce the design and the analysis of a control strategy, for semi-active suspensions in road vehicles, based on Model Predictive Control (MPC) strategies. It is a well known fact that the computation of such predictive control laws is a crucial task in every MPC application due to fact that an optimization problem has to be solved on line. In order to overcome this problem, a "Fast" Model Predictive Control (FMPC) implementation, based on nonlinear function approximation techniques, is introduced. Moreover, as the computation of the FMPC controller requires the knowledge of the system state variables, a suitable "robust" observer whose accuracy is not affected by variation of the system parameters (i.e. masses, damping and stiffness coefficients, etc.), will be employed to provide the estimation of such variables. The performances of the proposed FMPC technique will be compared with the well established "On-Off Sky-Hook" (see e.g [1]) semi-active control algorithm. In order to investigate in a reliable way the properties of the proposed FMPC control strategy, simulation results are performed using "benchmark" road profiles employed in standard industrial tests.

II. Problem formulation

In this paper, the attention will be focused on the half-car model reported in Figure 1.

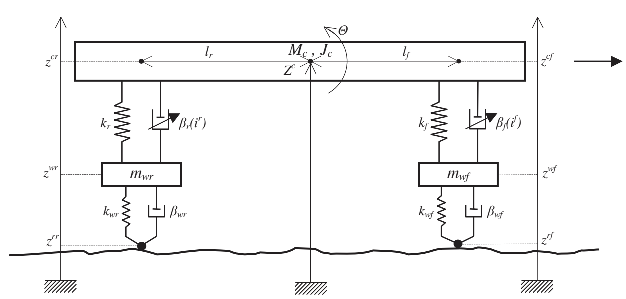

Fig. 1. Half-car semi-active suspension schematic

In the half-car model of Figure 1, the chassis, the engine and the wheels are modelled as rigid bodies, static nonlinear characteristics have been assumed for suspensions. The vehicle is assumed to run at a constant speed v. The parameters characterizing the model are:
- M_c: chassis (sprung) mass.
- J_c: chassis moment of inertia.
- m_{wf}, m_{wr}: front and rear wheels (unsprung) masses.
- k_{wf}, k_{wr}: front and rear tires stiffness coefficients.
- k_f, k_r: front and rear suspensions spring constants.
- β_{wf}, β_{wr}: front and rear tires damping coefficients.

- l_f, l_r: distance of chassis barycenter from front and rear suspensions.
The variables describing the model are:
- z^{rf}, z^{rr}: front and rear road profiles.
- z^{wf}, z^{wr}: front and rear wheels vertical positions.
- z^{cf}, z^{cf}: front and rear chassis vertical positions.
- Z^c chassis barycenter vertical position.
- Θ: chassis pitch angle.
- i^f, i^r: control currents of front and rear suspensions.
- $\beta_f(\cdot)$, $\beta_r(\cdot)$: variable damping coefficients of front and rear suspensions. The half-car model provides a quite accurate description of the suspension system including the effects of both heave and pitch accelerations on the vehicle body mass. This allows to take into account the coupling effects between the front and the rear suspension forces of the vehicle and to provide reliable evaluations of the achievable performances of the proposed design procedure. Indeed, to perform the design of the control algorithm in an effective way, simplified models of the suspension such as quarter car are usually employed. The main limitation in using quarter car models is the fact that front-rear coupling effects can not be taken into account. On the other hand, several simulation results based on the non linear model described in [3], have shown that the front suspension command influences the rear vehicle dynamics for a percentage less than the 10% (the same holds for the effect of the rear command on the front of the vehicle). Given such negligible coupling properties of half-car system it appears quite reasonable to perform the control design procedure in a decentralized fashion on the basis of two separate quarter car models describing the front and the rear suspensions. The decoupling properties of half car models into quarter models in an optimal control context using quadratic performance index have also been studied in [4], where, in particular, conditions on the weights of the quadratic performance index and on the physical structure of the half-car model are given under which the resulting optimal control strategy has a decoupled structure. Then, the design of the control system will be worked out considering separately two different quarter-car models having the structure depicted in Figure 2. However, the testing of the obtained FMPC design and the other considered techniques will be worked out using the half-car model.

In the quarter car scheme m_c is the sprung mass (quarter of car body and passenger mass), m_w is the unsprung mass (wheel, tire, and other suspension components) and z^c, z^w, z^r are the vertical positions of the sprung mass, the unsprung mass, and the road profile, respectively. Moreover, k_w and β_w are the tire stiffness and damping coefficient respectively, k is the suspension spring constant and $u(t)$ is the damper force. As the proposed design methodology is identical for the front and the rear suspension, the developments that follow will be worked out considering the generic quarter-car system of Figure 2 without distinctions. In semi-active suspensions systems the damper force is $u(t) = \beta(t)\left(\dot{z}^w(t) - \dot{z}^c(t)\right)$, where the damping coefficient $\beta(t)$ is variable. Variable damping can be obtained in many different ways. As an example, standard hydraulic dampers

