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A Dissipative Approach to Control of Biological Wastewater Treatment Plants Based on Entire Nonlinear Process Models

Hiroshi Ito

***Abstract*— This paper proposes an approach to control design of biological wastewater treatment(WWT) plants based on rigorous treatment of the complex mathematical models from a nonlinear control theoretical viewpoint. Without resorting to order reduction, localization and linearization of process models, this paper provides a promising avenue to model-based control design for necessary innovations of modern WWT. Fundamental properties of the activated sludge WWT plant are investigated, and a dissipation property of the entire plant is derived from precise integration of all components of the plant. For the purpose of efficient removal of carbon and nitrogen, control laws are proposed so that the dissipation of the entire plant is preserved in the presence of control inputs. This paper demonstrates that utilization of the natural principle, the dissipation, is very effective in extracting compact global information of the large-scale plant, and it enables us to accomplish a model-based control design taking into account the whole behavior of the WWT plant.**

I. INTRODUCTION

Due to stricter effluent legislation set by various municipalities and nations, essential innovations are becoming necessary for control of wastewater treatment(WWT) plants. Activated sludge WWT plants are the most common type of modern WWT. The activated sludge process consist of numerous biochemical reactions behaving nonlinearly. The reactions include mechanisms which are useful for reduction of carbonaceous materials and other undesirable compounds in wastewater[1]. In order to develop reliable mathematical models having the capability of simulating activated sludge WWT plants, a task group was formed in the International Association on Water Quality(IAWQ) and a simulation benchmark framework has been presented[9], [11]. Naturally, models of WWT plants become very complex and involve huge numbers of variables, parameters, equations and nonlinearities. The development of automatic control based on the models have been very difficult although there are researchers who are aware of the necessity of model-based control design for essential improvement of WWT[10]. Automatic control has never been installed in a satisfactory way that the full capacity of WWT plants is utilized efficiently.

It has been widely believed that the complex models describing biochemical reactions and sedimentation processes are very difficult to handle exactly for control purposes. Researchers have been seeking reduced complexity models, and there are a number of linear and nonlinear approximate models[5], [7], [2], [12]. Usually, models are eventually linearized when we compute control laws, otherwise control laws are constructed by focusing only on individual local processes without paying attention to the behavior of other materials in other parts which may be affected by the local controls. Recently, many control strategies have been reported in these directions. There are few studies which propose

H. Ito is with Department of Control Engineering and Science, Kyushu Institute of Technology, 680-4 Kawazu, Iizuka, Fukuoka 820-8502, Japan. hiroshi@ces.kyutech.ac.jp

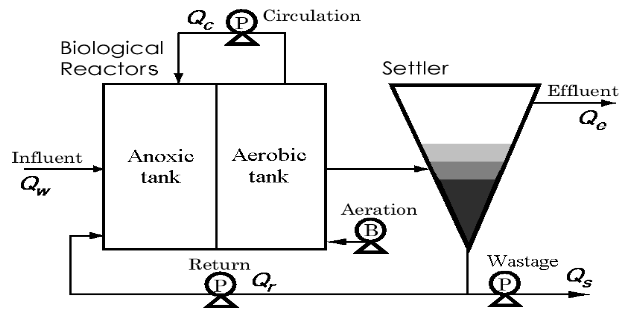

Fig. 1. Wastewater treatment plant in pre-denitrification layout.

and justify control strategies directly using the complex nonlinear model depicting the whole behavior of the system.

The primary purposes of this paper are twofold. One is to demonstrate feasibility of rigorous treatment of the entire biological WWT plant from a control theoretical viewpoint. The other is to demonstrate how we can design controllers directly using total integration of detailed models. These novel standpoints enable us

- to avoid delicate issues of order reduction of complex models
- to avoid linear approximation for unnecessary technical simplification
- to avoid the use of linearizing control mechanisms which are often wasteful and sensitive to parameter uncertainty and perturbation.
- to avoid unnecessary hierarchical structure of control consisting of set point supervision and local controls.
- to design individual control laws taking into account the behavior of the entire plant.

We focuses on a property called dissipation, and a dissipation equation is calculated rigorously from the large complex model describing the entire plant. The dissipation equation is compact information containing fundamental properties of the behavior of the *entire* system. The dissipativity of the *entire* plant is a natural consequence of combination of natural principles that govern all *individual* components of the plant. As one would expect, this paper obtains the dissipation in the form of mass balance held for the *entire* plant. This paper rediscovers the usefulness of the mass balance for control design taking the plant globally into account. In the pre-denitrification layout, control laws for carbon and nitrogen removal are selected so that the dissipation of the entire plant is preserved even in the presence of all control inputs. Simulation results are presented to illustrate the effectiveness of the control laws designed via the dissipative control strategy.

II. MODEL OF BIOLOGICAL WASTEWATER TREATMENT PLANT

A task group of IAWQ and the European COST actions 682 and 624 conducted simulation studies of Biological WWT plants consisting of bioreactors and settlers[9], [11]. In their benchmark, Activated Sludge Model No.1 (ASM1)[3] describes each bioreactor, and Takács model[4] simulates sedimentation of each settler.

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TABLE I

STATE VARIABLES OF ASM1

Description	State variable	Symbol	Unit
Soluble inert organic matter	z_1	S_I	gCOD/m^3
Readily biodegradable substrate	z_2	S_s	gCOD/m^3
Particulate inert organic matter	z_3	X_I	gCOD/m^3
Slowly biodegradable substrate	z_4	X_s	gCOD/m^3
Active heterotrophic biomass	z_5	X_{BH}	gCOD/m^3
Active autotrophic biomass	z_6	X_{BA}	gCOD/m^3
Particulate products arising from biomass decay	z_7	X_P	gCOD/m^3
Oxygen	z_8	S_O	gCOD/m^3
Nitrate and nitrite nitrogen	z_9	S_{NO}	gN/m^3
Ammonia	z_{10}	S_{NH}	gN/m^3
Soluble biodegradable organic nitrogen	z_{11}	S_{ND}	gN/m^3
Particulate biodegradable organic nitrogen	z_{12}	X_{ND}	gN/m^3
Alkalinity	z_{13}	S_{ALK}	mol/m^3

