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Studies on Structural Vibration Control with MR Dampers Using μGA

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***Abstract*—This paper presents a new approach to reducing the seismic responses of spatial structures with magneto-rheological (MR) dampers using Genetic Algorithm with small populations (μGA). μGA is used to obtain the controller for the MR dampers for structural vibration reduction that is difficult to be solved by the classical optimal control. The advantages of μGA are of global property and having little requirement of the conditions of the optimal function. Numerical results demonstrate the effectiveness of the proposed method in reducing the responses of structure to earthquake ground motions.**

I. Introduction

TMagneto-rheological (MR) fluid is a controllable fluid and belongs to the family of smart materials [1], which is with suspensions of micro-sized and magnetizable particles. Under normal conditions, an MR Fluid is a free-flowing liquid with a consistency similar to that of motor oil. However, exposure to a magnetic field can transform the fluid into a near-solid status in milliseconds. This is because the particles in the fluid acquire dipole moments aggregate to form chains parallel to the field direction. Just as quickly, the fluid can be returned to its liquid state with the removal of the field. The change can appear as a very large change in effective viscosity. The MR fluid along with electromagnets is often used to design controllable dampers, brakes, and couplers. Also, the MR fluid technology can provide flexible control capabilities in designs that are far less complicated and more reliable than conventional electro-mechanical products. Consequently, the MR dampers are ideal tools for semi-active structural control [2,3].

In the application of semi-active control of MR damper, the control force of controller is normally passive and limited [4, 5]. Thus, the optimization of performance index for control is a complex problem. The classically optimal method (e.g. gradient descent algorithm etc.) is often not suitable in this case. A new optimization algorithm, Genetic Algorithm (GA), provides a new approach to this kind of issue, which is a form of random search based on the Darwin's Evolution theory. Since it is an algorithm that simulates the evolution rule, it solves the computational problem by simulating the evolution rule like "The survival of the fittest", "The weak will perish" etc. Thereby, GA is an effective approach to solving the complex problem especially for the optimization problem that is difficult to solve or cannot be effectively solved by the classically optimal method. This approach does not require much information of the system and not require derivative operation, either. Therefore, it becomes very effective when the optimization function is not differentiable or has no gradient information. In a standard GA algorithm, the scale of population is about 30 to 200. In theory, if the scale is small, the information cannot be processed completely. And it will easily fall into the local optimum. However, the small population has the advantage of simple computation, fast convergence, etc. Here, GA with small population, called as μGA, is proposed to obtain the global optimum in a much faster fashion [6].

GA has already been widely applied in civil engineering. Normally, it is mainly applied to calculate the control force [7,8] and optimizes the weights of neural network [9]. In this paper, μGA is applied in the structural vibration control to solve the optimization problem in the semi-active control that is difficult to be solved by the classic control method. The advantages of μGA are of global property and having little requirement of the conditions of the optimal function. Numerical results demonstrate the effectiveness of the proposed method in reducing the responses of structure to earthquake ground motions.

II. Problem of Structural Vibration Control

The equation of motion for the linear structure of a multi-degrees-of-freedom system with control device is as follows

$$[M]\{\ddot{U}\}+[C_d]\{\dot{U}\}+[K]\{U\}=-[M][H_g]\{\ddot{X}_g\} + [H_c]\{F\} \tag{1}$$

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where $\{U\}$ is the displacement vector of structure; $[M]$, $[C_d]$ and $[K]$ are the mass, damping and stiffness matrices, respectively; $[H_g]$ denotes the position matrix of seismic actions; $\{\ddot{X}_g\}=\{\ddot{X}_{Xg},\ddot{X}_{Yg},0\}^T$ means the seismic acceleration vector with X and Y components without rotation; $[H_c]$ implies the position matrix of control forces; $\{F\}$ represents the vector of the control forces.