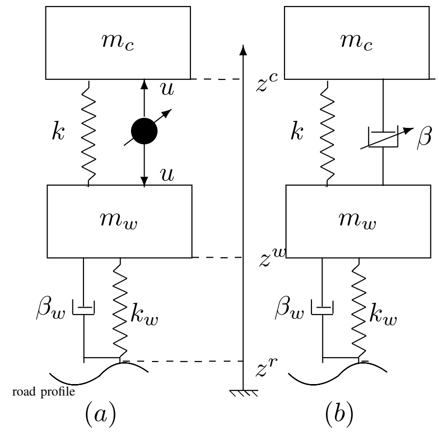

Fig. 2. (a) Quarter-car suspension schematic. (b) Quarter-car semi-active suspension schematic

exploit the quantity of fluid that flows through suitable valves. By controlling the opening of the valves it is possible to vary the damping coefficient so that the only power required for the damper is the relatively small power needed to control the valves. In particular, the valves opening can be actuated by a suitable servo-mechanism driven by an appropriate current $i(t)$. The determination of such driving current is realized through a "force-current map" which gives the dependence of force $u(t)$ at each time as a function of current $i(t)$ and relative speed $v_{wc}(t) = \dot{z}^w(t) - \dot{z}^c(t)$, i.e. $u = u_i(v_{wc})$. In Figure 3 the behavior of a typical force-current map for a commercial damper is shown. For sake of simplicity, in Figure 3, only the maximum and the minimum curves $u_{i,max}(v_{wc})$ and $u_{i,min}(v_{wc})$ are represented.

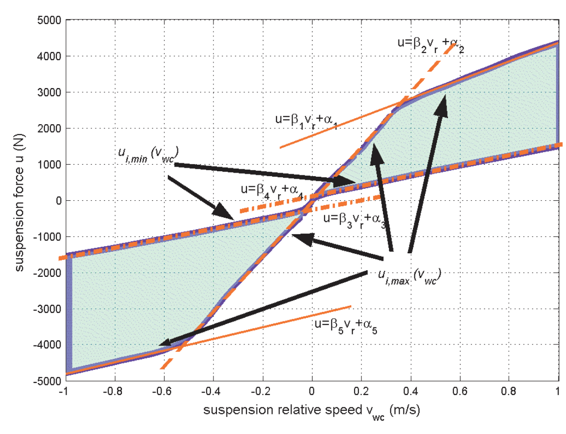

Fig. 3. Damper map

Now, observing Figure 3, it can be seen that the straight lines having cartesian equation

$$\begin{cases} u = \beta_1 v_{wc} + \alpha_1 \\ u = \beta_2 v_{wc} + \alpha_2 \\ u = \beta_3 v_{wc} + \alpha_3 \\ u = \beta_4 v_{wc} + \alpha_4 \\ u = \beta_5 v_{wc} + \alpha_5 \end{cases} \tag{1}$$

for suitable values of the real parameters $\beta_i > 0$ and α_i, for $i = 1, \ldots, 5$ define the shadow bounded region in which the control force $u(t)$ must lay. Such a region establishes the allowable values of the control force $u(t)$ that can be actuated by the semi-active damper device. Then, in order to ensure the feasibility of the suspension forces, the semi-active control strategy must be computed guaranteeing the satisfaction of the passivity constraint:

$$u_{i,min}(v_{wc}) \leq u \leq u_{i,max}(v_{wc})$$

which can be written in a more detailed form as:

$$\begin{array}{ll} \text{if } \ v_{wc} \geq 0 & \left\{ \begin{array}{l} u \leq \beta_1 v_{wc} + \alpha_1 \\ u \leq \beta_2 v_{wc} + \alpha_2 \\ u \geq \beta_4 v_{wc} + \alpha_4 \end{array} \right. \\ \text{if } \ v_{wc} < 0 & \left\{ \begin{array}{l} u \leq \beta_3 v_{wc} + \alpha_3 \\ u \geq \beta_2 v_{wc} + \alpha_2 \\ u \geq \beta_5 v_{wc} + \alpha_5 \end{array} \right. \end{array} \tag{2}$$

At present, a widely used semi-active techniques is the 'On-Off Sky-Hook" control strategy (see e.g. [5]), where the damper is adjusted at maximum or minimum damping as follows:

$$u = \left\{ \begin{array}{ll} u_{i,max}(v_{wc}) & \text{if } \dot{z}^c(\dot{z}^c - \dot{z}^w) \geq 0 \\ u_{i,min}(v_{wc}) & \text{if } \dot{z}^c(\dot{z}^c - \dot{z}^w) < 0 \end{array} \right. \tag{3}$$

The Sky-Hook semi active policy emulates the ideal body displacement control configuration of a passive damper hooked between the sprung mass and the sky.

It can be noted that the described Sky-Hook strategy satisfies the constraint at each current time, without considering its effects on future time. This may cause, as it will be shown by the presented simulation results, relevant limitations in achievable performances, because the dynamic evolution of the overall system is not taken into account. Model Predictive Control appears to be a more appropriate technique able to handle the control design accounting for both passivity constraints and dynamic evolution of involved variables (accelerations, velocities and positions).

