Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Modeling and control of railway networks

Ton van den Boom and Bart De Schutter

Abstract— **In this paper we consider the modeling and control of railway networks. The main aim of the control is to recover from delays in an optimal way by breaking connections and changing the departure of trains (at a cost). To model the controlled railway system we will use a switching max-plus-linear system description. We define the optimal control design problem for the railway network, and we show that solving this problem leads to an integer optimization problem. By solving an easier low-dimensional real-valued optimization problem we obtain good initial values for the integer optimization problem.**

I. INTRODUCTION

We discuss the modeling, analysis and control of railway and subway networks. Typical examples of discrete event systems are flexible manufacturing systems, telecommunication networks, parallel processing systems, traffic control systems, and logistic systems. The class of discrete event systems essentially consists of man-made systems that contain a finite number of resources (such as machines, communications channels, or processors) that are shared by several users (such as product types, information packets, or jobs) all of which contribute to the achievement of some common goal (the assembly of products, the end-to-end transmission of a set of information packets, or a parallel computation) [1]. In general, models that describe the behavior of a discrete event system are nonlinear in conventional algebra. However, there is a class of discrete event systems — the max-plus-linear discrete event systems — that can be described by a model that is "linear" in the max-plus algebra [1], [2], which has maximization and addition as its basic operations. The max-plus-linear discrete event systems can be characterized as the class of discrete event systems in which only synchronization and no concurrency or choice occurs. A typical example is a railway network with rigid connection constraints and a fixed routing schedule. Note that in this railway context, synchronization means that some trains should give pre-defined connections to other trains, and a fixed routing means that the order of departure is fixed. However, if one of the trains has a too large delay, then it is sometimes better — from a global performance viewpoint — to break a connection or to reschedule the train order, and to let a train depart anyway. In this way we prevent an accumulation of delays in the network. Of course, missed connections should lead to a penalty due to dissatisfied passengers. In [3], [4] we have considered the control of railway networks using breaking connections only as control measure. In this paper we will extend the control handles and reschedule the trains by breaking connections as well as changing train order. We will model a controlled railway system using a switching max-plus-linear system description. In this description we use a number of max-plus-linear system descriptions, each description corresponds to a specific mode, describing the network by a different set of connection and order constraints. We control the system by switching between different modes, allowing us to break train connections and to change the order of trains. We will define a control algorithm to optimize the performance of the network.
Other work in connection with the modeling and control of railway networks (mainly in a discrete event context) can be found in [5], [6], [7], [8], [9], [10].

We will first derive a model for a railway system using a switching max-plus-linear systems description. Next, we define a control design problem for such a system where we can break connections if delays occur, and change the order of trains if this leads to a better global performance. We will optimize the systems behavior. In general this will lead to an integer optimization problem, which we will solve using genetic algorithms. Computational experiments show that (for small sized problems or for a small control horizon) the genetic algorithm approach yields good results.

T.J.J. van den Boom and B. De Schutter are with the Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands, email: {t.j.j.vandenboom,b.deschutter}@dcsc.tudelft.nl

II. MODEL

Consider a railway operations system, which follows a schedule with period T. In nominal operation mode, we assume that all the trains follow a pre-scheduled route, with fixed train order and pre-defined connections. If for some reason we have to break connections or change the train order, we will operate in a perturbed mode. With every new schedule we can associate a perturbed mode. First we will discuss the nominal operation.

A. Nominal Operation

Consider a railway operations system which is operating in nominal operation mode.

Each track of the railway network has a number and a train allocated to it. For the sake of simplicity we will say "(virtual) train j" to denote the (physical) train on track j, and "station j" to denote the station at the beginning of track j (cf. Fig. 1). Let n be the number of "virtual" tracks in the network. We say virtual to denote that some of the virtual tracks may actually be the same physical track (corresponding to different trains using the same track). This means that the number of tracks is usually smaller than n. Let $x_j(k)$ be the time instant at which train j departs from

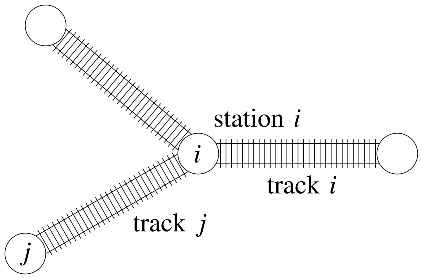

Fig. 1. A part of a railway network.

station j for the kth time. Let $d_j(k)$ be the departure time for this train according to the time schedule.

