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Observer Design for Linear Switched Control Systems

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***Abstract*— Full and reduced order observers for a class of linear switched control systems (LSCS) are studied in this paper. A "sub-observer" is first designed for the $i-$th subsystem. Then, a switching observer for an LSCS is constructed by simply picking the $i-$th sub-observer whenever the $i-$th subsystem is active. In the case of a full order observer, when subsystems are detectable, the state estimation error can converge to zero if the dwell time is large enough. Under certain conditions, the state estimation error may even converge to zero exponentially for arbitrary switching. Unlike classical linear systems where full order and reduced order observer can be designed under the same conditions, the design of a reduced order observer for an LSCS, besides detectability/observability, requires additional condition that the gains for all reduced order sub-observers need to be chosen the same. In such a case, similar stability results as those of full order observers are obtained for reduced order observers. Finally, examples and simulation results are given to show the effectiveness of the proposed observers.**

I. INTRODUCTION

As a special class of hybrid systems, switched systems have received a great deal of attention. The stability and stabilization problems for these systems have been studied extensively and useful results are now available. Detailed achievements in this research field can be found in survey papers by Decarlo,et.al. [1] and Liberzon and Morse [2].

Unlike stability and stabilization problems, observer design problem has received less attention and only a few results are available. Some researchers have designed switching observers for non-switched systems. The main idea is to use switching to solve the observer design problem for more complex systems, see [4], and/or to improve estimation performance, see [5].

An observer for continuous-time linear switched control system based on co-prime factorization approach is designed in [6]. The main idea is to construct a common observer for all subsystems. Inspired by the common Lyapunov function method for stabilization of switched control systems [2], observers are designed for discrete-time linear switched control systems in [7] and for both continuous- and discrete-time linear switched control systems in [8]. The design problem is reduced to solving a group of linear matrix inequalities (LMIs) for a common solution. The advantage of this observer design is that the stability of error dynamics can be guaranteed for arbitrary switching. One problem however is that the common Lyapunov function may not exist in some cases.

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Although the observability of switched control systems is important and has been studied in recent years (see [9] and the references list therein), it is not always necessary for the design of observers. To estimate the states, we can design observers directly without analyzing the observability if we can guarantee the state estimation error will converge to zero. This is the proposed strategy of this paper. We study the observer design problem for a class of linear switched control systems (LSCS). Both full order and reduced order observers will be designed without using a common Lyapunov function based observer design approach. The state estimation error dynamics resulting from both full order observers and the reduced order observers will be shown to be globally asymptotically stable when the dwell time is large enough. The stability results for both classes of observers under arbitrary switching will also be presented. This is achieved under certain conditions which up to now were enjoyed only by common Lyapunov function based observer design approach.

II. SWITCHED CONTROL SYSTEMS

We consider a class of switched linear control systems (SLCS) with M subsystems described as

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}u, x \in R^n \\ y &= C_{\sigma(t)}x, u \in R^p, y \in R^m \end{aligned} \tag{1}$$

where x, u, and y are the system state, input and output, respectively. Matrices A_i, B_i and C_i with $i \in S = \{1, 2, \cdots, M\}$ are $n \times n$, $n \times p$ and $m \times n$ constant matrices, respectively. The function $\sigma(t) : [0, \infty) \to S$ is a piecewise constant function of time and/or outputs, called a *switching rule*. The corresponding system for $\sigma(t) = i \in S$ is called the $i-$th subsystem. In such a case, we also say that the $i-$th subsystem is "active".

Assumptions:

- A1– The state in (1) is continuous for any control input. That is, the state does not jump at the switching instants.
- A2– For each $i \in S$, the $i-$th subsystem is detectable or observable.

III. FULL ORDER OBSERVER DESIGN

In [8], a full order observer is designed for system (1). To ensure the stability of the state estimation error dynamics, a common Lyapunov function must be found by solving some LMIs. However, for some SLCSs, no common Lyapunov function may exist. In such a case, the observer design in

[8] can not be applied. In this section, we design a full order observer for the system (1), which can be used for the case where a common Lyapunov function does not exist. Some sufficient conditions under which the state estimation error dynamics is globally asymptotically stable are derived.

