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Feedback Time Delay as a Stabilizing Tool in Trajectory Tracking, Analysis And Experiments

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Abstract— **A trajectory-tracking problem is considered, represented by linear time invariant (LTI) dynamics of the error. Interesting complexity appears, both analytically and experimentally, when the feedback line is affected by a time delay. We analyze first, the influence of the delay on the system stability and then validate the findings via experiments and numerical simulations. In this effort the system dynamics and the control law are all kept fixed while the only varying parameter is taken to be the feedback time delay, τ. The analysis is conducted utilizing a recent framework of the authors, "cluster treatment of characteristic roots (CTCR)". We obtain the complete stability outlook of the system in the domain of τ and observe an interesting property: The feedback delay may be utilized as a stabilizing tool within the control. That is, when properly selected it may improve the quality of control. We validate these observations via an experimental study and a dynamic simulation work. Ultimately we propose a unique and interesting procedure to use the time delay as a stabilizing agent as well as a tool to enhance the control performance.**

I. Introduction And Problem Statement

WE study the classical trajectory-tracking problem from a novel perspective. The novelty resides in the presence of feedback time delays within the controlled dynamics. The aim is to understand the influence of the delay on the system behavior. Furthermore, we wish to manage the delay intelligently for the obvious objective of improving the controlled dynamics.

Trajectory tracking has been a favorite control problem within a broad range of practical applications. As the name suggests, a control strategy is determined to force the system output to follow a desired trajectory. If one considers a linear time-invariant (LTI) system with full state feedback, the controlled dynamics can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\ \mathbf{x} + \mathbf{B}'\ \mathbf{u} \tag{1}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$, $(\mathbf{A}, \mathbf{B}')$ is a controllable pair of matrices of appropriate dimensions. Let us consider a desired trajectory, $\mathbf{x}_d$ for this system. Because of the controllability assumption there exists a control action, $\mathbf{u}_d$, which satisfies

$$\dot{\mathbf{x}}_d = \mathbf{A}\ \mathbf{x}_d + \mathbf{B}'\ \mathbf{u}_d \tag{2}$$

Subtracting (1) from (2) we obtain, what is called the "error dynamics", as

$$\dot{\mathbf{e}} = \mathbf{A}\ \mathbf{e} + \mathbf{B}'\ (\mathbf{u}_d - \mathbf{u}) = \mathbf{A}\ \mathbf{e} + \mathbf{B}'\ \mathbf{u}' \tag{3}$$

where $\mathbf{e} = \mathbf{x}_d - \mathbf{x}$ is the error vector, $\mathbf{u}'$ is the control action on the error dynamics, of which the definition is self-evident. We consider a linear full-state feedback control law in order to drive $\mathbf{e} \to 0$ asymptotically. The control law, which is predetermined and fixed, is given as

$$\mathbf{u}' = \mathbf{K}\ \mathbf{e} = \mathbf{u}_d - \mathbf{u} \tag{4}$$

These selections obviously require the negative definiteness of the matrix $\mathbf{A} + \mathbf{B}'\ \mathbf{K}$. That is, the system given in (3) should be asymptotically stable. The selection of $\mathbf{K}$ can be done via various methods (LQR, pole assignment, etc.), and it is kept outside the scope of this text. Regardless of the initial conditions, the dynamics given in (3) is driven to $\mathbf{e} \to 0$ asymptotically using the control action, **u**, defined by (4). This defines a desirable tracking operation.

The trajectory-tracking problem for an LTI system, as described above is classical. The arbitrary feature of this study comes at this point: The error state, **e**, is made available to the controller only with a time delay, τ. Then the error dynamics given in (3) becomes

$$\dot{\mathbf{e}} = \mathbf{A}\ \mathbf{e} + \mathbf{B}\ \mathbf{e}(t-\tau) \tag{5}$$

where $\mathbf{B} = \mathbf{B}'\mathbf{K}$.