If the state vector is taken as $\{X\}=\{U,\dot{U}\}^T$, the motion system described in (1) can be written as the following state space equation:

$$\{\dot{X}\}=[A]\{X\}+[B]\{F\} \tag{2}$$

where,

$$[A]=\begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C_d] \end{bmatrix}$$

$$[B]=\begin{bmatrix} 0 \\ [M]^{-1}[H_c] \end{bmatrix}$$

To show the control effectiveness, the performance index is taken as

$$J\ (F)=\max_{t,i}\left(\frac{|x_i(t)|}{x^{\max}}\right)\quad (|F_i|\le F_{i\max}) \tag{3}$$

where, $x_i(t)$ means the displacement of the i^{th} floor with control; $x^{\max}$ denotes the maximum of the i^{th} floor without control; F_i represents the control force of which the i^{th} controller can provide and $F_{i\max}$ is the maximum control force. In the semi-active control presented in this paper, the control forces are provided by the MR dampers.

The performance index function J described in (3) is not differentiable, thus it cannot be solved directly by the classically optimal control method. Here, a method to obtain an approximate solution is proposed as follows.

The precise solution of the optimal problem in (3) can be described as the following nonlinear feedback:

$$\{F\}=-F(\{X\}) \tag{4}$$

Generally, the larger the dimension of system is, the more complex the system becomes. Thus, the nonlinear feedback in (4) is difficult to obtain. Therefore, the system will here be linearized to obtain the approximate solution.

Equation (4) can be rewritten as an expended component form:

$$\begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{Bmatrix}=-\begin{Bmatrix} f_1(x_1,x_2,...,x_{3n}) \\ f_2(x_1,x_2,...,x_{3n}) \\ \vdots \\ f_m(x_1,x_2,...,x_{3n}) \end{Bmatrix} \tag{5}$$

where n is the number of structural floors and m is the number of controller, i.e. the number of the degrees of freedom to be controlled. Here, the nonlinear functions $f_1, f_2, \ldots, f_m$ will be linearized by the Taylor's expansion formula of the multi-variables function. And their processes of the linearization are given by

$$f_1=f_1(x_1,x_2,...,x_{3n})\approx f_{10}+\left.\frac{\partial f_1}{x_1}\right|_{x_1=0}\cdot x_1+\left.\frac{\partial f_1}{x_2}\right|_{x_2=0}\cdot x_2$$
$$+\ldots+\left.\frac{\partial f_1}{x_m}\right|_{x_m=0}\cdot x_m=f_{10}+\left.\left(\frac{\partial f_1}{x_1},\frac{\partial f_1}{x_2},...,\frac{\partial f_1}{x_m}\right)\right|_{\{X\}=\{0\}}\cdot\{X\} \tag{5}$$

$$f_2=f_2(x_1,x_2,...,x_{3n})\approx f_{20}+\left.\frac{\partial f_2}{x_1}\right|_{x_1=0}\cdot x_1+\left.\frac{\partial f_2}{x_2}\right|_{x_2=0}\cdot x_2$$
$$+\ldots+\left.\frac{\partial f_2}{x_m}\right|_{x_m=0}\cdot x_m=f_{20}+\left.\left(\frac{\partial f_2}{x_1},\frac{\partial f_2}{x_2},...,\frac{\partial f_2}{x_m}\right)\right|_{\{X\}=\{0\}}\cdot\{X\} \tag{6}$$

Indeed, the values for high order variables in (6) are relatively small and enough to be omitted so that one-order terms can be chosen as the approximate values for functions. Hence, substitution of (6) into (5) results in the following equation:

$$\begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{Bmatrix}=-\begin{Bmatrix} f_{10}+\left.\left(\frac{\partial f_1}{x_1},\frac{\partial f_1}{x_2},...,\frac{\partial f_1}{x_m}\right)\right|_{\{X\}=\{0\}}\cdot\{X\} \\ f_{20}+\left.\left(\frac{\partial f_2}{x_1},\frac{\partial f_2}{x_2},...,\frac{\partial f_2}{x_m}\right)\right|_{\{X\}=\{0\}}\cdot\{X\} \\ \vdots \\ f_{m0}+\left.\left(\frac{\partial f_m}{x_1},\frac{\partial f_m}{x_2},...,\frac{\partial f_m}{x_m}\right)\right|_{\{X\}=\{0\}}\cdot\{X\} \end{Bmatrix}$$