Let $p_i(k)$ be the predecessor track of train i, and let $\mathscr{C}_j(k)$ be the set of trains to which the kth train j gives a connection. Let $\mathscr{F}_j(k)$ be the set of trains that move over the same track as train j, in the same direction as train j, and are scheduled behind train j. Let $\mathscr{W}_j(k)$ be the set of trains that move over the same track as train j, in the opposite direction of train j, and are scheduled behind train j. Furthermore, let $a_j(k)$ be the traveling time on track j, define a minimum connection time $c_{ij}^{\min}(k)$ for passengers to get from train j to train i for each train $j \in \mathscr{C}_i(k)$ and define a minimum stopping time $s_j^{\min}(k)$ of train j at station j to allow passengers to get off or on the train. Finally, define a minimum separation time $f_j^{\min}(k)$ between two different trains moving over the same track and in the same direction as train j, and a minimum separation time $w_j^{\min}(k)$ between two different trains moving over the same track and in the opposite direction.

Now we have the following constraints for the kth departure time $x_i(k)$ of train i:

- Time schedule constraint:
$$x_i(k) \geqslant d_i(k) \ .$$
- Continuity constraints: For train $j = p_i(k)$ we have
$$x_i(k) \geqslant x_j(k-\delta_{ij}^*) + a_j(k) + s_j^{\min}(k)$$
where the notation δ_{ij}^* is used to denote 1 if the $(k-1)$th train j continues as the kth train i, and 0 if the kth train j continues as the kth train i (and if some trips last longer than the twice the cycle time T of the schedule, δ_{ij}^* might be equal to 2, and so on — see also the example in Section IV). However, for the sake of simplicity, we will only consider δ_{ij}^*'s of either 0 or 1 in this paper.
- Connection constraints:
For each train $i \in \mathscr{C}_j(k)$ we have
$$x_i(k) \geqslant x_j(k-\delta_{ij}^*) + a_j(k) + c_{ij}^{\min}(k)$$
where the notation δ_{ij}^* is similar as for the continuity constraint, so $\delta_{ij}^* = 1$ if the $(k-1)$th train j gives a connection to the kth train i, and $\delta_{ij}^* = 0$ if the kth train j gives a connection to the kth train i.
- Follow constraints:
For each train $i \in \mathscr{F}_j(k)$ we have
$$x_i(k) \geqslant x_j(k-\delta_{ij}^*) + f_j^{\min}(k)$$
(δ_{ij}^* is defined similarly as above).
- Wait constraints:
For each train $i \in \mathscr{W}_j(k)$ we have
$$x_i(k) \geqslant x_j(k-\delta_{ij}^*) + a_j(k) + w_j^{\min}(k)$$
(δ_{ij}^* is defined similarly as above).

Since we let a train depart as soon as all connection conditions are satisfied, we have

$$\begin{aligned}
x_i(k) = \max\big(&d_i(k), \\
&(x_{p_i(k)}(k-\delta_{ip_i(k)}^*) + a_{p_i(k)}(k) + s_{p_i(k)}^{\min}(k)), \\
&\max_{j\in\mathscr{C}_i(k)} (x_j(k-\delta_{ij}^*) + a_j(k) + c_{ij}^{\min}(k)), \\
&\max_{l\in\mathscr{F}_i(k)} (x_l(k-\delta_{il}^*) + f_l^{\min}(k)), \\
&\max_{m\in\mathscr{W}_i(k)} (x_m(k-\delta_{im}^*) + a_m(k) + w_m^{\min}(k))\big)
\end{aligned} \tag{1}$$

Note that in a undisturbed, well-defined time schedule the term $d_i(k)$ in (1) will be the largest. However, if due to unforeseen circumstances (an incident, a late departure, etc.) one of the trains ($p_i(k)$,l or m) has a delay the corresponding term can become larger than the others, train i will depart later than the scheduled departure time $d_i(k)$ and will therefore also be delayed. Using successive substitution we can eliminate all right-hand terms with index k and by defining the appropriate matrix A^0, we can rewrite equation (1) as:

$$x_i(k) = \max\left(d_i(k) \, , \max_j \left(x_j(k-1) + A_{i,j}^0 \right) \right) \tag{2}$$

where $A_{i,j}^0$ is the (i,j)th entry of the matrix A^0.