Most control treatments (in a large volume of available literature) handle the problem of delay by stabilizing the delayed dynamics (using forecasting and estimation methods) and deploying the conditional estimate of the error $\hat{\mathbf{e}}(t) = E[\mathbf{e}(t)]\big|_{t-\tau}$ in the control. $\hat{\mathbf{e}}(t)$ is based on the information available until time $t-\tau$. These procedures have been proven successful in many applications [1, 5] to the cost of numerical overhead and

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the drawbacks of uncertainty carried along with this procedure. The treatment we adopt here is fundamentally different; in that the time delayed information is utilized in the control without any forecasting. Then the "stability" consequences of the time-delayed dynamics must be studied. In (5) the parameter τ, is the determining element of the system stability. Our aim is to assess the stability nature of the target tracking control for varying time delay values within the semi-infinite domain ($0 \le \tau < \infty$). Further aim of this study is to manipulate the time delay and assure the stability of the tracking dynamics. By "manipulating", we mean "prolonging the existing delays further" (the opposite action "cutting it shorter" would obviously be contrary to causality). This is a rather intriguing proposition from the perspective that, an undesirable entity, the delay, acts indeed as a utility for a meaningful purpose.

There is a large body of literature, especially in the last four decades, on the topic of time-delayed systems (TDS) [2-4,6-7,9]. They contain some wonderful theoretical milestones majority of which is on the stability question of LTI-TDS. In the present study we utilize one of these stability analyses named, "*the cluster treatment of characteristic roots (CTCR)*" as described in [7, 9]. CTCR reveals the complete set of stability intervals in $\tau \in \Re^+$. We then look at the 'novel ways of using the time delay as a control parameter'. We describe a procedure to achieve this goal in light of the findings from CTCR. In the first step we determine the exclusive stability disposition of the given dynamic system within the entire semi-infinite domain of $0 \le \tau < \infty$ using CTCR. Then, knowing all the stability intervals in $\tau \in \Re^+$ we propose a control law to benefit from the presence of delay. The CTCR specifies all the delay intervals (we alternately call them the "pockets") precisely, which render stability to the system. At any given instant if the present delay introduces instability, we can search for a larger delay, which may make the system recover the stability, without changing the existing control law. This procedure has a great practical potential in many applications. The determination of the stability pockets exhaustively is the key feature of this procedure enabling the control strategy to stabilize the dynamics. Again, the controller uses only "the time delay" as its control parameter to achieve this. This point is the main contribution of this study, which is presented analytically first, and then demonstrated experimentally and using numerical simulations. We deploy this new idea over two example case studies: *i)* a 2-D target-tracking problem, *ii)* a single-degree-of-freedom experimental study.

The text is composed as follows. In Section II we give an overview of our recent stability analysis method "the cluster treatment of characteristic roots". Example case studies are treated in Section III, both analytically and experimentally. Conclusions are in Section IV.

II. An Overview Of The "Cluster Treatment Of Characteristic Roots (CTCR)"

The objective of the control $\mathbf{u}'$ in (3) is to make the error, $\mathbf{e}$ approach to $\mathbf{0}$ and remain there. Resulting equation (5) represents an LTI-TDS (Linear Time Invariant, Time Delayed System). For this all the system eigenvalues have to be on the left half of the complex plane, i.e. they should be stable. Noting that the characteristic equation of (5)

$$CE(s,\tau) = \det(s\mathbf{I} - \mathbf{A} - \mathbf{B}\, e^{-\tau s}) = 0 \qquad (6)$$

is transcendental, possessing infinitely many roots, the stability assessment is not trivial. Furthermore when $rank(\mathbf{B}) > 1$ so called commensurate terms like $e^{-2\tau s}$, $e^{-3\tau s}$, etc. will also appear in $CE(s,\tau)$ as well as $e^{-\tau s}$. The stability analysis of (6) forms the main theme of many publications in the past four decades [2, 4, 6-7, 9]. We present an overview of one of them, CTCR, following [7, 9] next.

Again, $\mathbf{A}$ and $\mathbf{B}'$ matrices are taken as constant and fixed as well as the state feedback gain $\mathbf{K}$, while τ is the only parameter affecting the stability posture of the dynamics. The highlight of CTCR procedure is in the characterization of the infinitely many roots of (6), which we call "clustering". It groups those root sets, which have at least a pair of imaginary roots and treats them as the only points where the "stability switching" may occur between stability and instability. These values of τ and the corresponding root crossing frequencies form the complete set of root clusters upon which a structured routine, so called CTCR is performed [7, 9].

This procedure also has numerous unique features. CTCR can successfully detect the stability pockets of time delays **precisely** and **exclusively**. Unlike the peer routines [2, 4, 6], it can also depart from an unstable non-delayed dynamics ($\tau = 0$). The method also results in an explicit function of time delay, τ, for the number of unstable roots, NU, which is unique. Here, we only recite without proofs the two important propositions that form the base of CTCR. Interested readers are directed to [7] for details.