The dynamics of a biological reactor is modeled as

$$\frac{dz}{dt} = M^T r(z) + \frac{Q}{V}(w - z), \quad \forall t \in [0, \infty) \tag{1}$$

$$w(t) \geq 0, \quad \forall t \in [0, \infty) \tag{2}$$

$$z \in \mathbb{R}^{13}, \quad M = \left[\begin{array}{ccccc} m_1 & m_2 & \cdots & m_{12} & m_{13} \end{array} \right] \in \mathbb{R}^{8\times 13} \tag{3}$$

$$r(z) = \left[\begin{array}{c} \rho_1(z) \\ \rho_2(z) \\ \vdots \\ \rho_8(z) \end{array} \right] \in \mathbb{R}^8, \quad \begin{array}{c} z_i(0) \geq 0, \quad i = 1,2,...,13 \\ r(0) = 0 \\ \rho_1(z), \rho_2(z), ..., \rho_8(z) \geq 0, \ \forall z \in \mathbb{R}^{13}_+ \end{array} \tag{4}$$

Each element of the state vector z represents concentration of a component in wastewater contained in the reactor. The thirteen components considered in ASM1 is shown in Table I. The time is denoted by $t \in [0, \infty) = \mathbb{R}_+$. The vector $w(t) \in \mathbb{R}^{13}_+$ denotes the concentration of components contained in the inflow entering the reactor. Positive scalars Q and V denote the flow rate and the volume of the bioreactor, respectively. The constant matrix M is a matrix representation of stoichiometric coefficients, and the function $r(z)$ is a column vector representation of nonlinear process rate[3]. ASM1 contains eight very complex processes which are elements of $r(z)$. The matrix M has full row rank since each process is defined and distinguished in such a way.

Remark 1: The model of the bioreactor borrowed from [3], [9] does not exactly posses non-negativeness of state variables although the real reactor certainly have the non-negative property. The variable z_{10} (i.e.,S_{NH}) does not remain non-negative because of incompleteness of ρ_1 and ρ_2 which appear in dz_{10}/dt. This defect can be removed by simply introducing a switch mechanism which turns off when z_{10} reaches zero. It is reasonable naturally since it does not change dynamics in the positive domain. This paper employs $\rho_1(z)$ and $\rho_2(z)$ defined as

$$\rho_j(z) = \rho_{j,org}(z) \frac{S_{NH}}{K_{new} + S_{NH}}, \quad 0 < K_{new} \ll 1, \quad j = 1,2 \tag{5}$$

instead of $\rho_{1,org}(z)$ and $\rho_{2,org}(z)$ defined by ASM1. Due to this modification, all state variables are guaranteed to be non-negative for all $t \in \mathbb{R}_+$ mathematically. Negative concentration ASM1 produces has been also pointed out by SIMBA[9].

In the recent studies, high-order integration of one-dimensional models called layer models is accepted internationally as a tool for simulating settlers. Consider a settler whose volume is V_s. Let the settler divided into n layers fictitiously, and all layers are supposed to have the same volume v_s. We number layers from the top to the bottom. Let Q_e denote the rate of flow which spills out of the settler, and let Q_u denote the rate of flow pumped at the bottom. The model of the k-th layer describing the concentration of the i-th soluble component $z_{i,k}$ is written in the form of

$$\frac{dz_{i,k}}{dt} = \frac{Q_{k,in}}{v_s} z_{i,k,in} - \frac{Q_{k,out}}{v_s} z_{i,k}, \quad k = 1,2,\ldots,n \tag{6}$$

We use (6) for both upward flow and downward flow from the point the inflow is coming into. The pair $\{z_{i,k,in},\ Q_{k,in}\}$ is for the flow coming into the k-th layer. For layers below(above) the inflow point, the flow $\{z_{i,k,in},\ Q_{k,in}\}$ comes from a layer right above(below, respectively). The pair $\{z_{i,k},\ Q_{k,out}\}$ is for the flow going out of the k-th layer. For layers below(above) the inflow point, the flow $\{z_{i,k},\ Q_{k,out}\}$ goes into layer right below(above, respectively). The real flows Q_u and Q_e appear in (6) as follows:

$$Q_{m,in} = Q_e + Q_u, \quad Q_{1,out} = Q_e, \quad Q_{n,out} = Q_u \tag{7}$$

The inflow from outside is supposed to be located at the m-th layer, and $z_{i,m,in}$ represents the concentration in the inflow from outside. In the case of insoluble/particulate components, the k-th layer model is written in the form of

$$\frac{dz_{i,k}}{dt} = \frac{Q_{k,in}}{v_s} z_{i,k,in} - \frac{Q_{k,out}}{v_s} z_{i,k} + g_{s,i,k}(z_{k-1}, z_k, z_{k+1}) \tag{8}$$

$$g_{s,i,k}(z_{k-1}, z_k, z_{k+1}) \leq 0 \quad \text{if } z_{k-1} = 0 \tag{9}$$

$$g_{s,i,k}(0,0,0) = 0, \qquad q_{i,0} = 0, \quad q_{i,n} = 0 \tag{10}$$

$$g_{s,i,k}(z_{k-1}, z_k, z_{k+1}) = q_{i,k-1}(z_{k-1}, z_k) - q_{i,k}(z_k, z_{k+1}) \tag{11}$$

$$q_{i,k}(z_k, z_{k+1}) \geq 0, \quad \forall z_k, z_{k+1} \in \mathbb{R}_+ \tag{12}$$

for $k = 1,2,\ldots,n$. The Takács model employs $q_{i,k}(z_k, z_{k+1})$ determined by the double-exponential settling velocity function[4].

Remark 2: Jeppsson[6] and the COST benchmark definition [9] employ (8) for a single scalar variable representing the sum of all insoluble/particulate components in the k-th layer instead of each individual component $z_{i,k}$. They assume that the ratio of components in the inflow is instantaneously reflected in the spilling and pumped flows. The hypothesis is not rationalized since it neglects dynamics depending on the past proportion of components. To avoid this inadequacy, this paper employs (8) separately for each individual insoluble/particulate component $z_{i,k}$ instead of lumping all $z_{i,k}$ together into a single variable. The model is adopted by some simulation environments in [9] such as SIMBA.

Remark 3: The settler model borrowed from [4], [9] lacks the non-negative property which real settlers certainly posses. It is due to the gravity settling $g_{s,i,k}$ of insoluble/particulate components. For $k = 1,2,\ldots,n$, this paper replaces $q_{i,k}$ by

$$\bar{q}_{i,k}(z_k, z_{k+1}) = \begin{cases} 0 & \text{if } z_{i,k} = 0 \\ q_{i,k}(z_k, z_{k+1}) & \text{otherwise} \end{cases} \tag{13}$$

This modification is physically natural, and it does not change dynamics in the positive domain. Thanks to this modification, all variables $z_{i,k}(t)$, $i = 1,2,...,13$ are mathematically guaranteed to be non-negative for all t under the initial conditions $z_{i,k}(0) \geq 0$.