Now we introduce some notation from max-plus algebra. Define $\varepsilon = -\infty$ and $\mathbb{R}_\varepsilon = \mathbb{R} \cup \{\varepsilon\}$. The max-plus-algebraic addition ($\oplus$) and multiplication ($\otimes$) are defined as follows [1], [2]:

$$x \oplus y = \max(x,y) \qquad x \otimes y = x + y$$

for $x, y \in \mathbb{R}_\varepsilon$ and

$$\begin{aligned}
[A \oplus B]_{ij} &= a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij}) \\
[A \otimes C]_{ij} &= \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \max_{k=1,\ldots,n} (a_{ik} + c_{kj})
\end{aligned}$$

for $A, B \in \mathbb{R}_\varepsilon^{m\times n}$ and $C \in \mathbb{R}_\varepsilon^{n\times p}$. The matrix $\mathcal{E}$ is the max-plus-algebraic zero matrix: $[\mathcal{E}]_{ij} = \varepsilon$ for all i, j.

In max-plus notation, equation (2) becomes

$$x_i(k) = d_i(k) \oplus \bigoplus_{j=1}^{n} x_j(k-1) \otimes A_{i,j}^0$$

and in matrix-notation we obtain

$$x(k) = A^0 \otimes x(k-1) \oplus d(k) \tag{3}$$

B. Perturbed Operation

In the nominal operation we have assumed that some trains should give pre-defined connections to other trains, and the order of trains on the same track is fixed. However, if one of the preceding trains has a too large delay, then it is sometimes better — from a global performance viewpoint — to let a connecting train depart anyway or to change the departure order on a specific track. This is done in order to prevent an accumulation of delays in the network. In this paper we will consider the switching between different operation modes, where each mode corresponds to a different set of pre-defined or broken connection and a specific order of train departures. We allow the system to switch between different modes, allowing us to break train connections and to change the order of trains. Note that any broken connection or change of train order leads to a new model, similar to the nominal equation (3), but now with adapted system matrices (A^ℓ) for the ℓ-th model. We have the following system equation for the perturbed operation:

$$x(k) = A^\ell(k) \otimes x(k-1) \oplus d(k) \tag{4}$$

where the argument k of A^ℓ indicates that the system matrices change in time.

III. THE RAILWAY CONTROL PROBLEM

Switching max-plus-linear systems are different from conventional time-driven systems in the sense that the event counter k is not directly related to a specific time. A time instant t in cycle k (so $(k-1)T \leqslant t < kT$), some of the components of $x(k)$ may already be known while other components of $x(k)$ may still lie in the future (Recall that $x(k)$ contains the time instants at which the internal activities or processes of the system start for the kth time). Therefore, we will now present a method to address the timing issues in control of switching MPL systems.

We consider the case of full state information[1], since the components of $x(k)$ correspond to departure times, which are in general easy to measure.

Consider time instant t in cycle k, so $(k-1)T \leqslant t < kT$. We have measurements of departure times $x_{\text{past}}(k)$ and traveling times $a_{\text{past}}(k)$ of trains that have arrived at their destination. Sometimes there is information available about the estimated traveling time for trains that have not yet arrived at their destination at time t. With this information we can make an estimation $\hat{a}_{\text{est}}(k|t)$ (with the same dimension as $a(k)$) of the future traveling times. If no further information is available on a specific traveling time we take the nominal traveling time $[\hat{a}_{\text{est}}(k|t)]_i = a_{i,\text{nom}}$.