A. Full Order Observer

Due to Assumption A2, it is well known that one can design the following observer for the $i-$th subsystem.

$$\dot{\hat{x}} = A_i\hat{x} + B_i u + L_i(y - C_i\hat{x}) \quad (2)$$

where L_i is chosen such that $A_i - L_iC_i$ is a Hurwitz matrix, and moreover by choosing L_i properly, the eigenvalues of $A_i - L_iC_i$ can be assigned arbitrarily if each subsystem is observable.

With (2) at hand, the full order observer for SLCS (1) is given below.

$$\dot{\hat{x}} = A_{\sigma(t)}\hat{x} + B_{\sigma(t)}u + L_{\sigma(t)}(y - c_{\sigma(t)}\hat{x}) \quad (3)$$

where at any switching instant t_s, we let $\hat{x}(t_s) = lim_{t\to t_s^-}\hat{x}(t)$ such that $\hat{x}$ is continuous. This is very important for stability analysis later.

To design the full order observer, we need to find L_i for $1 \le i \le M$. To compute L_i for $1 \le i \le M$, we can use any pole placement technique. We can also use the LMI method proposed in [8] to obtain L_i for $1 \le i \le M$,. This is described below in a two step algorithm.

Step 1–For each i, solve the following LMI for a $P_i > 0$ and Y_i.

$$A_i^TP_i - C_i^TY_i^T + P_iA_i - Y_iC_i < 0 \quad (4)$$

Step 2–Compute L_i by letting $L_i = P_i^{-1}Y_i$.

Regarding the above method of computing L_i for $1 \le i \le M$, we have the following result.

Theorem 1: Under Assumption A2, the LMI (4) always has solutions for $P_i > 0$ and Y_i, moreover, $L_i = P_i^{-1}Y_i$ can make $A_i - L_iC_i$ a Hurwitz matrix.

Proof: It follows from A2 that, for each i we can find a matrix L_i such that $A_i - L_iC_i$ is a Hurwitz matrix. This ensures that there exists a $P_i > 0$ such that

$$(A_i - L_iC_i)^TP_i + P_i(A_i - L_iC_i) = -Q \quad (5)$$

where Q is any chosen positive definite matrix. If we let $Y_i = P_iL_i$, we can rewrite the above equation as

$$A_i^TP_i - C_i^TY_i^T + P_iA_i - Y_iC_i = -Q < 0 \quad (6)$$

This proves that the LMI (4) always has solutions for $P_i > 0$ and Y_i.

Now we prove the second conclusion. That is, for any solution $P_i > 0$ and Y_i of the LMI (4), $L_i = P_i^{-1}Y_i$ can make $A_i - L_iC_i$ a Hurwitz matrix. Obviously, we can rewrite (4) as

$$(A_i - L_iC_i)^TP_i + P_i(A_i - L_iC_i) < 0 \quad (7)$$

This proves that $A_i - L_iC_i$ is Hurwitz. ■

Remark 1: In [8], it is required that there must exist a common P for $i = 1, 2, \cdots, M$ such that $(A_i - L_iC_i)^TP + P(A_i - L_iC_i) < 0$. In some cases, this is impossible, and thus their design can not be carried out. In our design, the existence of P_i for $i = 1, 2, \cdots, M$ such that $(A_i - L_iC_i)^TP_i + P_i(A_i - L_iC_i) < 0$ can be guaranteed according to Theorem 1. Therefore, our design can always be carried out.

B. Stability of State Estimation Error Dynamics

If we let $e(t) = \hat{x} - x$, it then follows from (1) and (3)

$$\dot{e}(t) = (A_{\sigma(t)} - L_{\sigma(t)}C_{\sigma(t)})e(t) \quad (8)$$

For simplicity, we denote $\bar{A}_{\sigma(t)} = A_{\sigma(t)} - L_{\sigma(t)}C_{\sigma(t)}$, then we have

$$\dot{e}(t) = \bar{A}_{\sigma(t)}e(t) \quad (9)$$

where $e(t)$ is guaranteed to be continuous by the construction of our observer. Now the stability of state estimation error dynamics is reduced to the stability of a switched linear system given by (9). Because $\bar{A}_i$ for all i are Hurwitz, the problem becomes one of studying the stability of a switched linear system with all its subsystems being Hurwitz. This problem has been studied extensively in the literature, and many results are now available, see for example those cited in [1] and [2]. All those results are applicable here. However, in this paper, only two results will be given. One is based on the concepts of *dwell time* and a stability result given in [2]. The other is based on a new stability result that we shall present for a special class of (9).