III. SEMI-ACTIVE SUSPENSION CONTROL USING FMPC

Model Predictive Control (MPC) (see e.g. [6], and the references therein) is an optimization based control law. The optimal solution relies on a dynamic model of the process, satisfies given input and state constraints, and minimizes a quadratic performance measure. The quarter-car model dynamics are given by the following set of differential equations

$$\begin{array}{l} m_c \ddot{z}^c = -k\left(z^c - z^w\right) + u \\ m_t \ddot{z}^w = k\left(z^c - z^w\right) - u - k_w\left(z^w - z^r\right) - \beta_w\left(\dot{z}^w - \dot{z}^r\right) \end{array} \tag{4}$$

These equations can be rewritten in a state space form as:

$$\dot{x} = A_c x + B_c u \tag{5}$$

where $x = [z^c \; z^w \; \dot{z}^c \; \dot{z}^w]^T$ and A_c and B_c are suitable matrices. By choosing an appropriate sampling interval T and a discretization technique, a discrete time model may be obtained in state space form:

$$x_{t+1} = Ax_t + Bu_t \tag{6}$$

The objective is to find a control law u_t that optimizes the performances of the vehicle with regard to comfort and road handling, subject to the passivity constraint (2), and to the dynamic equation (6). By means of the damper map, the control force u_t computed by the semi-active control algorithm may be converted into the required driving current i_t. The performance criteria to be optimized include sprung mass acceleration $\ddot{z}^c$, suspension deflection $z^c - z^w$ and wheel acceleration $\ddot{z}^w$. These performance indexes can be included in an objective function J. By defining the prediction horizon N_p, the control horizon $N_c \leq N_p$ and positive definite matrices $Q = Q^T \succ 0$ and $R = R^T \succ 0$, the objective function J can be expressed by a quadratic function:

$$\begin{array}{l} J(U, x_{t|t}, N_p, N_c) = \\ \sum_{k=0}^{N_p-1} x_{t+k|t}^T Q x_{t+k|t} + \sum_{k=0}^{N_c-1} u_{t+k|t}^T R u_{t+k|t} \end{array} \tag{7}$$

where:
$x_{t+k|t}$ denotes the state predicted by the model (6), given the input sequence $u_{t|t}, \ldots, u_{t+k-1|t}$ and the "initial" state $x_{t|t} = x_t$, $U = \left[u_{t|t}, u_{t+1|t}, \cdots, u_{t+N_c-1|t}\right]^T$ is the vector of the control moves to be optimized. If $N_c < N_p$ the following choice is made: $u_{t+k|t} = u_{t+N_c-1|t}$ $k = N_c, N_c+1, \cdots, N_p-1$.

The passivity constraints can be written as linear inequalities on the control force u and the system state variables x. In particular, the relative velocity $v_{wc} = \dot{x}^w - \dot{x}^c$ between sprung and unsprung masses can be written as the product Cx with $C = [0 \; 0 \; -1 \; 1]$. Then, for every control instant $t+k|t$ such that $k = 0, 1, \cdots, N_c - 1$, the control move $u_{t+k|t}$ has to be computed taking into account the following constraints:

$$\begin{array}{ll} \text{if } \ Cx_{t+k|t} \geq 0 & \left\{ \begin{array}{l} u_{t+k|t} \leq \beta_1 Cx_{t+k|t} + \alpha_1 \\ u_{t+k|t} \leq \beta_2 Cx_{t+k|t} + \alpha_2 \\ u_{t+k|t} \geq \beta_4 Cx_{t+k|t} + \alpha_4 \end{array} \right. \\ \text{if } \ Cx_{t+k|t} < 0 & \left\{ \begin{array}{l} u_{t+k|t} \leq \beta_3 Cx_{t+k|t} + \alpha_3 \\ u_{t+k|t} \geq \beta_2 Cx_{t+k|t} + \alpha_2 \\ u_{t+k|t} \geq \beta_5 Cx_{t+k|t} + \alpha_5 \end{array} \right. \end{array} \tag{8}$$

The MPC control law is then obtained applying the following receding horizon strategy:

1) At time instant t, get x_t.
2) Solve the quadratic problem:

$$\begin{array}{ll} \min_U & J(U, x_{t|t}, N_p, N_c) \\ s.t. \; (8). & \end{array} \tag{9}$$

3) Apply the first element of the solution sequence U to the optimization problem as the actual control action $u_t = u_{t|t}$.
4) Repeat the whole procedure at time $t+1$.

It has to be noted that the passivity constraints (8) are defined in different ways according to the sign of the predicted suspension relative speed $v_{wc,t+k|t} = Cx_{t+k|t}$.

Thus the sign of $v_{wc,t+k|t}$ introduces, inside the prediction horizon, the necessity to switch between the constraints to be satisfied. This situation can be formulated as a predictive control scheme involving logic constraints whose solution can be computed by means of mixed integer programming techniques (see [7] for details).

An on-line application of the procedure cannot be actually performed, since it requires the solution of the optimization problem (9) at each sampling time, a task that cannot be performed on-line at the sampling periods required for this application which are of the order of $2 \div 5$ ms. This problem is overcome using the Set Membership approach to nonlinear function estimation proposed in [8] (the details can be found in [9]).