Next we have to define the set $\mathscr{U}(k|t)$ of possible future control actions (i.e. breaking connections or changing train order). Certain control actions are not feasible any more (e.g. If a connection has been broken in the past and the connecting train has already departed, it is impossible to 're-pair' this connection.). We define the vector $u(k|t) \in \mathscr{U}(k|t)$, where each element corresponds to a specific control action, so a specific (scheduled) connection or specific (scheduled) train order. We assume $u(k|t)$ to be binary, where $u_i(k|t) = 0$ corresponds to the nominal case, and $u_i(k|t) = 1$ to the perturbed case (the connection is broken or the order of two trains is switched).

To select the optimal set of possible future control actions, we define the following optimal control problem at time instant t ($(k-1)T \leqslant t < kT$):

$$\min_{\{u(k|t),u(k+1|t),u(k+2|t),\ldots\}} J(k|t) \tag{5}$$

where the performance index $J(k|t)$ is given by

$$J(k,t) = \sum_{j=0}^{\infty} \|Q\hat{e}(k+j|t)\|_2^2 + \|Ru(k+j|t)\|_2^2 \tag{6}$$

in which $\hat{e}(k+j|t)$ is the vector with the expected delays ($\hat{e}_i(k+j|t) = \hat{x}_i(k+j|t) - d_i(k+j)$), Q and R are weighting matrices, and $\|\cdot\|$ is the vector 2-norm. The first term of (6) is related to the sum of all predicted delays, and the second term denotes the penalty for all broken connections and switched train orders during cycle $(k+j)$.

To compute the predictions of $\hat{x}(k+j|t)$ we make use of the fact that at time t we have $a_{\text{past}}(k|t)$ and $\hat{a}_{\text{est}}(k+j|t)$ available and using that we can determine the estimates $\hat{A}^\ell(k+j|t)$ of all future $A^\ell(k+j)$. Now $\hat{x}(k+j|t)$ can be found by successive substitution

$$\hat{x}(k+j|t) = \hat{A}^\ell(k+j-1|t) \otimes \hat{x}(k+j-1|t) \oplus d(k+j)\ , \forall j \geqslant 1$$

In principle we have all elements to solve the optimal control problem (5). Note that if the railway network is well-defined and there is some margin in the schedule[2], there will always be an integer N such that in the nominal case ($u(k+j|t) = 0$ for all $j \geqslant 0$) the delays will have vanished ($\hat{e}(k+j|t) = 0$ for all $j \geqslant 0$). In the performance index (6) we may then replace the infinite sum by a finite one (with an optional constraint $\hat{e}(k+N|t) = 0$). We now have an integer optimal control problem with nN binary parameters. We can solve this problem efficiently with genetic algorithms [11] or with tabu search [12], [13], [14].

To find a good initial guess for the integer optimization we first solve an easier problem, in which we structure the input signal. This is done by defining a decision mechanism, where we use thresholds on (expected) delays to decide whether a connection should be broken or train orders should be switched. First consider the case where variable $u_l(k)$ is related to the connection of train j to train i, with nominal connection constraint

$$x_i(k) \geqslant x_j(k-\delta_{ij}^*) + a_j(k) + c_{ij}^{\min}(k)$$

[1]Note that measurements of occurrence times of events are in general not as susceptible to noise and measurement errors as measurements of continuous-time signals involving variables such as temperature, speed, pressure, etc.

[2]If the max-plus eigenvalue of the matrix A^0 is strictly smaller than 0 there is some margin in the schedule.

and let $d_i(k) > t$. Define $\hat{z}_j(k-\delta^*_{ij}|t) = \hat{x}_j(k-\delta^*_{ij}|t) + [\hat{a}_{\text{est}}]_j(k|t)$ as the expected arrival-time of train j. Now we choose

$$\begin{cases} u_l(k) = 0 & \text{if } \hat{z}_j(k-\delta^*_{ij}|t) + c^{\min}_{ij}(k) - d_i(k) \leqslant \tau \\ u_l(k) = 1 & \text{otherwise,} \end{cases}$$

where τ is a non-negative threshold. Next consider the case where variable $u_l(k)$ is related to the order of two trains j and i moving over the same track in the same direction, with nominal following constraint