The WWT plant considered in this paper is comprised of biological reactors and a settling tank, which is illustrated by Fig.1. The number of bioreactors are two. It is, however, purely for concise presentation, and results in this paper are applicable to plant models consisting of more than two bioreactors. All variables Q_w, Q_e, Q_r and Q_c of flow rate are non-negative. The effluent Q_e satisfies $Q_e = Q_w - Q_s$, so that we have a physical constraint $0 \leq Q_s \leq Q_w$. Let $z_{a,i}$, $z_{b,i}$ and w_i denote the concentration of the i-th material component of water contained in the anoxic tank, the aerobic tank and the influent, respectively. Let V_a, V_b and V_s denote the volume of the anoxic tank, the aerobic tank and the settler, respectively. The settler is divided into n layers. Let $z_{s,i,k}$ denote the concentration of the i-th component in the k-th layer. For example, $z_{s,i,1}$ and $z_{s,i,n}$ denote the concentration in the effluent and the wastage, respectively. The dynamics of the i-th components in wastewater over the plant is governed by

$$\frac{dz_{a,i}}{dt} = m_i^T r(z_a) + \frac{1}{V_a}\{Q_w w_i + Q_c z_{b,i} + Q_r z_{s,i,n} - (Q_w + Q_c + Q_r) z_{a,i}\} \quad (14)$$

$$\frac{dz_{b,i}}{dt} = m_i^T r(z_b) + \frac{Q_w + Q_c + Q_r}{V_b}(z_{a,i} - z_{b,i}) \quad (15)$$

$$\frac{dz_{s,i,k}}{dt} = \frac{Q_{k,in}}{v_s} z_{s,i,k,in} - \frac{Q_{k,out}}{v_s} z_{s,i,k} + \psi_{s,i,k}(z_{s,k-1}, z_{s,k}, z_{s,k+1}) \quad k = 1,2,\dots,n \quad (16)$$

$$w_i(t) \geq 0, \quad \forall t \in \mathbb{R}_+ \quad (17)$$

$$z_{a,i}(0) \in \mathbb{R}_+,\ z_{b,i}(0) \in \mathbb{R}_+,\ z_{s,i,k}(0) \in \mathbb{R}_+, k = 1,2,...,n \quad (18)$$

$$Q_{m,in} = Q_w + Q_r,\ Q_{1,out} = Q_w - Q_s,\ Q_{n,out} = Q_r + Q_s \quad (19)$$

$$z_{s,i,k,in} = \begin{cases} z_{s,i,k+1} & \text{if } 1 \leq k \leq m \\ z_{b,i} & \text{if } k = m \\ z_{s,i,k-1} & \text{if } m+1 \leq k \leq n \end{cases} \quad (20)$$

$$\psi_{s,i,k}(\dots) = \begin{cases} g_{s,i,k}(z_{s,k-1}, z_{s,k}, z_{s,k+1}) & \text{if } i \in \{3,4,5,6,7,12\} \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

Thirteen sets of the above equations $i = 1,2,...,13$ form the model of the entire plant. The state vector of the entire WWT plant is

$$x = \begin{bmatrix} z_a \\ z_b \\ z_s \end{bmatrix} \in \mathbb{R}^{26+13n},\ z_a \in \mathbb{R}^{13},\ z_b \in \mathbb{R}^{13},\ z_s \in \mathbb{R}^{13n} \quad (22)$$

Following the previous argument, we can verify that all components of $x \in \mathbb{R}^{26+13n}$ take non-negative values for all t.

III. Control design strategy

This section presents the main theoretical idea of control design proposed for the WWT plant in this paper.

A. Dissipativity

Consider the following general nonlinear system.

$$\frac{dx}{dt} = f(x,w), \quad e = h(x,w) \quad (23)$$

This system is said to be dissipative with respect to a supply rate $q(w,e)$ which is a continuous function with $q(0,0) = 0$ if there exists a continuously differentiable function $S(x)$ such that

$$0 \leq S(x) \quad (24)$$

$$\frac{\partial S}{\partial x} f(x,w) \leq q(w,e) \quad (25)$$

are satisfied for all x and w. Without loss of generality, $f(0,0) = 0$ and $h(0,0) = 0$ are assumed. The inequality (25) is called the dissipation inequality. The function $S(x)$ has the abstracted interpretation of stored energy in the system, and it is called the storage function. The vectors w and e represent the input and the output, respectively. The function $q(w,e)$ has the abstracted interpretation of net energy supply from outside. The dissipative system dissipates the energy since the stored energy is not larger than the amount of net energy supplied, which implies that the system is not destructive internally. The energy is supplied through w. The output e is usually defined as a vector through which the energy is extracted from the system, so that

$$q(0,e) \leq 0, \quad \forall e \in \mathbb{R}^{n_e} \quad (26)$$

holds. Natural choices of the storage function satisfy

$$S(x) \to +\infty \text{ as } \|x\| \to +\infty \quad (27)$$

since the stored energy should be unbounded for infinitely large magnitude of the system state. The dissipativity with (26) and (27) guarantees global boundedness of x for nil input $w = 0$. The dissipativity provides a more useful property that the bounded set

$$\mathscr{L}(c) = \left\{x \in \mathbb{R}^l : S(x) \leq c\right\} \quad (28)$$

is positively invariant for any $c > 0$. In other words, all trajectories starting from $\mathscr{L}(c)$ remain in the same set forever.

$$x(t_0) \in \mathscr{L}(c) \ \Rightarrow\ x(t) \in \mathscr{L}(c),\ \forall t \in [t_0, \infty) \quad (29)$$

A dissipative system does not generate energy internally. The dissipativity also provides invariance sets for nonzero w. Indeed,

$$\mathscr{L}(c,w) = \{x \in \mathscr{L}(c) : q(w,e) \leq 0\} \quad (30)$$

is positively invariant. If $\mathscr{L}(c,w)$ is empty, we can often modify $q(w,e) \leq 0$ in (30) appropriately. Although dissipation by itself may not guarantee the existence of invariant sets for violently large w, dissipative systems do not amplify disturbance w internally at least.