The MPC control u_t results to be a nonlinear static function of x_t, i.e.:

$$u_t = f(x_t)$$

The function f is not explicitly known, but the values of $f(x)$ may be known for a certain number of its argument by performing off-line the MPC procedure starting from initial conditions $\tilde{x}_k, \;\; k = 1, \ldots, M$, so that:

$$\tilde{u}_k = f(\tilde{x}_k), \;\; k = 1, \ldots, M$$

From these known values of $\tilde{u}_k$ and $\tilde{x}_k$, an approximation $\widehat{f}$ of f is derived as follows.

Let us define the functions:

$$\begin{aligned} f_u(x,\gamma) &\doteq \min_{k=1,\ldots,M} (\tilde{u}_k + \gamma \|x - \tilde{x}_k\|) \\ f_l(x,\gamma) &\doteq \max_{k=1,\ldots,M} (\tilde{u}_k - \gamma \|x - \tilde{x}_k\|) \end{aligned} \tag{10}$$

Compute:

$$\gamma^* = \inf_{\gamma : f_u(\tilde{x}_k) \geq \tilde{u}_k, \; k=1,\ldots,M} \gamma \tag{11}$$

The estimate of $f(x)$ is given by:

$$\hat{f}(x) = [f_u(x,\gamma^*) + f_l(x,\gamma^*)]/2 \tag{12}$$

Suppose for the sake of simplicity that the points $\tilde{x}_k$ are taken as a uniform grid of a rectangular region $X \subset \mathbb{R}^4$ where state $x = [z^c \; z^w \; \dot{z}^c \; \dot{z}^w]^T$ can evolve. Then it results:

$$\lim_{M \to \infty} |f(x) - \hat{f}(x)| = 0, \; \forall x \in X \tag{13}$$

Moreover, for given M, the estimation error $f(x) - \hat{f}(x)$ is bounded by

$$|f(x) - \hat{f}(x)| \leq [f_u(x,\gamma^*) - f_l(x,\gamma^*)]/2, \; \forall x \in X \tag{14}$$

This allows to check if $\hat{f}$ provides a sufficient approximation of f or if a larger value of M is needed.

Finally, the MPC control can be implemented on line, by simply evaluating the function $\hat{f}(x_t)$ at each sampling time:

$$u_t = \hat{f}(x_t)$$

This requires to know the state $x = [z^c \; z^w \; \dot{z}^c \; \dot{z}^w]^T$ at each sampling time. The most usual configuration of sensors for semi-active suspensions consists in accelerometers measuring $\ddot{z}^c$ and $\ddot{z}^w$, since measuring z^c, z^w, $\dot{z}^c$, and $\dot{z}^w$ requires too costly sensors. In this way, an estimate $\hat{x}$ of the system state x has to be provided. Sprung and unsprung masses positions and speeds can be obtained by suitable filtering actions of accelerometer signals. In particular, in order to remove DC offset effects, speeds are obtained by filtering the measured accelerations by means of suitable bandpass filters as described in [10]. Positions are then obtained via pseudo-integration of the estimated speed signals as proposed in [11].

IV. Simulation results

The simulations have been carried out using "benchmark" road profiles employed in standard industrial tests (see [3]). In particular, the following road profiles have been taken into account:

- Random: road with random profile, maximum amplitude of $0.015 \; m$ and run at $60 \; km/h$
- English Track: road with irregularly spaced holes and bumps, maximum amplitude of $0.025 \;\; m$ and run at $60 \; km/h$.
- Short Back: impulse road profile, maximum amplitude of $0.015 \; m$ and run at $30 \; km/h$.

In this way, the controlled suspensions behaviour is tested in different driving and road regularity conditions. The simulations were performed using a sampling time $T = 1/512 \; s$ and a simulation time of about $14 \; s$ for each profile type. The performance obtained by the proposed semi-active strategy based on MPC have been compared with the ones achieved by the Sky-Hook control technique.

The structural parameters of the half-car model are: $l_f = 1.18$ m, $l_r = 1.42$ m, $M_c = 792.5$ kg and $J = 1328$ kg·m^2. The FMPC design procedure has been applied to the quarter car systems representing the front and the rear suspensions with the parameter values contained in Table I.

TABLE I
Parameters values employed in the design

Front suspension	Rear suspension
m_{cf} = 432.82 kg	m_{cr} = 359.67 kg
k_f = 17200 N/m	k_r = 17200 N/m
m_{wf} = 40 kg	m_{wr} = 40 kg
β_{wf} = 10000 Ns/m	β_{wr} = 10000 Ns/m
k_{wf} = 200000 N/m	k_{wr} = 200000 N/m

Note that the sprung mass distribution over m_{cf} and m_{cr} has been chosen in such a way to satisfy the decoupling conditions given in [4]. The passivity constraint (2) has been taken into account by using the following parameters (for both the suspensions):

$$\begin{aligned} &\beta_3 = \beta_4 \hat{=} \beta_{min} = 1500 \; Ns/m \\ &\beta_1 = \beta_2 = \beta_5 \hat{=} \beta_{max} = 5000 \; Ns/m \\ &\alpha_i = 0, \;\; i = 1, \ldots, 5. \end{aligned} \tag{15}$$

For both the front and the rear suspensions the optimization problem (7) has been formulated by taking into account prediction and control horizons $N_p = N_c = 10$ and the following weighting matrices in the quadratic cost function J:

$$Q = \begin{pmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad R = 0.00001$$

First of all, and in order to put in evidence the comfort performances that can be achieved, the behaviour of the sprung mass barycenter heave and pitch accelerations $\ddot{Z}^c$, $\ddot{\Theta}$ have been considered. The comparison of selected samples of the typical behaviour of the different accelerations $\ddot{Z}^c$ and $\ddot{\Theta}$ obtained by using the MPC technique and the Sky-Hook control are depicted in Figures 4, 5 and 6 for the Random, the English track and the Short-Back profiles, respectively.