Let's first introduce the concept of dwell time.

Definition 1: For any switching signal $\sigma(t)$, if there exists a positive constant τ such that the interval between any two consecutive switching times is no smaller than τ, then τ is called **dwell time**.

With the help of the concept of dwell time consider the following result.

Theorem 2: Under Assumptions A1 and A2, if the dwell time τ is large enough, then the state estimation error dynamics (9) is globally asymptotically stable.

Proof: From Assumption A1, we know that x is continuous, this implies that $e(t)$ is continuous because $\hat{x}$ is continuous. For each $i \in \{1, 2, \cdots, M\}$, because $\bar{A}_i$ is Hurwitz, there exist $a_i \ge 0$ and $\lambda_i > 0$ such that for all $t \ge 0$ we have

$$\|e^{\bar{A}_it}\| \le e^{a_i - \lambda_i t} \quad (10)$$

where $\|A\| = \sqrt{\lambda_{max}(A^HA)}$, and H denotes the conjugate transpose. If

$$\tau > \max_{i=1,2,\cdots,M}\{\frac{a_i}{\lambda_i}\}$$

then all conditions needed in Lemma 2 in [10] are satisfied. Hence, by applying Lemma 2 in [10], the conclusion of the theorem follows. ■

Remark 2: Note that the requirement of $\hat{x}$ being continuous is crucial for the stability of the state error dynamics. Without it, the continuity of the state error can not be guaranteed. Therefore, the result in [10] can not be applied because it requires the states of the switched systems to be continuous. This is the main reason we require Assumption A1 and the continuity of $\hat{x}$.

Remark 3: In practical applications Theorem 2 poses two difficulties. One is the computation of $a_i \geq 0$ and $\lambda_i > 0$ for all i. To our knowledge, so far only their existence is guaranteed and no method for computing them exist. The other difficulty has to do with the requirement of having some knowledge of the dwell time. The dwell time is not generally available a priori. For these reasons, the result is of more theoretical than practical importance.

A method is given in the following theorem to solve the first difficulty mentioned in Remark 3 for a special class of matrices.

Theorem 3: For a matrix A, if

$$A = T\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix} T^{-1}$$

Then we have

$$\|e^{At}\| \leq e^{a+\lambda t} \tag{11}$$

where $a = ln(\|T\|\|T^{-1}\|)$ and $\lambda = max_{1\leq i\leq n}\{\lambda_i\}$

Proof: It is easy to see that

$$e^{At} = T\begin{pmatrix} e^{\lambda_1} & 0 & \cdots & 0 \\ 0 & e^{\lambda_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{\lambda_n} \end{pmatrix} T^{-1} \tag{12}$$

Note that $\lambda = max_{1\leq i\leq n}\{\lambda_i\}$ and $a = ln(\|T\|\|T^{-1}\|)$, it follows that

$$\begin{aligned} \|e^{At}\| &\leq \|T\|\left\|\begin{pmatrix} e^{\lambda_1 t} & 0 & \cdots & 0 \\ 0 & e^{\lambda_2 t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{\lambda_n t} \end{pmatrix}\right\|\|T^{-1}\| \\ &= \|T\|e^{\lambda t}\|T^{-1}\| \\ &= e^{a+\lambda t} \end{aligned} \tag{13}$$

This completes the proof. ■

Remark 4: If all subsystems are observable, then the eigenvalues of $\bar{A}_i, 1 \leq i \leq n$ can be arbitrarily assigned. If we assign the eigenvalues of each $\bar{A}_i$ into the left hand plane and at distinct locations, then all $\bar{A}_i, 1 \leq i \leq n$ satisfy the condition in Theorem 3. Hence $a_i \geq 0$ and $\lambda_i > 0$ for all i can be computed by using the formulas given in Theorem 3.

The first drawback in Remark 3 can be overcome for switched linear systems with observable subsystems. However, the second drawback still remains. In what follows, we plan to derive stability results for (9) under arbitrary switching. First, we give a new result on the stability of a special switching systems of the form (9).

Theorem 4: For a switching system of the form (9), assume that for all $1 \leq i \leq M$, $\bar{A}_i + \bar{A}_i^T$ are negative definite, then the error system is globally exponentially stable for arbitrary switching if (9) satisfies Assumption A1.