Control strategy utilizing this favorable property is dissipative control. Suppose that the plant (23) is dissipative in the sense of (24) and (25). The idea of dissipative control is to put the control input u in the original system (23) without changing the dissipation inequality (25). This strategy can be illustrated by

$$\frac{dx}{dt} = f(x,w) + \sum_{k \in \Psi} \psi_k u_k \quad (31)$$

$$\frac{\partial S}{\partial x}\{f(x,w) + \sum_{k \in \Psi} \psi_k u_k\} \leq q(w,e) \quad (32)$$

Here, ψ_k's are column vectors, and u_k's are scalars representing control inputs. The formula (32) does not provide specific answers of u_k in terms of improvement of water quality. The inequality (32) is a guideline for coordinating all inputs u_k each other so that the overall behavior of the system is not internally destructive.

B. Attenuating targets via positive control

It is impossible to take components away selectively from a bioreactor directly. We are only allowed to add substances or manipulate flow rates. There is no way to apply negative control input to the reactor. This subsection explains that we are still able to modify dynamics of target components in wastewater favorably by adding positive control inputs.

The equation for a component $z_{*,i}$ in a bioreactor with control action u_i at an integer $i \in [1,13]$ is in the form of

$$\frac{dz_{*,i}}{dt} = m_i^T r(z_*) + \{\text{in and out-flows}\} + u_i, \qquad \begin{matrix} u_i(t) \geq 0 \\ \forall t \in [0,\infty) \end{matrix} \quad (33)$$

This type of supplementary component input is represented by (31) with ψ_k whose elements are 0 except for an element which is 1. Putting a control in the dynamics of $z_{*,i}$ increases $z_{*,i}$. However, the contracting behavior of $z_{*,j}, j \neq i$ can be enhanced by forcing $z_{*,i}$ to take a value yielding more negative $m_j^T r(z_*)$ in

$$\frac{dz_{*,j}}{dt} = m_j^T r(z_*) + \{\text{in and out-flows}\} \quad (34)$$

at the expense of changing behavior of $z_{*,i}, i \neq j$. In fact, it is achievable if $m_j^T r(z_*)$ has the following property.

$$m_j^T r(z_*)\Big|_{z_{*,i}=\bar{c}} < m_j^T r(z_*)\Big|_{z_{*,i}=\underline{c}} \quad \forall z_* \in \mathbb{R}_+^{13} \setminus \{z_* : m_j^T r(z_*) = 0\} \quad \text{if } \bar{c} > \underline{c} \tag{35}$$

This inequality implies that larger $u_{*,i} > 0$ makes $z_{*,j}$ decrease faster or makes the trajectory $z_{*,j}$ bent toward the origin. If

$$u_i(z_*)\big|_{z_{*,j}=\bar{c}} > u_i(z_*)\big|_{z_{*,j}=\underline{c}} \quad \forall z* \in \mathbb{R}_+^{13} \setminus \{z_* : u_i(z_*) = 0\} \quad \text{if } \bar{c} > \underline{c} \tag{36}$$

holds additionally, the attenuation of $z_{*,j}$ is more effective.

Next, suppose that a control input u_k in (31) is flow-rate of return or circulation. Flow-rate inputs are again non-negative since pumps are unilateral. Increase of an outflow which removes substances implies increase of an inflow which supplies substances. Due to this flow balance, the vector ψ_k corresponding to the flow-rate input u_k consists of negative elements and positive elements, and the amount of them are balanced. In other words, we have $[1\ 1 \cdots 1]\psi_k = 0$ for ψ_k of each flow-rate u_k. Pumping out components at a place brings an equal amount of increase of the components at another place. The existence of reactions, however, provides us with ways to decrease the total amount of some components. The increase of components $z_{*,i}$, $\forall i \in I$ for some I caused by introduction of a flow is useful for reducing a component $z_{*,j}$ if it enhances $m_j^T r(z_*)$ in the sense of (35).

The argument in this subsection is justified only locally in the sense that we only look at $z_{*,i}$ and $z_{*,j}$ in a tank, and we forget other variables in the same tank and other parts of the plant. We do not take into account the effect of other reactors, settlers connected in a feedback way. Since local control has been common in control design of WWT plants. the idea described in this subsection is not completely unique. The uniqueness of this paper is to employ dissipative strategy which renders the local control inputs effective.

C. Dissipative control design

This paper propose a control design combining the ideas described in Subsection III-A and Subsection III-B. The objective of WWT is not reduction of all components in wastewater since we need to keep the bioreactors functioning. It is not asymptotic stabilization toward zero either. The objective is to reduce undesirable components, which we call the targets. The effort for the reduction brings increase of other components as described in Subsection III-B. Although a larger control effort aiming at reduction of a particular component seems to render that component smaller seemingly, the complex interaction between processes and reactors may result in the increase of the targeted particular component that we try to decrease. Therefore, controller design only based on local dynamics is dangerous, and controller design should take the whole plant into account. From an economical point of view, excess use of control efforts should be avoided. Excessive inputs only raise the energy level of the system which is never used. The energy should be supplied as much as it is consumed. To take all problems into account, this paper propose a dissipative control design for the WWT control which is summarized as follows.

- For each target component $z_{*,j}$ which is required to be made small, select i so that (35) holds.
- design non-negative control inputs $u_k(x)$ so that (36) holds individually, and that (32) holds together.

The subsequent sections show an appropriate choice of the storage function $S(x)$ and design individual control laws.

IV. Dissipativity of entire plant without control

This section investigates a dissipative property of the entire WWT plant. The dissipation is inherited naturally by many inartificial systems since it is a consequence of combination of natural principles. Mass balance which plays an important role in modeling individual bioreactors and settlers[3], [13] is one of such principles. Thus, sections of the WWT plant individually exhibit the dissipation in the sense of mass balance[13]. This section clarifies that the dissipative property does hold for the entire plant as one would expect. The rigorous derivation enables us to make the most of the dissipative property in control design.

A. Dissipation equation

This paper proposes the total mass of all components in the entire system in terms of of COD, nitrogen and electrical charges as the storage function of the entire plant.

$$S(x) = V_a \Phi z_a + V_b \Phi z_b + \frac{V_s}{n} \sum_{k=1}^{n} \Phi z_{s,-,k} \tag{37}$$

$$z_{s,-,k} = \left[\begin{array}{cccc} z_{s,1,k} & z_{s,2,k} & \cdots & z_{s,13,k} \end{array}\right]^T \tag{38}$$

$$\Phi = \left[\ 1\ 1\ 1\ 1\ 1{+}i_{XB}\ \ 1{+}i_{XB}\ \ 1{+}i_{XP}\ \ 1\ 1{+}\tfrac{1}{14}\ \ 1{-}\tfrac{1}{14}\ \ 1\ 1\ 1 \right] \tag{39}$$