$$x_i(k) \geqslant x_j(k-\delta^*_{ij}) + f^{\min}_j(k)$$

and let $x_i(k) \geqslant t$ (that means that at time t train $x_i(k)$ has not departed yet). Now we choose

$$\begin{cases} u_l(k) = 0 & \text{if } \hat{x}_j(k-\delta^*_{ij}|t) + f^{\min}_j(k) - d_i(k) \leqslant \phi \\ u_l(k) = 1 & \text{otherwise,} \end{cases}$$

where ϕ is a non-negative threshold. Finally consider the case where variable $u_l(k)$ is related to the order of two trains j and i moving over the same track in the opposite direction, with nominal waiting constraint

$$x_i(k) \geqslant x_j(k-\delta^*_{ij}) + a_j(k) + w^{\min}_j(k)$$

and let $d_i(k) > t$. Now we choose

$$\begin{cases} u_l(k) = 0 & \text{if } \hat{z}_j(k-\delta^*_{ij}|t) + w^{\min}_j(k) - d_i(k) \leqslant \omega \\ u_l(k) = 1 & \text{otherwise,} \end{cases}$$

where $\hat{z}_j(k-\delta^*_{ij}|t)$ is the expected arrival-time and ω is a non-negative threshold. In this structured-input case we end up with three parameters and a non-linear optimization problem over the variables (τ, ϕ, ω). In the worked example in the next section we first optimize over the structured inputs, and use the resulting sequence $u(k+j|t)$ as an initial value for the general case, solved with a genetic algorithm.

IV. WORKED EXAMPLE

Consider the railroad network of Fig. 2. There are 4 stations in this railroad network (A, B, C and D) that are connected by 6 single tracks. There are three trains available. The first train follows the route $D \to A \to B \to D$, the second train follows the route $A \to B \to C \to A$, and the third train follows the route $D \to A \to C \to D$. We assume that there exists a periodic time table that schedules the earliest departure times of the trains. The period of the time table is $T = 60$ minutes. So if a departure of a train from station B is scheduled at 5.30 a.m., then there is also scheduled a departure of a train from station B at 6.30 a.m., 7.30 a.m., and so on.

Table I summarizes the information in connection with the nominal traveling times and the departure times. All the times are measured in minutes. The indicated departure times are the earliest departure times in the initial station of the track expressed in minutes after the hour. The first period starts at time $t = 0$. At the beginning of the first period the first train is in station A and the second train is in station B.

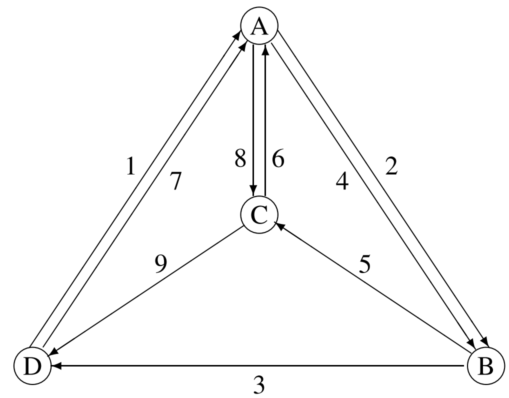

Fig. 2. The railroad network of the example of Section IV.

TABLE I
THE NOMINAL TRAVELING TIMES AND THE DEPARTURE TIMES FOR THE RAILROAD NETWORK OF THE EXAMPLE OF SECTION IV.

train	from-to	travel time	dep-arr	constraints
1	D-A	12	00-12	same train as 3^-, gives connection to 9^- follow 7^-,
2	A-B	12	15-27	same train as 1 , gives connection to 6^- follow 4^-,
3	B-D	20	30-50	same train as 2
4	A-B	12	19-31	same train as 6^-, follow 2, gives connection to 7^-
5	B-C	10	34-44	same train as 4
6	C-A	25	47-12	same train as 5 , wait until 8 has arrived
7	D-A	12	04-16	same train as 9^-, follow 1
8	A-C	25	19-44	same train as 7 , wait until 6^- has arrived
9	C-D	10	47-57	same train as 8 , gives connection to 5

