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A new robust delay-dependent stability criterion for a class of uncertain systems with delay

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***Abstract*— A new robust delay-dependent stability criterion for a class of uncertain systems with delay is established based-on Park inequality. The result for robust delay-dependent stability is presented in terms of linear matrix inequalities (LMIs) by using Lyapunov-Krasovskii functional method. By this result, we give an estimate of the maximum admissible delay, which can be transformed into a generalized eigenvalue problem. Furthermore, a mixed delay stability criterion for this class of uncertain systems with multi-delays is obtained. Finally, two numerical examples are also worked out to illustrate the efficiency and feasibility of the present result.**

Keywords: **Delay systems; Robust stability; Uncertainty; Linear matrix inequality (LMI).**

I. INTRODUCTION

In the mathematical description of a practical control process, one generally assumes that the process depends only on the present state. However, time-delay is ubiquitous in the practical control systems and may lead to destabilization. Studies of stability and stabilization of time-delay systems have become an important problem in control theory (see [1]). If an uncertain delay term is constant but unlimited, i.e., the time-delay belongs to $[0, \infty)$, researchers have provided some delay-independent stability criterion (see [2]–[6] and the references therein). It is necessary to study the delay-dependent stability if an unknown delay term is bounded. There are many valuable results on delay-dependent stability (see [7]–[13] and the references therein). Generally speaking, delay-dependent results are less conservative than delay-independent results authors. To the best of the authors' knowledge, very few results concerning the robust delay-dependent (or delay-independent, mixed delay) stability for the following system model are available. The aim of this paper is to present a new result on robust delay-dependent stability of delay systems.

We will study robust delay-dependent stability of a more general case of linear delay systems subject to norm-bounded uncertainties described by the section 2. By constructing an appropriate Lyapunov-Krasovskii functional, we derive an linear matrix inequality result on robust delay-dependent stability of the systems based on Park inequality. Furthermore, we will extend the result to the case of a mixed delay stability criterion for this class of uncertain systems with multi-delays. Finally, two numerical examples are worked out to illustrate the efficiency and feasibility of the present result.

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II. PRELIMINARIES

In the sequel, denote by A^T and A^{-1} the transpose and the inverse of matrix A (if it is invertible), and by I the unit matrix of appropriate dimensions. $\bar{\sigma}(\cdot)$ denotes the maximum singular value of matrix.

Consider the following uncertain linear system with delay described by Σ:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_\tau x(t-\tau) + Bp(t) \\ q(t) = Cx(t) + Dp(t) \\ p(t) = \Delta q(t), \qquad \|\Delta\| \leq 1, \quad \overline{\sigma}(D) < 1 \\ x(t_0 + s) = \psi(s),\ s \in [-\tau,\ 0], \end{cases} \tag{1}$$

where $x(\cdot) : \mathbf{R} \to \mathbf{R}^n$ is the state vector, p, q are the uncertain input and output of the plant, respectively. $(t_0,\ \psi) \in \mathbf{R} \times \mathcal{C}_{n,\tau}$, where $\mathcal{C}_{n,\tau} = \mathcal{C}([-\tau,\ 0], \mathbf{R}^n)$ is the Banach space consisting of all continuous vector-valued functions from $[-\tau,\ 0]$ to $\mathbf{R}^n$. The coefficient matrices $C_\tau, A, A_\tau, B, C, D$ are real constant matrices of appropriate dimensions. A description of such type of uncertainties can be found in [14], [15].

By Newton-Leibnitz formula:

$$x(t-d) = x(t) - \int_{t-d}^{t} \dot{x}(s)ds,$$

the above form can be rewritten as the following algebraic-differential equations including integration for descriptor systems with distributed delay in the variable y:

$$\begin{cases} \dot{x}(t) = (A + A_\tau)x(t) + Bp(t) - A_\tau \int_{t-\tau}^{t} \dot{x}(\xi)d\xi \\ q(t) = Cx(t) + Dp(t) \\ p(t) = \Delta q(t), \qquad \|\Delta\| \leq 1, \quad \overline{\sigma}(D) < 1 \\ x(t_0 + s) = \psi(s),\ s \in [-\tau,\ 0], \end{cases} \tag{2}$$

Similar transformations of a delay system into such a form have been used for delay-dependent stability analysis of retarded type systems [7].