The row vector Φ sums up all components in terms of COD, nitrogen and electrical charges. The elements of Φ are considered as conversion coefficients between different units. Supply of the i-th component in wastewater from outside is $Q_w w_i$, and the mass is extracted by the effluent and the wastage flow as $(Q_w - Q_s) z_{s,i,1} + Q_s z_{s,i,n}$. Thus, we define the supply rate of the entire plant as

$$q(w,e) = \Phi\left(Q_w w - (Q_w - Q_s) z_{s,-,1} - Q_s z_{s,-,n}\right) \tag{40}$$

$$e = [z_{s,-,1}^T, z_{s,-,n}^T]^T \tag{41}$$

respectively. The function $q(w,e)$ defined by (40) fulfills (26) for all $e \in \mathbb{R}_+^{26}$. The function $S(x)$ satisfies

$$S(0) = 0, \qquad S(x) > 0,\ \forall x \in \mathbb{R}_+^{26+13n} \setminus \{0\} \tag{42}$$

and (27). It can be verified through calculation that our WWT plant is dissipative, and the dissipative inequality

$$\frac{d}{dt} S(x) \le q(w,e) \tag{43}$$

holds along the solutions of the entire system. More precisely, we obtain the following dissipative equation.

$$\frac{d}{dt} S(x) = \Phi\left(V_a M^T r(z_a) + V_b M^T r(z_b)\right) + q(w,e) \tag{44}$$

The function $M^T r(z_*)$ is given by

$$\Phi M^T r(z_*) = -2\frac{1-Y_H}{Y_H}\rho_1(z_*) - \left(1+\frac{1}{2.86}\right)\frac{1-Y_H}{Y_H}\rho_2(z_*) - \left(\frac{4.57}{Y_A}-2\right)\rho_3(z_*) \tag{45}$$

$$\rho_1(z_*) = \hat{\mu}_H \left(\frac{S_s}{K_s+S_s}\right)\left(\frac{S_O}{K_{OH}+S_O}\right)\left(\frac{S_{NH}}{K_{new}+S_{NH}}\right) X_{BH} \tag{46}$$

$$\rho_2(z_*) = \hat{\mu}_H \left(\frac{S_s}{K_s+S_s}\right)\left(\frac{K_{OH}}{K_{OH}+S_O}\right)\left(\frac{S_{NO}}{K_{NO}+S_{NO}}\right) \times \left(\frac{S_{NH}}{K_{new}+S_{NH}}\right)\eta_g X_{BH} \tag{47}$$

$$\rho_3(z_*) = \hat{\mu}_A \left(\frac{S_{NH}}{K_{NH}+S_{NH}}\right)\left(\frac{S_O}{K_{OA}+S_O}\right) X_{BA} \tag{48}$$

where $Y_H = 0.67$ and $Y_A = 0.24$ are stoichiometric parameters, $\hat{\mu}_*$, $\eta_g > 0$ and $K_* > 0$ are kinetic parameters. Note that we have

$$\Phi M^T r(z_*) \le 0, \quad \forall z_* \in \mathbb{R}_+^{13} \tag{49}$$

Processes contributing to the negativity are ρ_1, ρ_2 and ρ_3 representing aerobic growth of heterotrophic bacteria, anoxic growth of heterotrophic bacteria and aerobic growth of autotrophic bacteria, respectively.

B. Behavior viewed from dissipativity

Due to $S(x)$ satisfying (27) and (42), the situation where water becomes completely clean, and biomass and oxygen become absent is represented by $S(x) \to 0$. The other extreme $S(x) \to \infty$ describes the situation where water becomes foul helplessly, and biomass and oxygen become excessive. The inequality $S(\underline{z}) < S(\overline{z})$ means that the water $\underline{z}$ is cleaner than the water $\overline{z}$.

From (44) and (49) it follows that the differential equation of the state vector $x \in \mathbb{R}_+^{26+13n}$ has a *unique* equilibrium at $x = 0$ for pure water influent, i.e., when $w(t) \equiv 0$ and $Q_w > Q_s > 0$ hold. The origin $x = 0$ clearly satisfies $dx/dt = 0$. The converse is verified by noting that $dx/dt = 0$ implies $dS(x)/dt = 0$, $dS_{ab}(z_a, z_b)/dt = d(V_a\Phi z_a + V_a\Phi z_b)/dt = 0$ and $dS_a(z_a)/dt = d(V_a\Phi z_a)/dt = 0$. An equilibrium at the origin is natural since all materials are washed away gradually. The uniqueness is favorable since it excludes the existence of traps at undesirably large x. For non-zero w, the deviation of the equilibrium from the origin is guaranteed to be continuous with respect to w. For small w, the equilibria where x is trapped are not very far from the origin in the continuous sense.

According to the dissipation equation (44), evolution of the total mass depends on neither the return flow Q_r nor the circulation flow Q_c. The flows Q_r and Q_c are recycle inside the plant. Every particulate/insoluble component usually satisfy $z_{s,i,1} \ll z_{s,i,n}$ since the gravity makes the component descend to the bottom of the settler. The magnitude of the negativity (49) consisting of Monod functions does not always surpass the inflow $Q_w w$. According to (44), particulate/insoluble components accumulate in the settler if Q_s is zero. Disposal of wastage in the settler, i,e, $Q_s > 0$, is necessary for preventing excess accumulation.

Under the condition of $w(t) \equiv 0$ and $Q_w > Q_s > 0$, $dS/dt = 0$ holds if and only if x belongs to the following set.

$$\mathscr{Z} = \left\{ x \in \mathbb{R}_+^{26+13n} : \begin{array}{l} z_{s,-,1} = z_{s,-,2} \cdots = z_{s,-,n} = 0. \\ z_a \text{ and } z_b \text{ satisfies} \\ \quad \{(S_O + S_{NO})S_s X_{BH} + S_O X_{BA}\} S_{NH} = 0. \end{array} \right\} \quad (50)$$