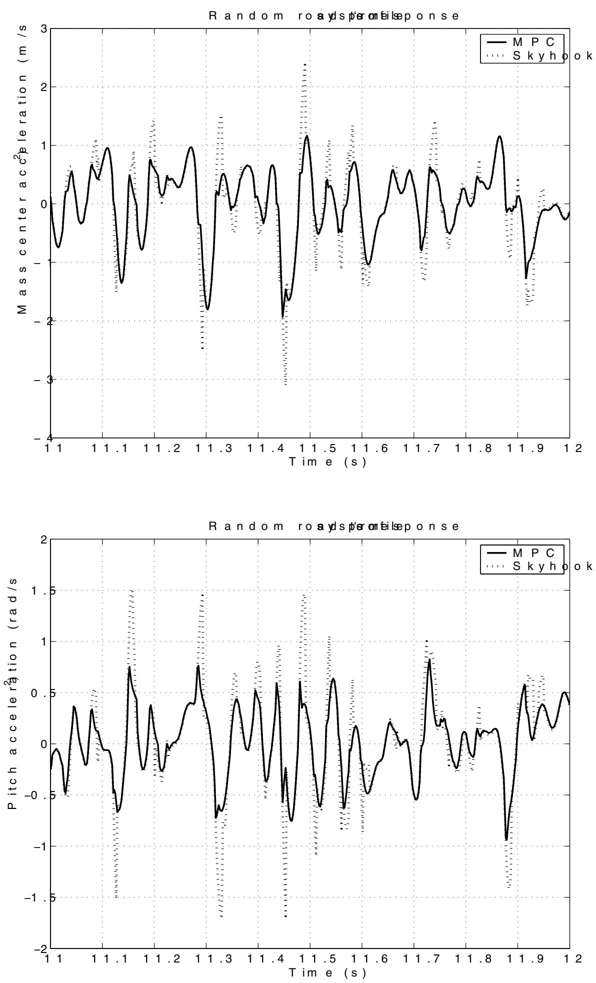

Fig. 4. Random profile. Up: Sprung mass barycenter heave acceleration. Down: Sprung mass barycenter pitch acceleration.

Observing Figures 4, 5 and 6 it can be easily noted the significant reductions on the peak amplitudes of the sprung mass accelerations introduced by the proposed predictive strategy over Sky-Hook. Similar improvements can be observed for the front and rear accelerations $\ddot{z}^{cf}$ and $\ddot{z}^{cr}$ variables.

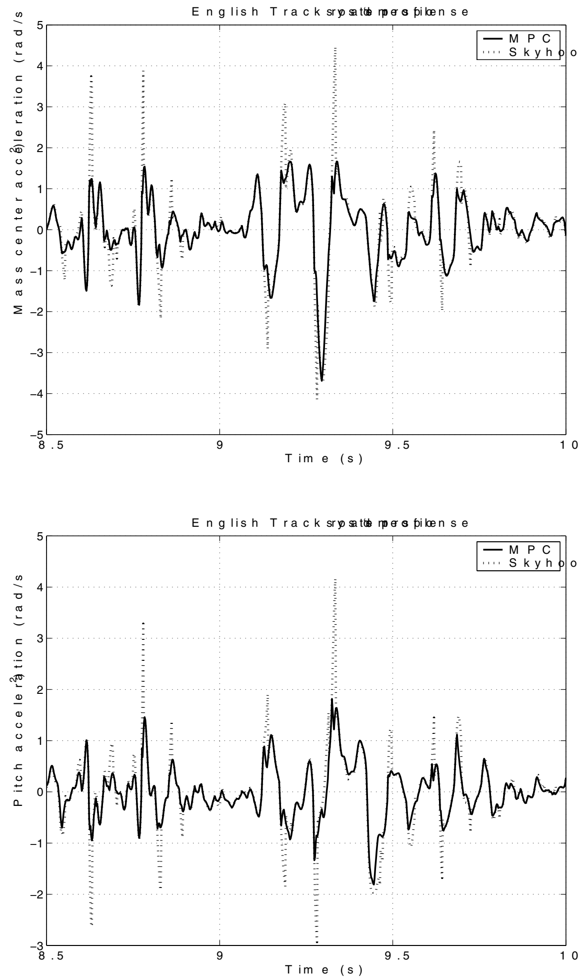

Fig. 5. English track profile. Up: Sprung mass barycenter heave acceleration. Down: Sprung mass barycenter pitch acceleration.