Proposition I:
The time-delayed dynamics (5) can exhibit only a finite number (say m) of purely imaginary characteristic roots $\pm\omega_k i$, $k = 1...m$ for all possible $\tau \in \Re^+$. These roots may be single or multiple roots for a given τ. Regardless, however, there are infinitely many characteristic roots that cross at each of these ω_k's at infinitely many equidistant time delays with the spread of $\Delta\tau_k = 2\pi/\omega_k$ in between, ($\tau_{k\ell}$, $k = 1..m$, $\ell = 1..\infty$) starting from $\tau_{k0} > 0$, $k = 1..m$.

Proposition II:
The infinitely many characteristic roots crossing at each of

these ω_k's for the delays $\tau_{k\ell}$, $k=1..m$, $\ell=1..\infty$ have unique directions (either from stable to unstable, or vice versa) for increasing values of τ. This direction is called the root tendency, RT_k, $k=1..m$, and it is **independent** of the time delays that cause the crossing.

CTCR deploys the two propositions above for clustering the characteristic roots. It ultimately generates a stability table as will be displayed in the example section. This table is sorted in ascending values of $\tau_{k\ell}$ ($k=1..m$, $\ell=0,1...$). The root cluster identifiers, the crossing frequencies (ω_k) and the respective root tendencies (RT_k), are also displayed on this table, along with the number of unstable roots (*NU*) in the intervals of τ. The intervals where $NU=0$ are obviously the stable regions. Remember that we can express *NU* as an explicit function of τ, as such the stable operating regions of τ are determined exhaustively. The exact bounds of these "stability pockets" are evaluated as well.

The stability table is exciting and important, because it reveals the option of more than one stability pocket to operate within. Assume that the state information is available to the pursuer with a particular time delay, which makes (5) unstable. The control logic may still be used maintaining stability, simply by imposing a longer delay on purpose, until the total delay reaches the next stable region. We propose this as intelligent management of time delay, which gives the controller a unique ability to pursue. It is clear that such a procedure is possible only with the knowledge of all the stability pockets.

III. Two Example Cases And Experiments

We present two example cases next

a) 2-D target-tracking problem (depicted in Fig. 1). The error dynamics of (5) is given with

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -30.5 & -2.7945 & -1 & 1\\ 0 & 0 & 0 & 1\\ -1 & 1 & -40 & -2\end{bmatrix}$$

$$B=-\begin{bmatrix}0 & 0 & 0 & 0\\ 14.514 & 1.285 & -22.941 & 0.283\\ 0 & 0 & 0 & 0\\ 14 & 1 & -23 & 0.283\end{bmatrix}$$

for which $\mathbf{e}^T=(\mathbf{e}_x, \dot{\mathbf{e}}_x, \mathbf{e}_y, \dot{\mathbf{e}}_y)=(\mathbf{x}_2-\mathbf{x}_1, \dot{\mathbf{x}}_2-\dot{\mathbf{x}}_1, \mathbf{y}_2-\mathbf{y}_1, \dot{\mathbf{y}}_2-\dot{\mathbf{y}}_1)$ with $(\mathbf{x}_1, \mathbf{y}_1)$ and $(\mathbf{x}_2, \mathbf{y}_2)$ representing the coordinates of the (controlled) follower (F) and the evader (E), respectively.

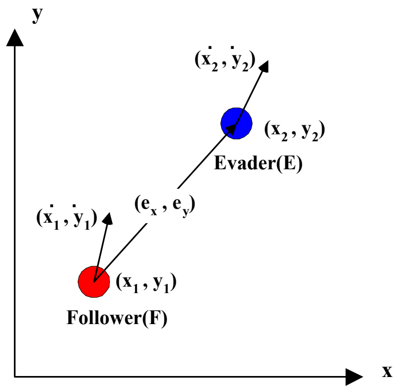

Figure 1. 2-D target-tracking problem

The coupling between the dynamics of (F) and (E) is apparent from the formation of matrix **A**. Notice that $rank(B)=2$ and therefore the characteristic equation

$$\begin{aligned}CE(s,\tau) &= s^4+4.794\,s^3+75.089\,s^2+174.781\,s+1219\\ &+(1.567\,s^3-3.8424\,s^2+14.549\,s-111.973)e^{-\tau s}\\ &+(0.0805\,s^2-6.463\,s-12.658)\,e^{-2\tau s}\end{aligned} \tag{7}$$

has commensurate transcendentality of order 2. The CTCR procedure for this equation results in the stability table, Table 1. The following observations can be made on this table:

- This system can have only 2 distinct imaginary roots for any $\tau\in\Re^+$ (according to proposition I) and the root tendencies are as marked in Table 1 at each of these crossings (according to proposition II). There are clearly three stability pockets

$$\begin{aligned}0 &< \tau < 0.514 \text{ sec}\\ 1.035 &< \tau < 1.416 \text{ sec}\\ 2.182 &< \tau < 2.318 \text{ sec}\end{aligned} \tag{8}$$

- If one studies the best stability margins in the first and second pockets, one determines $\tau_1^*=0.234$ sec, $\tau_2^*=1.24$ sec. Two critical observations on this point are: (i) $\tau_1^*=0.234$ sec delayed control performs better than $\tau=0$ sec (non-delayed case). (ii) The 3rd stability pocket offers no discernable improvement in between marginally stable bounds ($2.182<\tau<2.318$ sec). Thus it is not included. We display the impulse responses of the error dynamics of (5) for $\tau=0$ sec, $\tau_1^*=0.234$ sec, $\tau_2^*=1.24$ sec. in Fig. 2. It is interesting to observe that $\tau_1^*=0.234$ sec delay exhibits better settling time than $\tau=0$ sec. That is, longer delay creates better stability margin *for this dynamics*. So for any delay $0<\tau<0.234$ sec, the controller should prefer an artificially prolonged delay at $0.234\ \text{sec}$. Similar properties are also obtained for $\mathbf{e}_y$, which are not plotted here.

Time delay (τ) [sec]	NU	Stability posture	Crossing frequencies [rad/sec]	RT
0.000				
	0	Stable		
0.514			6.967	1
	2			
1.035			5.477	-1
	0	Stable		
1.416			6.967	1
	2	Unstable		
2.182			5.477	-1
	0	Stable		
2.318			6.967	1
	2	Unstable		

Table 1. Stability table of the error dynamics of example case 1

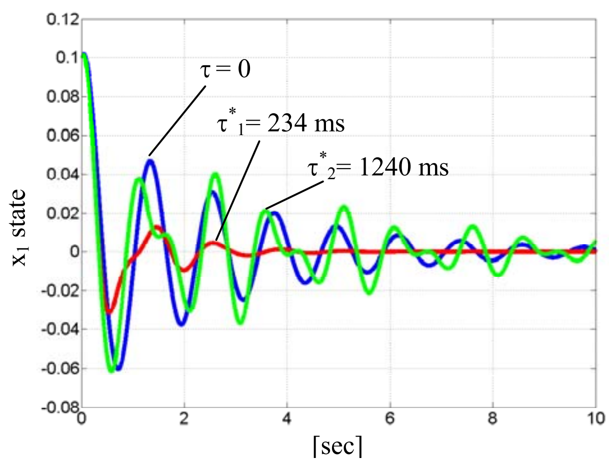

Figure 2. Impulse response of the dynamics for τ, τ^*_2 and τ^*_3

b) A single axis dynamics as shown in Fig 3, is taken. It contains a simple pendulum with a DC motor actuation. The pendulum is made of a rod of length ℓ and mass m. The angular position of the arm is measured via an encoder, and the motor is energized through a high-speed real time control loop. The objective is to make the pendulum arm to follow an arbitrary trajectory. The key constraint here is that the feedback information is intentionally delayed by τ sec. The control logic is kept the same and τ is varied within $0 \le \tau < \infty$. The stability features of the motion are studied first numerically then experimentally. And the findings are compared.

Assuming small angular motion, θ, the error dynamics same as (5) using the error vector $\mathbf{e}^T = \left(\theta_d - \theta,\, \dot{\theta}_d - \dot{\theta}\right)$ where $\theta =$ pendulum angle, θ_d, is the target trajectory, $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ \frac{-\ell mg}{2J} & \frac{-(k_m k_b + Rb)}{RJ} \end{pmatrix}$, $\mathbf{B}' = \begin{pmatrix} 0 \\ \frac{k_m}{RJ} \end{pmatrix}$, $\mathbf{K} = \left(k_1\, k_2\right)$ and $\mathbf{u}' = \mathbf{K}\,\mathbf{e}(t-\tau)$ is the control voltage created by the amplifier (see Fig 3) for the DC motor. k_m and k_b are the motor torque constant and the back electromotive force constant, respectively. R is the armature resistance of the motor, J is the rotational inertia of the pendulum about the pivot point. b is the viscous friction torque constant imposed on the motor shaft. The CTCR stability analysis is performed next on this LTI-TDS revealing the stability regions over $\tau \in [0, \infty)$.