Note: 3^- denotes train 3 in the previous cycle

The continuity constraints are that the trains on tracks 1, 2 and 3 are physically the same train, and the same holds for the trains on tracks 4, 5 and 6 and for the trains on tracks 7, 8 and 9. Connection constraints are introduced to allow the passengers to change trains:

- train 1 has to wait for train 9 with minimum connection time $c^{\min}_{19}(k) = 3$,
- train 2 has to wait for train 6 with minimum connection time $c^{\min}_{26}(k) = 3$,
- train 4 has to wait for train 7 with minimum connection time $c^{\min}_{47}(k) = 3$,
- train 9 has to wait for train 5 with minimum connection time $c^{\min}_{95}(k) = 3$.

Follow constraints are introduced to guarantee sufficient separation time between two trains on the same track (moving in the same direction):

- train 4 is scheduled behind train 2 with a minimum separation time $f^{\min} = 4$,
- train 2 is scheduled behind train 4 in the previous cycle with a minimum separation time $f^{\min} = 4$,
- train 7 is scheduled behind train 1 with a minimum separation time $f^{\min} = 4$,
- train 1 is scheduled behind train 7 in the previous cycle with a minimum separation time $f^{\min} = 4$,

Finally, a wait constraint is introduced to guarantee that two trains (moving in opposite direction) are not on the same track at the same time:

- train 6 is scheduled behind train 8 with a minimum separation time $w^{\min} = 1$.
- train 8 is scheduled behind train 6 in the previous cycle with a minimum separation time $w^{\min} = 1$.

The minimum stopping time of train j at station j to allow passenger to get off or on the train is fixed at $s_{ij}^{\min}(k) = 1$.

Each train departs as soon as all the connections are guaranteed (except for a connection when it is broken), the passengers have gotten the opportunity to change over and the earliest departure time indicated in the time table has passed. We assume that in the first period all the trains depart according to schedule. Recall that $x_j(k)$ is the time instant at which the train on track j departs from the initial station of the track for the kth time.

Now we write down the equations that describe the evolution of the $x_j(k)$'s.

First we consider the train on track 1 and we determine $x_1(k)$, the time instant at which this train departs from station A for the kth time. The train has to wait at least until the train has arrived in station A for the $(k-1)$th time[3] and the passengers have got the time to get out of the train so we have $x_1(k) \geqslant x_3(k-1) + a_3(k-1) + 1$. Furthermore, the train on track 1 has to wait for the passengers of the train on track 9 in the $(k-1)$th cycle, which arrives in station B at time instant $x_9(k-1) + a_9(k-1)$. The passengers have $c_{19}^{\min} = 3$ minutes to change trains. Further the train on track 1 has to follow the train on track 7 in the previous cycle with a minimum separation time $f^{\min} = 4$. According to the time table the train on track 1 can only depart after time instant $00 + k60$. Hence, we have

$$\begin{aligned} x_1(k) &= \max(x_3(k-1) + a_3(k-1) + s^{\min}, \\ &\qquad x_7(k-1) + f^{\min}, \\ &\qquad x_9(k-1) + a_9(k-1) + c_{19}^{\min}, d_1(k)) \\ &= \max(x_3(k-1) + 21, x_7(k-1) + 4, \\ &\qquad x_9(k-1) + 13, k60) \end{aligned} \tag{7}$$

for $k = 1, 2, \ldots$ with $x_3(0) = x_9(0) = -\infty$.

[3]Under nominal operations the kth train on track 1 (e.g., the one that departs from station D at 10.00 a.m.) proceeds to the $(k-1)$th train on track 3 (which has departed from station B at 9.30 a.m.) and *not* to the kth train on track 3 (which will depart from station B at 10.30 a.m.).