Remark 5: In the literature, stability for switching systems with arbitrary switching is analyzed by the so-called common Lyapunov function approach [2]. Here for a special class of switching systems, by investigating the solution directly, we give a new stability result for arbitrary switching without using the common Lyapunov function approach though the condition is quite restrictive. This result will be used later to analyze the stability of state estimation error dynamics of both full and reduced order observers.

As a special case of Theorem 4, the following result is obviously true.

Theorem 5: Under assumptions A1, if we can design an observer of the form (3) such that $\bar{A}_i$ is Hurwitz and $\bar{A}_i^T = \bar{A}_i$ for all $1 \leq i \leq m$, then the state estimation error dynamics (9) is globally exponentially stable for arbitrary switching.

To use Theorem 5, we need to design the observer gain L_i (3) such that $\bar{A}_i = A_i - L_iC_i$ is Hurwitz and $\bar{A}_i^T = \bar{A}_i$ for all $1 \leq i \leq m$. This is not always possible. A sufficient condition for the existence of the observer gain L_i is given in the following theorem.

Theorem 6: Let $C_i = \begin{bmatrix} 0 & I_{p\times p} \end{bmatrix}$, $L_i = \begin{pmatrix} L_1^i \\ L_2^i \end{pmatrix}$ and $A_i = \begin{pmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{pmatrix}$, where A_{12}^i is an $n-p$ by p matrix. If A_{11}^i is negative definite, then L_i can be chosen such that $\bar{A}_i$ is negative definite.

Proof: Proof: Since $\bar{A}_i = A_i - L_iC_i$, we have

$$\bar{A}_i = \begin{pmatrix} A_{11}^i & A_{12}^i + L_1^i \\ A_{21}^i & A_{22}^i + L_2^i \end{pmatrix}$$

If we choose

$$L_1^i = (A_{21}^i)' - A_{12}^i$$

and

$$L_2^i = P_{22}^i - A_{22}^i + A_{21}^i(A_{11}^i)^{-1}(A_{21}^i)'$$

where P_{22}^i can be any negative definite matrix, then it can be shown that $\bar{A}_i$ is negative definite. ■

IV. REDUCED ORDER OBSERVER DESIGN

In this section, we first design a reduced order observer for the system (1). Then, we derive sufficient conditions under which the state estimation error dynamics resulting from reduced order observer is stable. Although the design is quite similar to that of full order observer, the entire design procedure is presented in order to illustrate the fact that the conditions for stability of the the two types of observers are different. This is a new phenomenon in switched linear systems which is quite different from conventional linear systems where the stability conditions are the same.

To design reduced order observer, we need the following assumption.

A3 – There exist a common nonsingular matrix T such that $C_1T = \cdots = C_MT = [0\ I]$, where I is $m \times m$ identity matrix.

A. Reduced Order Observer

Because the case for $T \neq I$ in A3 can be transformed into $T = I$ through a state transformation, for simplicity, we only present the reduced observer design for $T = I$. In this case, we have $C_1 = \cdots = C_M = [0 \quad I]$. For each i, we partition the state vector as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, x_1 \in R^{n-m}\ x_2 \in R^m \quad (14)$$

Because $C_i = [0 \quad I]$, we have $y = x_2$. Therefore, we do not need to estimate x_2, and only x_1 need to be estimated. To estimate x_1, we define $\bar{x}_1 = x_1 - L_iy$. If we can estimate $\bar{x}_1$, x_1 can be estimated. Now, with the help of the definition of $\bar{x}_1$, the reduced observer for the $i-$th subsystem can be designed as following

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= (A_{11}^i - L_iA_{21}^i)\hat{\bar{x}}_1 + (A_{11}^i - L_iA_{21}^i)L_iy \\ &+ (A_{12}^i - L_iA_{22}^i)y + (B_1^i - L_iB_2^i)u \\ \hat{x}_1 &= \hat{\bar{x}}_1 + L_iy \end{aligned} \quad (15)$$