In this paper, we shall focus on the problem of the delay-dependent robust stability of the system (1) with respect to (w.r.t.) the uncertainty Δ (i.e., finding robust stability conditions containing delay information). The following result will be useful in the sequel.

Recall Schur complement formula (see [14], [15]), namely

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} < 0 \ (\leq 0)$$

if and only if one of the following conditions holds:

1) $R_{22} < 0$ and $R_{11} - R_{12}R_{22}^{-1}R_{12}^T < 0 \ (\leq 0)$;
2) $R_{11} < 0$ and $R_{22} - R_{12}^T R_{11}^{-1} R_{12} < 0 \ (\leq 0)$.

Lemma 1. (see [11]) For any positive definite matrix X and any matrix M of appropriate dimensions, the following inequality holds

$$\pm 2x^T y \leq \begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} X & XM \\ M^T X & (2,2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix},$$

where $(2,2) =: (M^T X + I)X^{-1}(XM + I)$ and x, y are arbitrary dimensional vectors.

III. Robust delay-dependent stability

To analyze delay-dependent robust stability of the system (1), the Lyapunov-Krasovskii functional for the system (1) can be chosen as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t). \tag{3}$$

where

$$V_1(t) = x^T P x, P > 0; \tag{4}$$

$$V_2(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(\xi) A_\tau^T X A_\tau \dot{x}(\xi) d\xi d\theta, \ X > 0; \tag{5}$$

$$V_3(t) = \int_{t-\tau}^{0} x^T(s) Q x(s) ds, \ Q > 0; \tag{6}$$

$$V_4(t) = \int_{0}^{t} q^T(s) q(s) - p^T(s) p(s) ds \ . \tag{7}$$

Notice that $V(t)$ is radially unbounded with respect to $x(t)$, and $V_4(t) \geq 0$ since $\|\Delta\| \leq 1$.

From (4)–(7), it follows that

$$\begin{aligned} \tfrac{dV_1}{dt}|_{(1)} = \ & 2x^T P(A + A_\tau)x + 2x^T PBp(t) - \\ & 2x^T PA_\tau \int_{t-\tau}^{t} \dot{x}(\xi) d\xi \end{aligned} \tag{8}$$

$$\frac{dV_2}{dt}|_{(1)} = \tau \dot{x}^T(t) A_\tau^T X A_\tau \dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^T(\xi) A_\tau^T X A_\tau \dot{x}(\xi) d\xi \ , \tag{9}$$

$$\frac{dV_3}{dt}|_{(1)} = x^T(t) Q x(t) - x^T(t-\tau) Q x(t-\tau) \ , \tag{10}$$

and

$$\begin{aligned} \tfrac{dV_4}{dt}|_{(1)} = \ & q^T(t) q(t) - p^T(t) p(t) \\ = \ & \begin{bmatrix} x \\ p \end{bmatrix}^T \begin{bmatrix} C^T C & C^T D \\ D^T C & D^T D - I \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix} . \end{aligned} \tag{11}$$

Moreover, by Lemma 1,

$$\begin{aligned} & -2\int_{t-t}^{t} x^T(\xi) P^T A_\tau \dot{x}(\xi) d\xi \\ \leq \ & \int_{t-t}^{t} \begin{bmatrix} A_\tau \dot{x}(\xi) \\ Px \end{bmatrix}^T \begin{bmatrix} X & XM \\ M^T X & (2,2) \end{bmatrix} \begin{bmatrix} A_\tau \dot{x}(\xi) \\ Px \end{bmatrix} d\xi \\ = \ & \tau x^T P (M^T X + I) X^{-1} (XM + I) P x \\ & + 2x^T P M^T X A_\tau + \int_{t-\tau}^{t} \dot{x}(\xi) ds\xi \\ & + \int_{t-\tau}^{t} \dot{x}^T(\xi) A_\tau^T X A_\tau \dot{x}(\xi) d\xi. \end{aligned}$$