Using a similar reasoning, we find that the other departure times are given by

$$\begin{aligned} x_2(k) &= \max(x_1(k) + 13, x_4(k-1) + 4, \\ &\qquad x_6(k-1) + 28, 15 + k60) \\ x_3(k) &= \max(x_2(k) + 13, 30 + k60) \\ x_4(k) &= \max(x_2(k) + 4, x_6(k-1) + 26, \\ &\qquad x_7(k-1) + 14, 19 + k60) \\ x_5(k) &= \max(x_4(k) + 13, 34 + k60) \\ x_6(k) &= \max(x_5(k) + 11, x_8(k) + 26, 47 + k60) \\ x_7(k) &= \max(x_9(k-1) + 11, x_1(k) + 4, 4 + k60) \\ x_8(k) &= \max(x_7(k) + 13, x_6(k-1) + 26, 19 + k60) \\ x_9(k) &= \max(x_8(k) + 26, x_5(k) + 13, 47 + k60) \end{aligned}$$

for $k = 1, 2, \ldots$ with $x_j(0) = -\infty$ for $j = 1, 2, \ldots, 9$.

We solve the optimal control problem (5) for the structured input case and the general case (without structuring). In the last optimization we use the result of the structured input as an initial value to start the optimization. We assume the system is at nominal schedule for $k < 0$ and we introduce a perturbation at time $t = 0$:

$$a_3(-1|0) = 13, \ a_7(-1|0) = 23 \ \text{and} \ a_9(-1|0) = 22.$$

We first optimize the threshold values $\{\tau, \phi, \omega\}$, and compute the corresponding optimal structured input signal $u_{\text{structured}}(k + j|t)$, $j \geqslant 0$. We find $(\tau, \phi, \omega)^* = (8.6, 3.4, 25)$. Note that $\omega^* = 25$ means that switching the departure order on track 5 will not give any improvement of the cost criterion J for the given initial perturbations. With a genetic algorithm we now optimize the (unstructured) input signal, using the sequence $u(k + j|t)$ for the optimal $(\tau, \phi, \omega)^*$ as an initial value. No improvement is found, and so we conclude that the control signal based on $(\tau, \phi, \omega)^*$ is already optimal.

In Fig. 3 the maximum delay $e_{\max}(k) = \max(e(k))$ in each cycle k is given for both the uncontrolled case (so $u(k + j|t) = 0$ for all $j > 0$) and for the optimal controlled case (with $(\tau, \phi, \omega)^*$). We see that the delay in the controlled case decays much faster than the uncontrolled case.

V. DISCUSSION

We have presented a control design method for a railway network. The control action consists in breaking certain connections or changing the order of departure to prevent delays from accumulating. These control moves can only be done at a certain cost. We have also shown that the resulting optimization problem can be solved using integer optimization methods, for example genetic algorithms or tabu search.

Good initial values for the integer optimization are obtained by first solving an easy real-valued optimization problem using a structuring input sequence. This structured input sequence is based on a decision mechanism, where we use thresholds on (expected) delays to decide whether a connection should be broken or train orders should be switched.

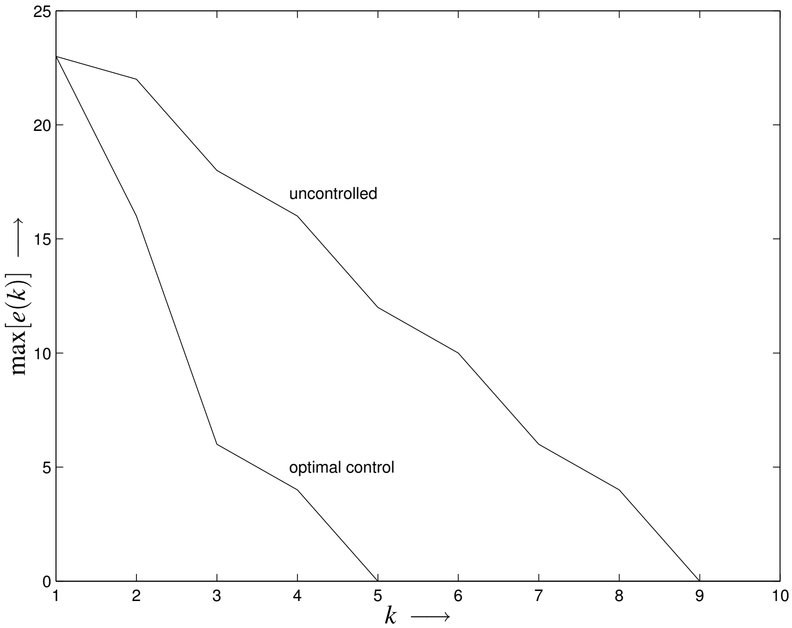

Fig. 3. Maximum delay for uncontrolled and optimal controlled railway system

Due to the use of a future horizon this method can be used in on-line applications and it can deal with (predicted) changes in the system parameters. So if we can predict the delays that will occur due to an incident or to works, then we can include this information when determining the optimal control input for the next cycles of the operation of the network.