Because of Assumption A2, (A_{21}^i, A_{11}^i) is detectable or observable. Hence L_i can be chosen such that $A_{11}^i - L_iA_{21}^i$ is a Hurwitz matrix. A_{jk}^i and B_j^i for $j = 1, 2; k = 1, 2$ are defined as

$$A_i = \begin{pmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{pmatrix}, B_i = \begin{pmatrix} B_1^i \\ B_2^i \end{pmatrix} \quad (16)$$

where $A_{11}^i \in R^{(n-m)\times(n-m)}$ and $B_1^i \in R^{(n-m)\times p}$. If we use the same idea for the full order observer design, with (15) at hand, the reduced order observer for SLCS (1) would be given as

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= (A_{11}^{\sigma(t)} - L_{\sigma(t)}A_{21}^{\sigma(t)})\hat{\bar{x}}_1 \\ &+ (A_{11}^{\sigma(t)} - L_{\sigma(t)}A_{21}^{\sigma(t)})L_{\sigma(t)}y \\ &+ (A_{12}^{\sigma(t)} - L_{\sigma(t)}A_{22}^{\sigma(t)})y + (B_1^{\sigma(t)} - L_{\sigma(t)}B_2^{\sigma(t)})u \\ \hat{x}_1 &= \hat{\bar{x}}_1 + L_{\sigma(t)}y \end{aligned} \quad (17)$$

where at any switching instant t_s, we let $\hat{\bar{x}}_1(t_s) = lim_{t\to t_s^-}\hat{\bar{x}}_1(t)$ such that $\hat{\bar{x}}_1$ is continuous.

Now, the question is: Would the above reduced order observer have a satisfactory performance? Generally, the answer is no. This is point is clearly illustrated in the simulation results later. The reason for poor quality performance of the reduced order observer as opposed to the full order observer is that if we do not have $L_1 = L_2 = \cdots = L_M$, $\bar{x}_1 = x_1 - L_{\sigma(t)}y$ is not continuous at any switching instant t_s. If we let $e(t) = \hat{\bar{x}}_1 - \bar{x}_1$, then we know that $e(t)$ is not continuous because $\hat{\bar{x}}_1$ is while $\bar{x}_1$ is not. Therefore, the results given in last sections for continuous switching systems can not be applied anymore, and thus the exponential stability of the observer is not guaranteed.

Based on the above discussions, we conclude that to make the reduced order observer perform satisfactorily, we need to ensure that $e(t)$ is continuous, or equally that $\bar{x}_1 = x_1 - L_{\sigma(t)}y$ is continuous. It is easy to see that $L_1 = L_2 = \cdots = L_M$ makes $e(t)$ continuous. Now, the problem is to find a common L such that $A_{11}^i - LA_{21}^i$ for $1 \leq i \leq M$ are Hurwitz. The existence of such an L is equivalent to simultaneous observability [6]. Hence any conditions for simultaneous observability can be used here to check the existence of L. When L exists, the question is to how to find the common L. For this, we can use the LMI based method given in [8]. Once a common L is found, we can use the reduced order observer given by (17). Because $L_1 = L_2 = \cdots = L_M = L$, we have:

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= (A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)})\hat{\bar{x}}_1 + (A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)})Ly \\ &+ (A_{12}^{\sigma(t)} - LA_{22}^{\sigma(t)})y + (B_1^{\sigma(t)} - LB_2^{\sigma(t)})u \\ \hat{x}_1 &= \hat{\bar{x}}_1 + Ly \end{aligned} \quad (18)$$

Remark 6: Unlike full order observer in the last section, the design of the reduced order observer requires the existence of a common observer gain. That is, simultaneous observability of (A_{21}^i, A_{11}^i) for $1 \leq i \leq M$ is needed. This is quite different from the observer design for classical linear systems, where both full order and reduced order observer can be designed under the same conditions.

B. Stability of State Estimation Error Dynamics of Reduced Observer

In this subsection, we show that $\hat{x}_1$ converges to x_1. By definition, we only need to show that $\hat{\bar{x}}$ converges to $\bar{x}_1$, that is, $e(t) = \hat{\bar{x}}_1 - \bar{x}_1$ converges to zero. Noticing that the definition of $\bar{x}_1$, it follows from (1) and (18) that

$$\dot{e}(t) = (A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)})e(t) \quad (19)$$

For simplicity, if let $\bar{A}_{\sigma(t)} = A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)}$, then we have

$$\dot{e}(t) = \bar{A}_{\sigma(t)}e(t) \quad (20)$$

Note that the above error dynamics has exactly the same form as that derived from full order observer. Therefore, we can prove similar results for reduced order observer, which are given below.