Regarding the remaining system variables related to the road handling performances such as unsprung masses accelerations, $\ddot{z}^{wf}$ and $\ddot{z}^{wr}$, suspension deflections ($z^{cf} - z^{wf}$, $z^{cr} - z^{wr}$) and tire deflections ($z^{wf} - z^{rf}$, $z^{wr} - z^{rr}$), similar behaviors have been obtained by using the two compared control techniques and for all the considered road profiles. As an example, selected samples of the obtained simulation results are reported in Figure 7 for the unsprung mass accelerations $\ddot{z}^{wf}$ and $\ddot{z}^{wr}$, in Figure 8 for the front and rear suspension deflections $z^{cf} - z^{wf}$ and $z^{cr} - z^{wr}$ and in Figure 9 for the front and rear tyre deflection $z^{wf} - z^{rf}$ and $z^{wr} - z^{rr}$ considering a road random profile.

In order to study in a more quantitative way the ride comfort performances achieved by the proposed predictive technique and to compare them with the ones achieved by the Sky-Hook strategy, the RMS value of the sprung mass acceleration $\ddot{Z}^{c,RMS}$, normalized with respect to the gravity acceleration g, according to [12] can be considered:

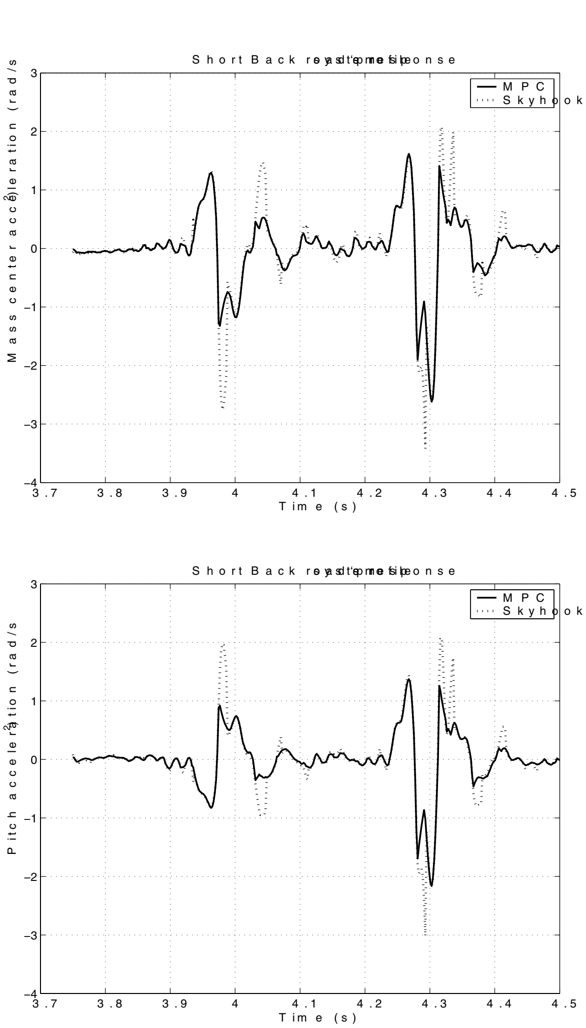

Fig. 6. Short-back profile. Up: Sprung mass barycenter heave acceleration. Down: Sprung mass barycenter pitch acceleration.

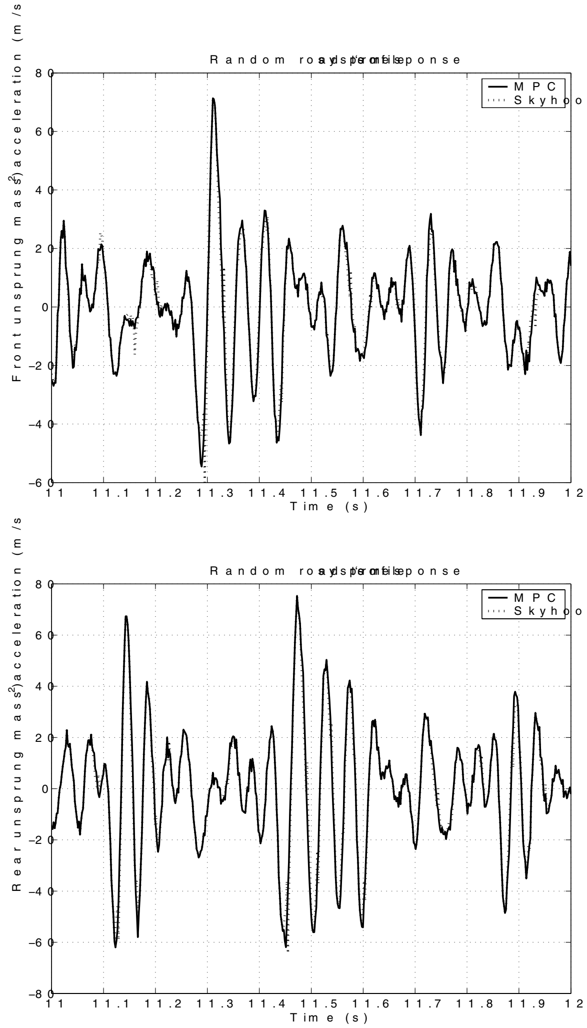

Fig. 7. Random Profile.Up: Front unsprung mass acceleration. Down: Rear unsprung mass acceleration.