Although $x = 0$ is not asymptotic stable, it is globally stable. In the case of $w(t) \equiv 0$ and $Q_w > Q_s > 0$, the set $\mathscr{L}(c)$ is positively invariant for any $c \geq 0$. If $x(t_0) \in \mathscr{L}(c) \setminus \mathscr{Z}$ holds at some $t_0 \in \mathbb{R}_+$, there exist $d < c$ and $T > t_0$ such that $x(t) \in \mathscr{L}(d)$ holds for all $t \in [T, \infty)$. We can use $\mathscr{L}(c)$ to characterize invariant sets even for non-zero w by defining l as the infimum of α such that

$$0 < \Phi\left(V_a M^T r + V_b M^T r\right) + q(w,e), \ \forall x \in \left\{ x \in \mathbb{R}_+^{26+13n}, \ S(x) < \alpha \right\}$$

holds. It is verified that such $l \geq 0$ exists if there exist constants $e > 0$ and $0 \leq g < l$ such that

$$\min_{i \in \{2,5,6,8,10\}} \{z_{a,i}(t), z_{b,i}(t)\} \geq e, \ S_{part} = \underline{\Phi}(V_a z_a + V_b z_b) \leq g \quad (51)$$

are satisfied for all $t \in \mathbb{R}_+$ and if w is sufficiently small. The vector $\underline{\Phi} = [1\ 0\ 1\ 1\ 0\ 0\ 1{+}i_{XP}\ 0\ 0\ 0\ 1\ 1\ 1]$ is selection of S_I, X_I, X_s, X_P, S_{ND}, X_{ND} and S_{ALK} which do not contribute to the dissipativity. The set $\mathscr{L}(c)$ is positively invariant for any c satisfying $c \geq l(w,e,g)$. The thicker the influent w is, the larger l is. The requirement $e > 0$ implies the necessity of minimum amounts of X_{BA}, X_{BH}, S_O, S_s and S_{NH} to keep bacteria acting. The requirement $g \geq 0$ implies the necessity of wastage disposal for preventing accumulation. For the existence of positively invariant sets, the limit of w is inevitable since there is constraint on processing speed of bioreactors which has saturating characteristics. For any w, there always exists a constant $d(w)$ such that $z_{z,-,1}(t) + z_{z,-,n}(t) \leq d$ holds for all $t \in \mathbb{R}_+$.

The dissipation is a natural consequence of conservation of mass and balance of flows. This section has reconfirmed its usefulness for obtaining compact information of a very complicated system.

V. Design of dissipative control laws

Following the strategy proposed in Section III, this section derives particular solutions of individual control laws. The control objective is carbon and nitrogen removal. We want to reduce the concentration of S_s, S_{NH} and S_{NO}. Note that (32) is identical to

$$\begin{bmatrix} V_a\Phi & V_b\Phi & v_s\Phi & \ldots & v_s\Phi \end{bmatrix} \sum_{k\in\Psi} \psi_k u_k \\ \leq -\Phi\left(V_a M^T r(z_a) + V_b M^T r(z_b)\right) \quad (52)$$

The consumption of S_{NH} and S_s by biomass is described as

$$m_{10}^T r = -i_{XB}\rho_1 - i_{XB}\rho_2 - (i_{XB} + \frac{1}{Y_A})\rho_3 + k_a S_{ND} X_{BH} \quad (53)$$

$$m_2^T r = -\frac{1}{Y_H}\rho_1 - \frac{1}{Y_H}\rho_2 + \rho_7 \quad (54)$$

According to $\rho_3(z)$, the larger the dissolved oxygen S_0 is, the faster the ammonia S_{NH} is consumed by the autotrophic biomass X_{BA}. The process is called the nitrification. According to $\rho_1(z)$ and $\rho_3(z)$, the larger S_0 is, the faster S_s is consumed by X_{BA} and the heterotrophic biomass X_{BH}. If the oxygen is supplied, the anoxic growth ρ_2 of X_{BH} is very small compared with $\rho_1(z)$ and $\rho_3(z)$. In addition, in the aerobic circumstance where the oxygen increases the biomass X_{BH}, the hydrolysis of entrapped organics $\rho_7(z)$ is very small compared with $\rho_1(z)$. Thus, in the aerobic tank, we have (35) for each of $z_{b,10} = S_{NH}$ and $z_{b,2} = S_s$ with respect to the oxygen input $u_{b,8}$. The oxygen supply is modeled by

$$\frac{dz_{b,8}}{dt} = m_8^T r(z_b) + \frac{Q_w + Q_c + Q_r}{V_b}(z_{a,8} - z_{b,8}) + u_{b,8} \quad (55)$$

$$u_{b,8} = K_L a(S_{O,sat} - z_{b,8}) \quad (56)$$

According to (55) and $m_8^T r(z_b) \leq 0$, the oxygen $z_{b,8} = S_O$ is never larger than the saturated dissolved oxygen concentration $S_{O,sat} > 0$. Instead of $u_{b,8}$, we manipulate the non-negative coefficient $K_L a(t)$ for aeration. Using (52), we choose a control law of aeration as

$$u_o(z_b) = k_{o1} 2 \frac{1 - Y_H}{Y_H} \rho_1(z_b) + k_{o2}\left(\frac{4.57}{Y_A} - 2\right)\rho_3(z_b) \quad (57)$$

$$K_L a(z_b) = \min\left\{\frac{u_o(z_b)}{S_{O,sat} - z_{b,8}}, \ K_{o.sat}\right\} \quad (58)$$

where $0 < k_{o1} < 1$ and $0 < k_{o2} < 1$ are parameters which can be tuned by operators. The number $K_{o.sat} > 0$ represents the inevitable limitation of the oxygen transfer rate due to a compressor. The control input (56)-(58) satisfies (36) in the sense that larger S_{NH} and S_s imply larger $u_{b,8}$. It is stressed that the dissipation inequality (43) holds in the presence of the aeration. The aeration coefficient $K_L a$ and the oxygen supply $u_{b,8}$ are non-negative all times.

The variation of the nitrate S_{NO} is described by

$$m_9^T r = -\frac{1 - Y_H}{2.86 Y_H}\rho_2 + \frac{1}{Y_A}\rho_3 \quad (59)$$

In order to enhance the denitrification process(decrease of S_{NO}), the increase of readily biodegradable substrate S_s is effective in an anoxic circumstance since it renders $\rho_2(z)$ large. Note that ρ_3 is zero in the absence of S_O. Thus, in the anoxic tank, we have (35) for $z_{a,9} = S_{NO}$ with respect to carbon(S_s) input $u_{a,2}$. We utilize (52) again, and a control law of external carbon is obtained as

$$u_{car}(z_a) = k_{car}\left(1+\frac{1}{2.86}\right)\frac{1-Y_H}{Y_H}\rho_2(z_a) \qquad (60)$$

$$u_{a,2}(z_a) = \min\{u_{car}(z_a),\ U_{car.sat}\} \qquad (61)$$

where $0 < k_{car} < 1$ is a tunable parameter. The saturating function (61) takes into account the limitation of flowrate and the available amount of external carbon. The control input $u_{a,2}(z_a)$ satisfies (36) in the sense that larger S_{NO} implies larger $u_{a,2}$. The dissipation inequality (43) is retained for the entire plant in the presence of the carbon dosage. The input $u_{a,2}$ is non-negative all times.