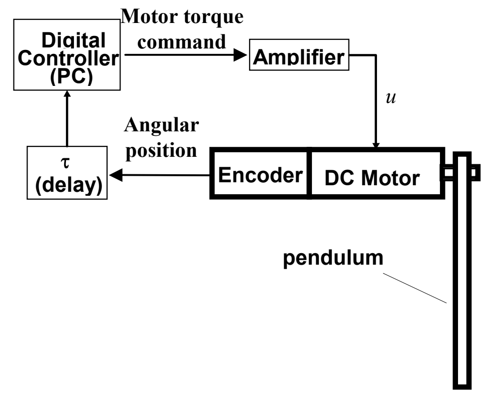

Figure 3. SISO Dynamics

The numerical values for the experimental set up shown in Fig 3 are $\ell = 0.33\ m$, m = 0.13 kg. , J = 4.72 10^{-3} kg m^2 and $\omega_{natural}$ = 6.677 rad/sec (close to 1 Hz) . DC motor controlling the pendulum has armature resistance of R= 3.4 Ω, and its inductive reactance is negligible compared to R. Motor torque constant k_m=0.058 N×m/Ampere, and back electro-motive-force constant k_b=0.06 Volt × sec. Parameter identification effort resulted in a hub friction constant (considered viscous) b = 0.0041 Nmsec . The state and control matrices are formed as follows

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -44.5814 & -1.0855 \end{pmatrix},\ \mathbf{B}' = \begin{pmatrix} 0 \\ 3.6141 \end{pmatrix} \tag{9}$$

The selection of the feedback gain vector as $\mathbf{K} = \left(3.5575 \quad -0.09246\right)$ places the non-delayed system's poles at $s_{1,2} = -0.375 \pm 7.57\,i$. Corresponding to this, the delayed state matrix of equation (5) appears as

$$\mathbf{B}'\,\mathbf{K} = \begin{pmatrix} 0 & 0 \\ 12.8575 & -0.3342 \end{pmatrix} \tag{10}$$

The characteristic equation of (6) becomes

$$CE = s^2 + 1.0854\,s + 44.5814 - (0.3341\,s - 12.857)\,e^{-\tau s} \tag{11}$$

Notice the single delay term appearing in this equation. The commensurate delay term ($e^{-2\tau s}$) does not exist because $\mathbf{B}'\mathbf{K}$ is of 'rank 1'. CTCR does not require this simplification; nevertheless the numerical process becomes easier. We present the ensuing stability table in the following segment jointly with the experimental findings.

In addition to the system parameters given earlier we list the following control system description: sampling speed is 2 KHz., optical encoder has 4096 pulses/ revolution, and the delay within the digital control loop (sensing to actuation) is 8 ms (this is accounted for as a part of the delay τ). Using these numerical values a stability table is created (Table 2) following the steps of CTCR. The τ, ω, RT and NU are evaluated from this study. The rest of the Table 2 is populated from the experimental part explained in what follows. According to the Table, there are 3 stability regions of which the bounds show acceptably small discrepancies comparing analytical and experimental findings. The corresponding resonance (stability switching) frequencies are also given in a comparative form. Most of the results fall within 3% of error level in comparison, which reinforces the validity of the CTCR.

Similarly to the earlier discussions we find three best time delays in the three stability pockets: $\tau_1^* = 0$, $\tau_2^* = 706\ ms$ and $\tau_3^* = 1666\ ms$. For instance in the second stable delay interval ($432 < \tau < 911\ ms$), $\tau_2^* = 706\ ms$ offers the fastest dominant characteristic roots. The exponential decay rate of the error vector is the highest for this τ_2^*. Obviously the boundary points of each stability pocket (e.g., $\tau = 432\ ms$ and $\tau = 911\ ms$) yield marginally stable behavior. Thus the controller should stay away from these points to offer a stability margin. But, how far away? This question requires a rigorous "dominant root" analysis over (11). We have a numerical code, which achieves this. In essence it follows a path described in [4, 8]. Similar effort is repeated for intervals 1,2 and 3 to find τ_1^*, τ_2^* and τ_3^*. In the meantime the real parts of the dominant roots are determined as a part of this routine. They are $\sigma_1^* = -0.3756$, $\sigma_2^* = -0.855$ and $\sigma_3^* = -0.12$ for τ_1^*, τ_2^* and τ_3^* above, respectively.