$$\ddot{Z}_{rel}^{c,RMS} = \sqrt{\frac{1}{\tau}\int_{t=0}^{\tau}\left(\frac{\ddot{Z}^c(t)}{g}\right)^2 dt} \qquad (16)$$

Regarding the handling performances, they depend on the forces that are exchanged between tire and road which are given by $F_{z^{w,f}} = k_{wf}(z^{wf} - z^{rf}) + \beta_{wf}(\dot{z}^{wf} - \dot{z}^{rf})$ and $F_{z^{w,r}} = k_{wr}(z^{wr} - z^{rr}) + \beta_{wr}(\dot{z}^{wr} - \dot{z}^{rr})$ for the front and the rear suspension respectively. Then, the RMS values of the dynamic forces $F_{z^{w,f}}$ and $F_{z^{w,r}}$ normalized with respect to the static forces acting on the wheels $F_{z^{w,f}}^{\text{stat}} = (m_{cf} + m_{wf})g$ and $F_{z^{w,r}}^{\text{stat}} = (m_{cr} + m_{wr})g$ according to [12] can be conveniently used to evaluate the handling performance levels:

$$F_{z^{w,f}}^{RMS} = \sqrt{\frac{1}{\tau}\int_{t=0}^{\tau}\left(\frac{F_{z^{w,f}}(t)}{F_{z^{w,f}}^{\text{stat}}}\right)^2 dt}$$
$$F_{z^{w,r}}^{RMS} = \sqrt{\frac{1}{\tau}\int_{t=0}^{\tau}\left(\frac{F_{z^{w,r}}(t)}{F_{z^{w,r}}^{\text{stat}}}\right)^2 dt} \qquad (17)$$

In Tables II, III and IV the achieved values of the performance indexes (16) and (17) for the two considered control strategies are reported and compared for the Random, English track and Short-back road profile respectively.

TABLE II
RMS PERFORMANCE ANALYSIS FOR RANDOM PROFILE

	MPC	Sky-Hook
$\ddot{Z}_{rel}^{c,RMS}$	0.0428549	0.0557125
$F_{z^{w,f}}^{RMS}$	0.0361358	0.0407019
$F_{z^{w,r}}^{RMS}$	0.0469727	0.0512242

The analysis of the results reported in Tables II, III and IV puts in evidence the improvements on the comfort and handling characteristics introduced by proposed predictive control technique over the Sky-hook methodology. In particular, it can be noted that the comfort index (16) is improved of about the 20%. Regarding the handling measures (17) the improvements introduced by the predictive control ranges from the 5% to the 12%.

The indexes (16) and (17) allow to evaluate the comfort

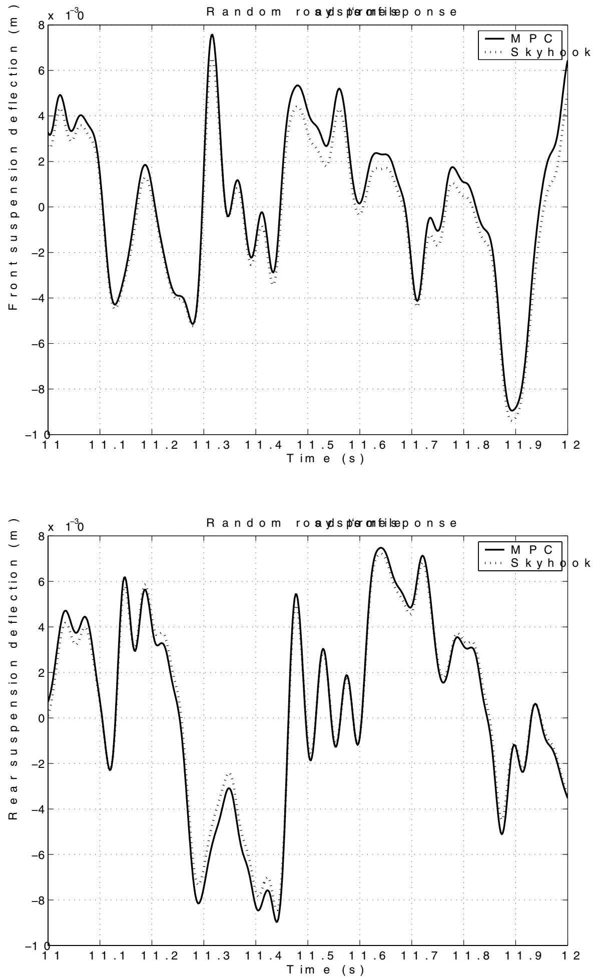

Fig. 8. Up: Front suspension deflection for random profile. Down: Rear suspension deflection for random profile.