For better nitrification we need large X_{BA}, which is characterized by $\rho_3(z_b)$. For better denitrification, we need large X_{BH}, which is characterized by $\rho_2(z_a)$. Since particulate components flow toward the settler, we need to return X_{BH} and X_{BA} in the settler to bioreactors where they are spent. The accumulated sludge at the bottom of the settler is a rich source of X_{BA} and X_{BH}. A large return flow renders $m_{10}^T r$, $m_2^T r$ and $m_9^T r$ more negative, so that we have (35). The return is ineffective if the sludge at the bottom of the settler is thin. Hence, a choice of the return flow rate Q_r is

$$Q_r = \begin{cases} \min\{c_r(X_{B,s}-X_{B,a}),\ Q_{r.sat}\} & \text{if } X_{B,s}-X_{B,a}>0 \\ 0 & \text{otherwise} \end{cases} \qquad (62)$$

$$X_{B,s} = z_{s,5,n}+z_{s,6,n}, \quad X_{B,a} = z_{a,5}+z_{a,6} \qquad (63)$$

where $c_r > 0$ is a design parameter. The constant $Q_{r.sat} > 0$ takes into account limited capability of the pump. The dissipation equation (44) is independent of Q_r. Thus, (52) holds automatically.

The anoxic tank fundamentally lacks S_{NO} to be denitrificated unless S_{NO} is fed back from the aerobic tank. Since nitrogen can be removed from the wastewater by only denitrification, the component S_{NO} in the aerobic tank should be put in the anoxic tank. The increase of S_{NO} by the circulation actually accelerates $m_9^T r$ for nitrogen removal, which implies (35). Let $S_{NO,a} = z_{a,9}$ and $S_{NO,b} = z_{b,9}$. We choose the following law for the circulation.

$$Q_c = \begin{cases} \min\{c_c(S_{NO,b}-S_{NO,a}),\ Q_{c.sat}\} & \text{if } S_{NO,b}-S_{NO,a}>0 \\ 0 & \text{otherwise} \end{cases} \qquad (64)$$

The constant $c_c > 0$ is a design parameter. The dissipation equation (44) is independent of Q_c, so that (52) holds.

Due to sedimentation, particulate and insoluble components accumulate in the settler and they become redundant for WWT. The dissipation equation (44) implies that the disposal of the sludge directly decreases the mass. Thickness of the sludge at the bottom of the settler is an important information for the necessity of disposal. Thus, a reasonable control law of the wastage flow is

$$Q_s = \begin{cases} \min\{c_s(X-e_{qs}),\ Q_{s.sat},\ Q_w\} & \text{if } X-e_{qs}>0 \\ 0 & \text{otherwise} \end{cases} \qquad (65)$$

$$X = X_s+X_{BH}+X_{BA}+X_P+X_{ND}+X_I \quad \text{at the bottom of the settler}$$

where $c_s > 0$ is a design parameter. If the wastage flow is so large that the sludge becomes very thin, the bioreactors lack bacteria to process the influent. The proportional law $c_s(X-e_{qs})$ with the parameter $e_{qs} > 0$ prevents such a situation.

We finally modify some of the above control laws to prevent washout in extraordinary circumstances. The oxygen supply (57) tends to zero as S_0 goes to zero. Oxygen remains zero regardless of the amount of S_s and S_{NH} if $S_O = 0$ happens to hold. The carbon dosage (60) tends to zero as S_s goes to zero. The carbon remains zero regardless of S_{NO} if $S_s = 0$ happens to hold. It is seen from ASM1 that X_{BH} and X_{BA} monotonously decrease toward zero under $S_O = S_s = 0$. As described in Section IV, the presence of non-zero S_O, S_s, X_{BH} and X_{BA} is necessary not only for keeping bioreactor acting, but also for the existence of invariance sets. In order to keep S_O and S_s from being extraordinarily small in the emergent situations, we replace (57) and (60) with

$$u_o(z_b) = \begin{cases} \text{Eq.(57)}+\alpha_o(e_o-S_O) & \text{if } e_o-S_O>0 \\ \text{Eq.(57)} & \text{otherwise} \end{cases} \qquad (65)$$

$$u_{car}(z_a) = \begin{cases} \text{Eq.(60)}+\alpha_{car}(e_{car}-S_s) & \text{if } e_{car}-S_s>0 \\ \text{Eq.(60)} & \text{otherwise} \end{cases} \qquad (66)$$

respectively. The positive parameters α_o and α_{car} are selected to be so large that the small positive scalars e_o and e_{car} become lower bounds of S_O and S_s, respectively. It is verified form ASM1 that the emergent supply of S_O and S_s yield X_{BH} and X_{BA}. The modifications (65) and (66) make the dissipation inequality (43) violated only in a small neighborhood of the origin $x = 0$.

Remark 4: For a non-zero constant inflow w, components in the bioreactors and the settler have non-zero steady-state values depending on the parameters $\{\ k_{car},\ k_{o1}\ ,\ k_{o2}\ ,\ \alpha_{car},\ \alpha_o,\ e_{car},\ e_o,\ c_r\ ,\ c_c\ ,\ c_s\ ,\ e_{qs},\ Q_{r.sat},\ Q_{c.sat},\ Q_{s.sat},\ K_{o.sat},\ U_{car.sat}\ \}$. Although the inflow w never be ideal constant in practical operation, there may be representative values of the average inflow depending on the weather and seasonal condition, and other circumstance of the regional community. A useful way to pick design parameters is to select the parameters so that the concentration of components takes desirable steady-state values in such representative circumstances.