A set of experiments was conducted on our setup to verify this point. They are given in Fig 4. On the pendulum, we deploy a fixed intensity impulsive torque, which disrupts the equilibrium. $\mathbf{x}_d(t) = 0$ is set as the objective. We monitor the speed of return to this equilibrium $\mathbf{x}(t) = 0$, after the impulse. Decay times (or settling times) can be measured from Fig 4 as $t_{1,sett} = 4.27$ sec, $t_{2,sett} = 1.85$ sec and $t_{3,sett} = 4.6$ sec. Obviously $t_{2,sett}$ is the shortest and it suggests the preferred time delay as τ_2^*. Notice the δ indicating impulsive torques, which are applied at 5th, 15th and 25th seconds of the exercise and the control torques appear τ^* seconds later. The decay rate is determined from that point onwards. This is an exciting finding for such LTI-TDS that one may be able to execute better control using larger delays. Two suggestions are made for managing the time delay in this specific control application: *(i)* The controller should prolong the delay to $\tau_2^* = 706\ ms$ if the existing (unavoidable) delay is less than this. For $706 < \tau < 911 ms$ the controller keeps the delay as is. The operation is still in the second stable pocket of time delay. *(ii)* If $911 < \tau < 1666\ ms$ the controller enforces $\tau_3^* = 1666\ ms$. And for $1666 < \tau < 1759\ ms$, the delay is maintained as is. Obviously for $\tau > 1759\ ms$ controlled system cannot achieve stable tracking, and therefore should not be used.

τ [ms] Analytical	τ [ms] Experimental	% error	Frequency [rad/s] Analytical	Frequency [rad/s] Experimental	% error	RT	NU
0							
STABLE							0
63.5	63.5	0.00	7.41	7.46	-0.67	+1	
unstable							2
432.4	425	1.71	5.76	5.8868	-2.20	-1	
							0
911.4	928	-1.82	7.41	7.42	-0.36	+1	
unstable							2
1523	1426	6.37	5.76	5.96	-3.47	-1	
							0
1759	1849	-5.12	7.41	7.28	1.75	+1	
unstable							2

Table 2. Comparison of the analytical and experimental findings

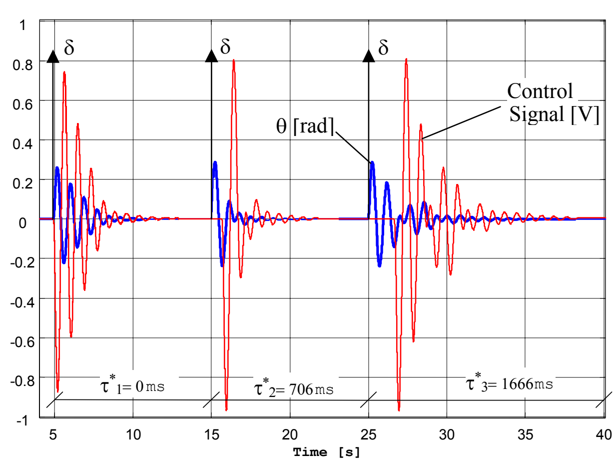

Figure 4. Impulse response of the dynamics for τ^*_1, τ^*_2, τ^*_3 and the control signals

IV. Conclusions

Trajectory-tracking problem for linear time invariant time delayed systems (LTI-TDS) is considered. The time delay on the feedback line introduces some interesting properties to the dynamics. We follow a recent procedure CTCR for the stability assessment. It is shown that the time delay has to be selected within some distinct stability intervals in order to assure a successful tracking. We also describe how a "faster tracking" control law can be achieved simply by manipulating the delay. This leads to an "intelligent management" of time delay in a control structure, which is a unique proposition for a TDS. We then present two case studies to demonstrate the findings of the CTCR framework. One example is a 2-D target-tracking control, and the other is a single-axis arm control. Both cases reinforce the highlight claim of this work, that the delay could be effectively used to enhance the quality of the control, and it may be a complementary tool for the control designer.

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