Theorem 7: Under Assumptions A1-A3, if the **dwell time** τ is large enough, then the state estimation error dynamics (20) is globally asymptotically stable.

Theorem 8: Under Assumptions A1-A3, if a common L can be found such that $\bar{A}_i = A_{11}^i - LA_{21}^i$ are negative definite for all $1 \le i \le m$, then the state estimation error dynamics (20) is globally exponentially stable for arbitrary switching.

V. ILLUSTRATIVE EXAMPLE

In this section, both full order and reduced order observers for an LSCS are designed. We then give some simulation results to see the effectiveness of these observers. Consider the following LSCS.

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}u \\ y &= C_{\sigma(t)}x \end{aligned} \quad (21)$$

where A_i, B_i, C_i are given as

$$\begin{aligned} A_1 &= \begin{pmatrix} -1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -0.45 \end{pmatrix} \\ B_1 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_1 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \\ A_2 &= \begin{pmatrix} 0 & 1 & 0 \\ -10 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \\ B_2 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \ C_2 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (22)$$

with $\sigma(t) \in \{1, 2\}$ defined as

$$\begin{aligned} \sigma(t) &= 1 \ if \ t \in [20kT \quad 10(2k+1)T) \\ \sigma(t) &= 2 \ if \ t \in [10(2k+1)T \quad 20(k+1)T) \end{aligned} \quad (23)$$

where $k = 1, 2, \cdots$, and T is a constant which determines how fast the switching signal switches. Both of the subsystems are open-loop unstable, to design full order observer, we need to choose L_i such that $A_i - L_iC_i$ is Hurwitz for each $i \in \{1, 2\}$. For the above system, it is easy to check $A_1 - L_1C_1$ and $A_2 - L_2C_2$ are Hurwitz if we choose $L_1 = [11\ 15\ 7.55]^T$ and $L_2 = [6.6\ 17\ 8]^T$. With L_1 and L_2, the full order observer is designed as

$$\dot{\hat{x}} = A_{\sigma(t)}\hat{x} + B_{\sigma(t)}u + L_{\sigma(t)}(y - c_{\sigma(t)}\hat{x}) \quad (24)$$

Simulation results for the full order observer with $T = 0.001$ are presented in Figure 1. The switching is very fast, but from Figure 1, we see that that the estimates of states converge to the states of the switched systems asymptotically.

Unlike the full order observer, for reduced order observer design, we need further requirement on the sub-observer gains. That is, all sub-observer gains must be chosen equal. Otherwise, the stability of the state estimation error dynamics can not be guaranteed. To see this, we design two reduced order observers. The first one is with different sub-observer gains; the second one is with the same sub-observer gain.

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= (A_{11}^{\sigma(t)} - L_{\sigma(t)}A_{21}^{\sigma(t)})\hat{\bar{x}}_1 \\ &+ (A_{11}^{\sigma(t)} - L_{\sigma(t)}A_{21}^{\sigma(t)})L_{\sigma(t)}y \\ &+ (A_{12}^{\sigma(t)} - L_{\sigma(t)}A_{22}^{\sigma(t)})y + (B_1^{\sigma(t)} - L_{\sigma(t)}B_2^{\sigma(t)})u \\ \hat{x}_1 &= \hat{\bar{x}}_1 + L_{\sigma(t)}y \end{aligned} \quad (25)$$

where $L_1 = [0 \quad 4]^T$ and $L_2 = [0 \quad 7]^T$.

$$\begin{aligned} \dot{\hat{\bar{x}}}_1 &= (A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)})\hat{\bar{x}}_1 + (A_{11}^{\sigma(t)} - LA_{21}^{\sigma(t)})Ly \\ &+ (A_{12}^{\sigma(t)} - LA_{22}^{\sigma(t)})y + (B_1^{\sigma(t)} - LB_2^{\sigma(t)})u \\ \hat{x}_1 &= \hat{\bar{x}}_1 + Ly \end{aligned} \quad (26)$$

where $L = [0 \quad 8]^T$ makes both $A_{11}^1 - LA_{21}^1$ and $A_{11}^2 - LA_{21}^2$ have negative and distinct eigenvalues.