Acknowledgments

Research partially funded by the Dutch Technology Foundation STW projects "Model predictive control for hybrid systems" (DMR.5675), and "Multi-agent control of large-scale hybrid systems" (DWV.6188), by the European IST project "Modelling, Simulation and Control of Nonsmooth Dynamical Systems (SICONOS)" (IST-2001-37172), and by the TU Delft spearhead program "Mobility of People and Transportation of Goods (TRM)". The authors like to thank Zoltan Petres for providing the simulation plots.

References

[1] F. Baccelli, G. Cohen, G. Olsder, and J. Quadrat, *Synchronization and Linearity*. New York: John Wiley & Sons, 1992.

[2] R. Cuninghame-Green, *Minimax Algebra*, ser. Lecture Notes in Economics and Mathematical Systems. Berlin, Germany: Springer-Verlag, 1979, vol. 166.

[3] B. De Schutter and T. van den Boom, "Model predictive control for railway networks," in *Proceedings of the 2001 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM'01)*, Como, Italy, July 2001, pp. 105–110.

[4] B. De Schutter, T. van den Boom, and A. Hegyi, "A model predictive control approach for recovery from delays in railway systems," *Transportation Research Record*, no. 1793, pp. 15–20, 2002.

[5] J. Braker, "Max-algebra modelling and analysis of time-table dependent transportation networks," in *Proceedings of the 1st European Control Conference*, Grenoble, France, July 1991, pp. 1831–1836.

[6] ——, "Algorithms and applications in timed discrete event systems," Ph.D. dissertation, Department of Technical Mathematics and Informatics, Delft University of Technology, Delft, The Netherlands, Dec. 1993.

[7] ——, "An extended algorithm for performance evaluation of timed event graphs," in *Proceedings of the 2nd European Control Conference*, Groningen, The Netherlands, June 1993, pp. 524–529.

[8] R. de Vries, B. De Schutter, and B. De Moor, "On max-algebraic models for transportation networks," in *Proceedings of the International Workshop on Discrete Event Systems (WODES'98)*, Cagliari, Italy, Aug. 1998, pp. 457–462.

[9] P. de Waal, A. Overkamp, and J. van Schuppen, "Control of railway traffic on a single line," in *Proceedings of the European Control Conference (ECC'97)*, Brussels, Belgium, paper 230, July 1997.

[10] R. Minciardi, M. Paolucci, and R. Pesenti, "Generating optimal schedules for an underground railway line," in *Proceedings of the 34th IEEE Conference on Decision and Control*, New Orleans, Louisiana, Dec. 1995, pp. 4082–4085.

[11] L. Davis, Ed., *Handbook of Genetic Algorithms*. New York: Van Nostrand Reinhold, 1991.

[12] F. Glover, "Tabu search: A tutorial," *Interfaces*, vol. 20, no. 4, pp. 74–94, 1990.

[13] F. Glover, E. Taillard, and D. de Werra, "A user's guide to tabu search," *Annals of Operations Research*, vol. 41, pp. 3–28, 1993.

[14] F. Glover and M. Laguna, *Tabu Search*. Boston: Kluwer Academic Publishers, 1997.

[15] B. De Schutter and T. van den Boom, "MPC for discrete-event systems with soft and hard synchronisation constraints," *International Journal of Control*, vol. 76, no. 1, pp. 82–94, 2003.

[16] ——, "Connection and speed control in railway systems — A model predictive control approach," in *Proceedings of the 6th International Workshop on Discrete Event Systems (WODES'02)*, M. Silva, A. Giua, and J. Colom, Eds., Zaragoza, Spain, Oct. 2002, pp. 49–54.