$$
= -\begin{Bmatrix} f_{10} \\ f_{20} \\ \vdots \\ f_{m0} \end{Bmatrix} - \left. \begin{Bmatrix} \dfrac{\partial f_1}{x_1}, \dfrac{\partial f_1}{x_2}, \ldots, \dfrac{\partial f_1}{x_m} \\ \dfrac{\partial f_2}{x_1}, \dfrac{\partial f_2}{x_2}, \ldots, \dfrac{\partial f_2}{x_m} \\ \vdots \\ \dfrac{\partial f_m}{x_1}, \dfrac{\partial f_m}{x_2}, \ldots, \dfrac{\partial f_m}{x_m} \end{Bmatrix} \right|_{\{X\}=\{0\}} \cdot \{X\} \quad (7)
$$

Consequently, the simplification of (7) can be rewritten as the following expression:

$$
\{F\} \approx -\{F_0\} - [F_1]\{X\} \quad (8)
$$

where, $\{F_0\}$ and $[F_1]$ are the constant vector and constant matrix to be determined as next section, respectively.

III. Application of μGA to structure control

In a standard GA, the knowledge about searching space is not needed. Whether the searched solution is good or not is judged only by the fitness function. In this case, the seismic response of structure during the whole time should be computed before the fitness function is calculated. This is the most time-consuming part in the application of GA to the optimization problem of structure control. The standard GA has a large scale of generations and the computation is complicated. To simplify the computation, the scale of generation may be decreased and new solutions are introduced time by time. Since the goal of the optimization problem is to find the minimum of the performance index, the fitness function used here is defined as[6]:

$$
Adaptality = \begin{cases} J_{\max} - J & \text{if } J < J_{\max} \\ 0 & \text{Otherwise} \end{cases} \quad (9)
$$

The keys to apply GA to the optimal problem are the coding method of the variables and the computation of the fitness function. The process of the coding method and computation of the fitness function are defined as follows:

Coding rules: in the algorithm presented here, every element in the vector F_0 and matrix F_1 is coded. Each solution is stored as the real data of structure and its number represents an element. Binary decoding is implemented only during the genetic operation. In another word, each element is transferred into the binary string, and the strings are combined one by one to form a complete string. After computation, the strings will be coded into real numbers. Thereby, the coding in this way can avoid the inaccuracy that is accompanied by a short string.

The steps of computation of fitness function are as follows:

(1) The strings for F_0 and F_1 are decoded into the real vector $\{F_0\}$ and matrix $[F_1]$;

(2) Substituting $\{F_0\}$ and $[F_1]$ into equation (8), the control force $\{F\}$ can be obtained;

(3) Then, the response of system over the whole simulating period is computed;

(4) The performance index J is computed according to (3), and then J is substituted into (9) to get the value of fitness function.

According to the above discussions, the computational process for μGA to optimize the structural control can be described below:

(1) Five populations are randomly chosen, or four of them are randomly chosen and the one comes from the searching result one step before.

(2) The so-called "elitist choice" is adopted, which means the result that has the best fitness value is inherited directly to next generation to keep the information of good diagram.

(3) The other four solutions are chosen according to the determined contest rule. Since the scale of population is very small, the choice is completely determined because the average rule is meaningless.

(4) The crossover operation is implemented with the probability of 1 so as to accelerate the production of diagram.

(5) Check the convergence criterion. If not satisfied, the computation process is returned to step (2) and the same computational process is again carried out until the convergence criterion is satisfied.