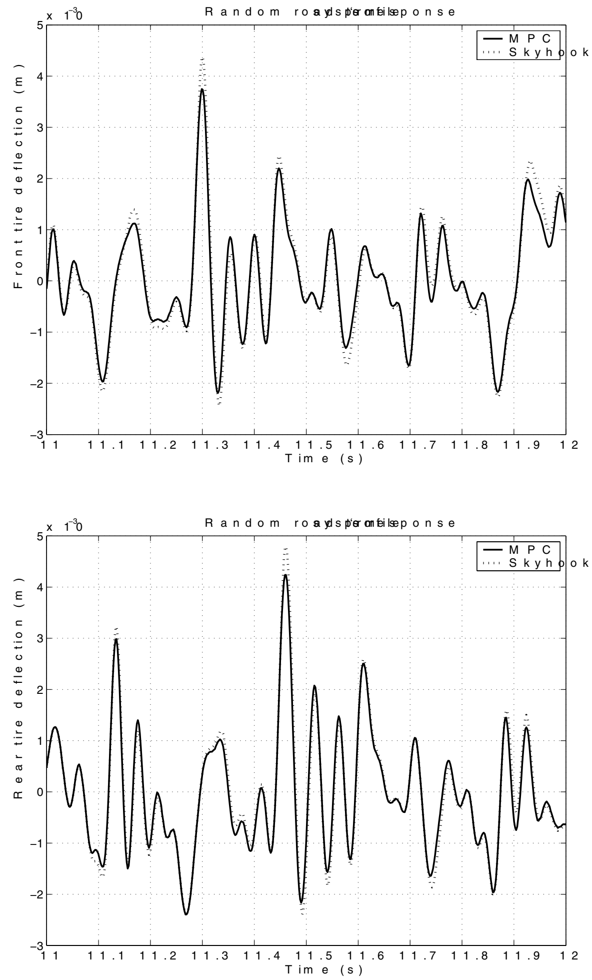

Fig. 9. Random Profile. Up: Front tire deflection. Down: Rear tire deflection.

TABLE III

RMS PERFORMANCE ANALYSIS FOR ENGLISH TRACK PROFILE

	MPC	Sky-Hook
$\ddot{Z}^{c,RMS}_{rel}$	0.0648871	0.0839407
$F^{RMS}_{zw,f}$	0.0525302	0.0584488
$F^{RMS}_{zw,r}$	0.0736751	0.0805891

TABLE IV

RMS PERFORMANCE ANALYSIS FOR SHORT-BACK PROFILE

	MPC	Sky-Hook
$\ddot{Z}^{c,RMS}_{rel}$	0.0360557	0.0467124
$F^{RMS}_{zw,f}$	0.0257788	0.0295034
$F^{RMS}_{zw,r}$	0.0454733	0.0488869

performances of the controlled suspension system on the basis of its averaged behavior over a time interval. However, it is a well known fact that, besides the averaged time behavior, also the vibration peaks influence in a significant way the passenger comfort during the ride. Then, it is interesting to evaluate the behavior of the system in correspondence of the most significant acceleration peaks of the sprung mass. To this end, the following variables may be conveniently considered:

$$\begin{aligned}\varrho_{\ddot{Z}^c}(t) &= \frac{\ddot{Z}^c_{SH}(t) - \ddot{Z}^c_{MPC}(t)}{\ddot{Z}^c_{MPC}(t)} \\ \varrho_{\ddot{\Theta}^c}(t) &= \frac{\ddot{\Theta}^c_{SH}(t) - \ddot{\Theta}^c_{MPC}(t)}{\ddot{\Theta}^c_{MPC}(t)}\end{aligned} \qquad (18)$$

where the subscripts SH and MPC stands for the simulated variables obtained using the Sky-Hook and MPC strategies respectively.

In Table V, the maximum over the most significant peak values of the variables considered in (18) are reported for the Random, the English Track and the Short-Back profiles, respectively. It has to be noted that the data reported in Table V reflect the overall behaviour in the considered road profile and not the data in the samples reported in the Figures.

TABLE V
PEAK PERFORMANCE ANALYSIS MPC VS. SKY-HOOK

	Random	English Track	Short-back
max $\varrho_{\ddot{Z}c}$	2.69405	3.01218	2.46355
max $\varrho_{\ddot{\Theta}c}$	6.16404	5.46583	2.12165

The analysis of the results presented in Table V evidences that the predictive technique decreases significantly the maximal peak accelerations of the sprung mass leading to a more comfortable behavior. In particular, it can be noted that the MPC strategy is able to reduce the peaks on the sprung mass accelerations from two to six times with respect to Sky-Hook control.

V. CONCLUSIONS

A control approach based on Model Predictive Control techniques for semiactive suspension systems has been proposed. In order to show the effectiveness of the proposed procedure, performance comparison with the well established semi-active control strategies Sky-Hook control has been presented. Simulation results have shown that the introduced approach allows to reach good performance levels in terms of car comfort by reduction of the RMS values of the sprung mass accelerations and a significant attenuation of their extremal values. Moreover also the vehicle handling characteristics are slightly improved as witnessed by the reduction of the RMS values of the forces dynamically exchanged between tire and road. The proposed suspension control strategy appears also feasible from a practical point of view as computational complexity related to the MPC formulation can be overcome by using a suitable “fast” MPC implementation technique.

VI. ACKNOWLEDGMENTS

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