VI. Simulation

This section presents simulation results carried out in a MATLAB/Simulink environment. The following values are used.

$$V_1 = 2000[\mathrm{m}^3], \quad V_2 = 4000[\mathrm{m}^3], \quad V_s = 6000[\mathrm{m}^3]$$
$$n = 10, \quad m = 5, \quad S_{o.sat} = 8.0[\mathrm{gCOD/m}^3]$$

The influent data, values of stoichiometric and kinetic parameters and settler parameters given in [9] are used. Response of the proposed dissipative control system to the dry weather influent is shown in Fig.2 for the following parameters of control laws.

$$K_{o.sat} = 360[1/\mathrm{day}], \quad U_{car.sat} = 100000[\mathrm{gCOD/m^3day}]$$
$$Q_{r.sat} = 40000[\mathrm{m}^3/\mathrm{day}], \quad Q_{c.sat} = 100000[\mathrm{m}^3/\mathrm{day}]$$
$$Q_{s.sat} = 2000[\mathrm{m}^3/\mathrm{day}], \quad k_{car} = 0.4, \quad k_{o1} = 0.76$$
$$k_{o2} = 0.76, \quad e_{car} = 0.1, \quad \alpha_{car} = 5000, \quad e_o = 0.6$$
$$\alpha_o = 9000,\ c_r = 6.4,\ c_c = 6800,\ c_s = 0.043,\ e_{qs} = 200$$

For an illustrative comparison, response of the proportional flow control is shown in Fig.3. The proportional flow control is set as

$$Q_c = 3Q_w[\mathrm{m}^3/\mathrm{day}], \quad Q_r = Q_w[\mathrm{m}^3/\mathrm{day}]$$
$$Q_s = 0.021Q_w[\mathrm{m}^3/\mathrm{day}], \quad K_La = 300[1/\mathrm{day}], \quad u_{car} = 0$$

The airflow is kept constant. The proposed dissipative control is better than the proportional flow control in various points. The concentration of S_{NO} achieved by the proposed dissipative control is considerably lower. The effectiveness of the dissipative design is that the level of S_s in the effluent is almost the same as the control without the external carbon dosage. Generally, the concentration of other components are also at almost the same level. According to Fig.4, the proposed dissipative control achieves them in an efficient manner. The magnitude of Q_c of the proposed control is significantly smaller than that of the proportional flow control. Peaks of other flow rates Q_r and Q_s are lowered very much by the proposed control. The aeration is operated efficiently by the

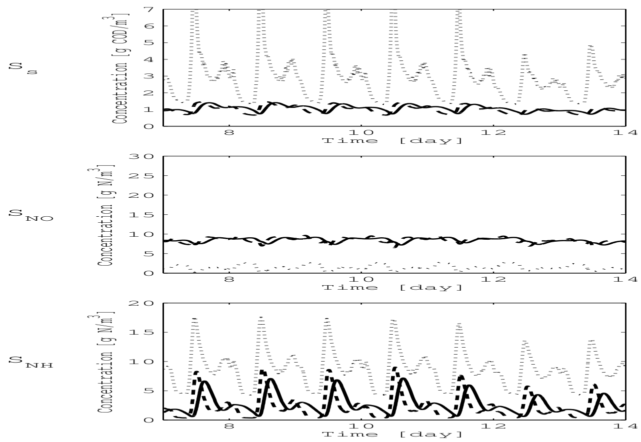

Fig. 2. Response to dry weather influent: proposed dissipative control. (dotted: anoxic tank, dashed: aerobic tank, solid: effluent)

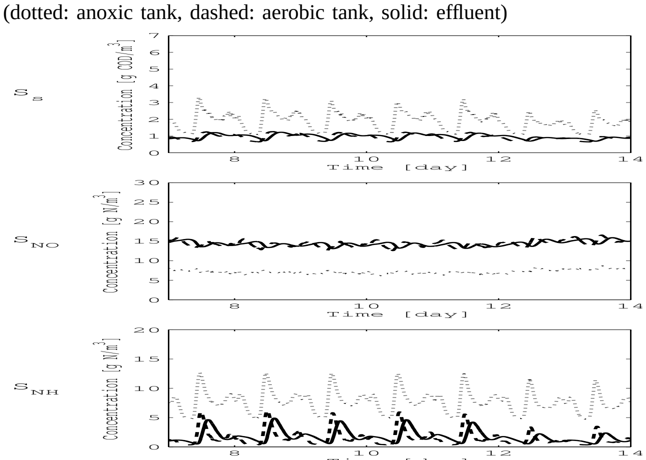

Fig. 3. Response to dry weather influent: proportional flow control. (dotted: anoxic tank, dashed: aerobic tank, solid: effluent)

dissipative control in the way that we do not need to aerate the reactor constantly at a very high level. Responses to rainy weather influent and stormy weather influent are omitted. The advantages of the dissipative control pointed out for the dry weather are evidently observed in the rainy and stormy weather.

VII. Concluding remarks

In this paper, an approach to control design of biological WWT plants has been presented through rigorous treatment of complex process models from a nonlinear control theoretical viewpoint. This paper resorts to neither order reduction nor linearization of the models. Fundamental properties of the WWT plant are investigated through a dissipation property held with respect to the total mass of the entire plant. In contrast to the literature in which it has been too difficult to apply formulas of control theory to the entire plant model of huge and complicated equations, this paper demonstrates that the utilization of the dissipativity enables us to successfully perform model-based control design taking account of the behavior of the entire plant. Control laws of aeration, external carbon dosage, sludge recycle, internal recycle and wastage extraction are proposed based on the idea of preserving dissipation. Simulation results have demonstrated their effectiveness. This paper has not taken into account sensors available for reliable on-line measurement. The study of estimation of on-line unmeasurable variables is a topic of another upcoming article.

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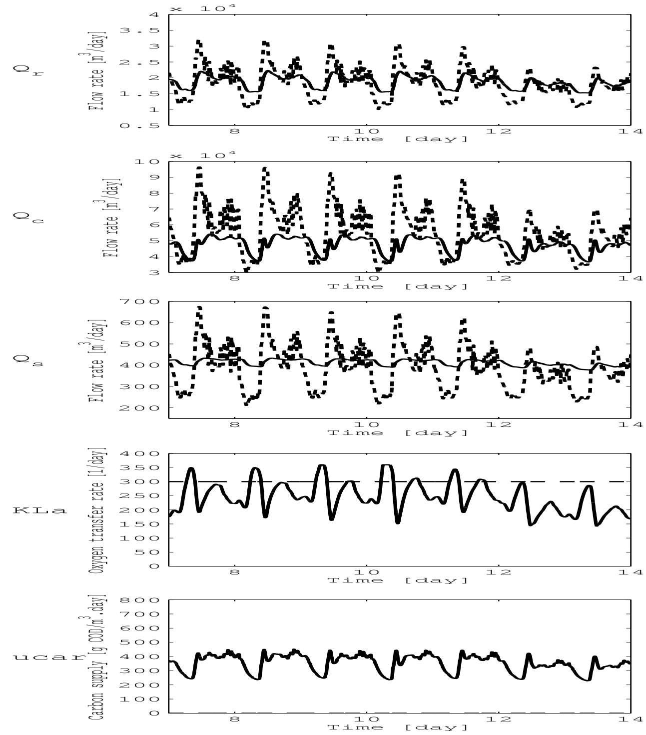

Fig. 4. Control inputs for dry weather influent. (solid: proposed dissipative control, dashed: proportional flow control)

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