IV. Numerical example for structural control

A two-story eccentric structure as shown in the Fig. 1 is chosen as numerical example. The floor-frame model in computation is applied to obtain the structural response to seismic excitation. The in-plane stiffness of floor is assumed to be infinite and the mass of each story is concentrated on the corresponding floor. The parameters of structure for computation are: the masses are m_1 = 50000kg and m_2 = 20000kg; rotation inertial moments J_1 = 8000 kgm and J_2 = 4600 kgm; eccentric distances E_{x1}=0.5m, E_{y1}=E_{x2}=E_{y2}= 0; translational and rotational stiffness Kx_1 = Ky_1 = 1.0020×10^8N/m, Kx_2 = Ky_2 = 0.6680×10^8 N/m, $K_{\theta1}$ = 1.1690×10^8 Nm and $K_{\theta2}$ = 0.3340×10^8 Nm; the height of layer is 3.6m.

The El Centro earthquake wave recoded on May 18, 1945 in USA is used in the numerical simulation. This wave has bi-directional components along X and Y directions. Four MR dampers are taken as the damping devices installed in the first floor (Fig.1). The support is connected with the bottom of supporting column. The plane diagram is shown in Fig. 2. Each damper can provide a maximum force of 20KN.

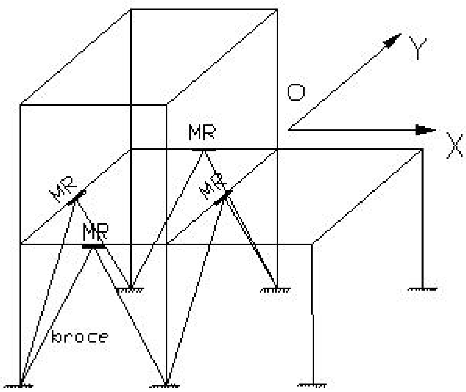

Fig. 1. Diagram of structure with MR Dampers

The El Centro earthquake wave recoded on May 18, 1945 in USA is used in the numerical simulation. This wave has bi-directional components along *X* and *Y* directions. Four MR dampers are taken as the damping devices installed in the first floor (Fig.1). The support is connected with the bottom of supporting column. The plane diagram is shown in Fig. 2. Each damper can provide a maximum force of 20KN.

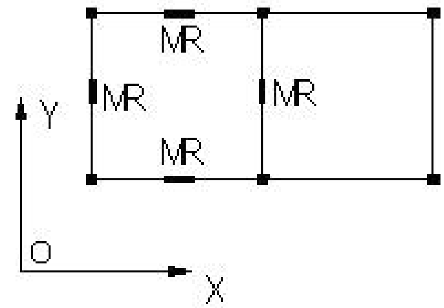

Fig. 2. Plane Placement of MR damper

The periods of the structure is obtained as 0.2015s, 0.1793s, 0.0926s, 0.0851s, 0.0821s and 0.0431s, respectively. The displacements in X and Y directions and the angular displacement with and without controls are shown in Figs. 3-5, respectively. It is clearly seem from these figures that structural vibrations are effectively reduced using the MR dampers with the control forces computed based on the proposed μGA algorithm.

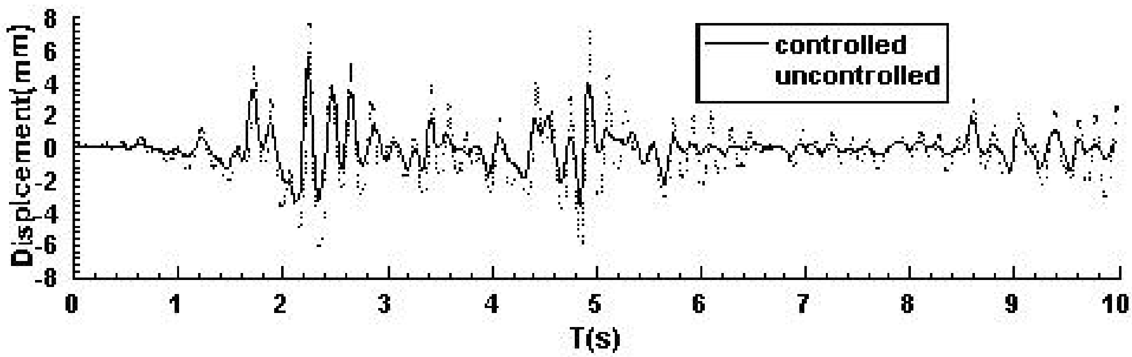

Fig. 3 Comparative time history of effectiveness of vibration reduction in *X* direction