Simulation results for the first reduced order observers is given in Figure 2, from which we see that the state estimation error dynamics is not stable without a common observer gain. The simulation results of the second reduced order observer for $T = 0.001$ are given in Figure 3. Though the switching is very fast, from Figures 3, we see that that the estimates of states converge to the states of the switched systems asymptotically. The initial conditions for Figures 1-3 are $(x_1(0), x_2(0), x_3(0)) = (0, 0, 0)$, $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)) = (0.6, 0.6, 0.6)$, and $(\hat{\bar{x}}_1(0), \hat{\bar{x}}_2(0)) = (0.6, 0.6)$.

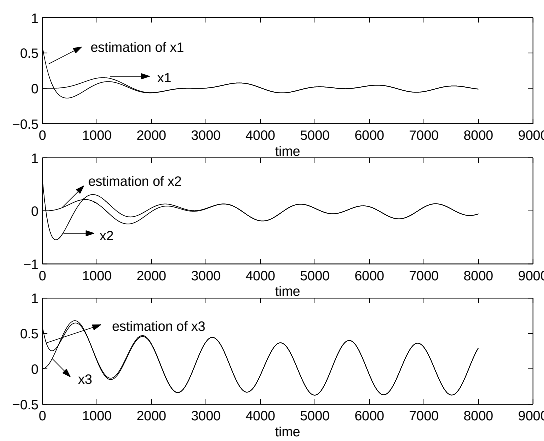

Fig. 1. Full order observer

VI. CONCLUSIONS AND FUTURE WORK

A. Conclusions

A procedure for designing both full as well as reduced order observers for a class of LSCSs was presented in this paper. The design strategy is based on designing a sub-observer for each subsystem, then the overall observer

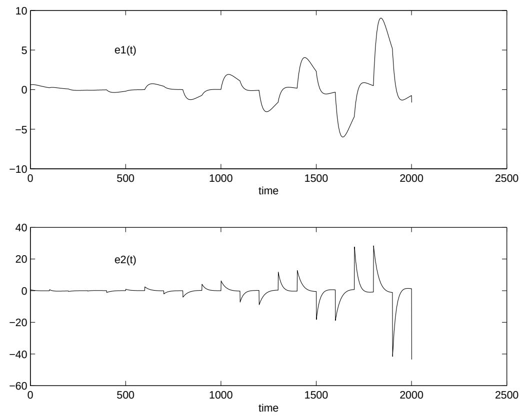

Fig. 2. Reduced order observer with different observer gains

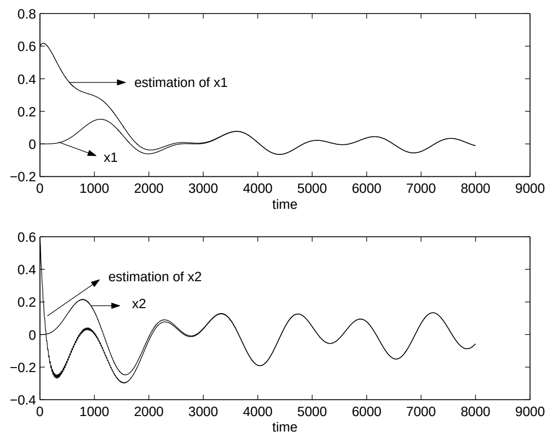

Fig. 3. Reduced order observer with common observer gain

for the switching system is obtained by picking the corresponding sub-observer according to the switching signal. It was established that unlike observer design for a typical linear system, full order and reduced order observer design for switching systems require different design conditions. It turns out that in the case of switching systems the design of reduced order observer requires stricter conditions than that of full order observer. The stability of the state estimation error dynamics of both classes of observers are analyzed. It was shown that so long that detectability condition for each subsystem is satisfied, the proposed full order observer design can guarantee the state estimation error dynamics is globally asymptotically stable if the dwell time is large enough. When the full order can be designed such that the state estimation error dynamics is a switched system with negative definite matrices, the state estimation error dynamics is globally exponentially stable for arbitrary switching which is preferable. For reduced order observer, however, it was shown that similar stability results can be obtained under stricter conditions.

B. Future Work

The observers designed in this paper require the switching signal to be known a priori. How to design stable observers for systems where the switching signal is not known is an interesting and unsolved problem. Currently work is being done to extend our results to switched systems which are not continuous in their states.

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