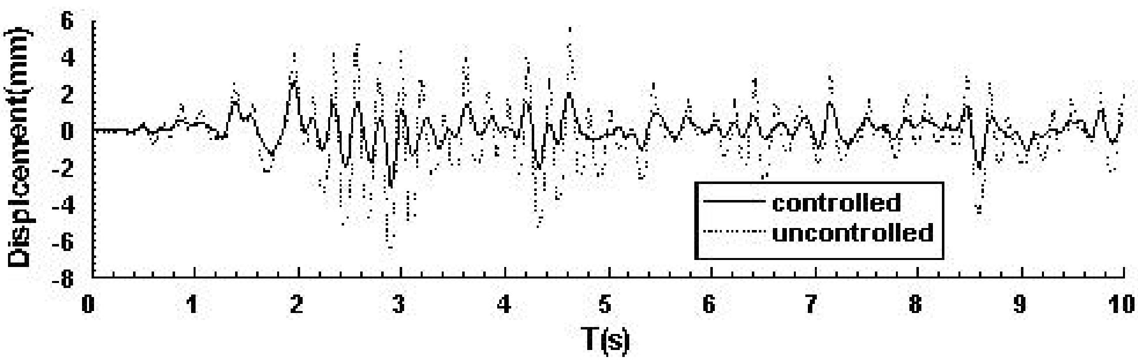

Fig. 4 Comparative time history of effectiveness of vibration reduction in *Y* direction

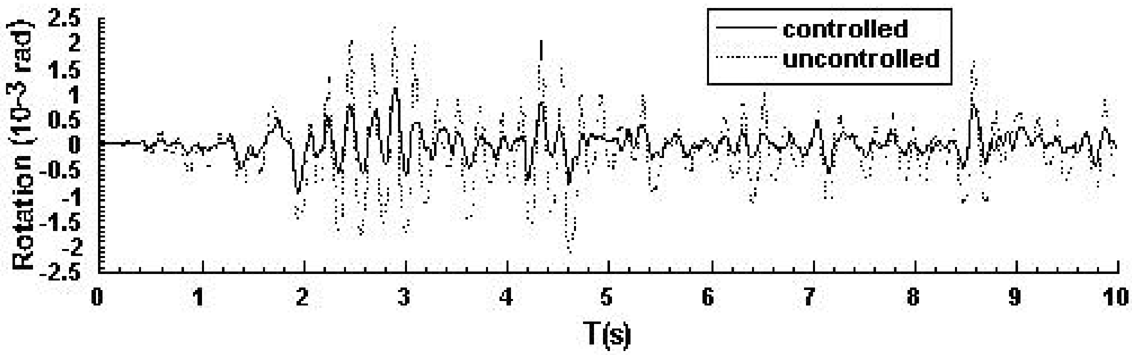

Fig. 5 Comparative time history of effectiveness of vibration reduction in rotational direction

V. Conclusions

In this paper, the optimization problem of a complex control system of a spatial structure with MR dampers is presented by using the μGA approach, which can suitably deal with not only the system of large dimensions, but also limited control force. And the performance index is not differentiable. And the control force is the complicated nonlinear feedback of state variables. To obtain the approximate solution, the nonlinear system is firstly linearized, and then the μGA is applied to solve the problem. A real computational case is given and it has been shown that the proposed control method is effective in structural vibration reduction using MR dampers based on the proposed μGA

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