Thus, from (8)–(11) and the above inequality, we can obtain

$$\begin{aligned} \tfrac{dV}{dt}|_{(1)} = \ & \tfrac{dV_1}{dt}|_{(1)} + \tfrac{dV_2}{dt}|_{(1)} + \tfrac{dV_3}{dt}|_{(1)} + \tfrac{dV_4}{dt}|_{(1)} \\ \leq \ & x^T [P(A + A_\tau) + (A + A_\tau)^T P \\ & + \tau P (M^T X + I) X^{-1} (XM + I) P] x \\ & + 2x^T PBp + 2x^T P M^T X A_\tau (x(t) - x(t-\tau)) \\ & + \tau \dot{x}^T(t) A_\tau^T X A_\tau \dot{x}(t) + x^T(t) Q x(t) \\ & - x^T(t-\tau) Q x(t-\tau) \\ & + \begin{bmatrix} x \\ p \end{bmatrix}^T \begin{bmatrix} C^T C & C^T D \\ D^T C & D^T D - I \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix} \\ = \ & \pi^T \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} \\ \Xi_{13}^T & \Xi_{23}^T & \Xi_{33} \end{bmatrix} \pi \end{aligned}$$

where $\pi = [x(t) \quad x(t-\tau) \quad p(t)]^T$,

$$\begin{aligned} \Xi_{11} = \ & P(A + A_\tau) + (A + A_\tau)^T P \\ & + C^T C + Q + \tau A^T A_\tau^T X A_\tau A \\ & + \tau P (M^T X + I) X^{-1} (XM + I) P \\ & + P M^T X A_\tau + A_\tau^T X M P, \\ \Xi_{12} = \ & \tau A^T A_\tau^T X A_\tau A - P M^T X A_\tau \\ \Xi_{13} = \ & C^T D + PB + \tau A^T A_\tau^T X A_\tau B \\ \Xi_{22} = \ & -Q + \tau A_\tau^T A_\tau^T X A_\tau A_\tau \\ \Xi_{23} = \ & \tau A_\tau^T A_\tau^T X A_\tau B \\ \Xi_{33} = \ & D^T D - I + \tau B^T A_\tau^T X A_\tau B. \end{aligned}$$

Taking $W =: XMP$, one has that

$$\begin{aligned} \Xi_{11} = \ & P(A + A_\tau) + (A + A_\tau)^T P + C^T C + Q \\ & + \tau A^T A_\tau^T X A_\tau A + \tau (W^T + P) V^{-1} (W + P) \\ & + W^T A_\tau + A_\tau^T W, \end{aligned}$$

Thus, it is guaranteed the negativeness of $\dot{V}(t)$ whenever π is nonzero if the following linear matrix inequality holds:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & \Pi_{15} \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} & 0 & 0 \\ \Pi_{13}^T & \Pi_{23}^T & D^T D - I & \tau B^T A_\tau^T X & 0 \\ 0 & 0 & \tau X A_\tau B & -\tau X & 0 \\ \Pi_{15}^T & 0 & 0 & 0 & -\tau X \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{aligned}\Pi_{11} =:& \ P(A+A_\tau)+(A+A_\tau)^TP+C^TC\\ &+Q+\tau A^TA_\tau^TXA_\tau A+W^TA_\tau+A_\tau^TW,\\ \Pi_{12} =:& \ \tau A^TA_\tau^TXA_\tau A-W^TA_\tau\\ \Pi_{13} =:& \ C^TD+PB+\tau A^TA_\tau^TXA_\tau B\\ \Pi_{15} =:& \ W^T+P\\ \Pi_{22} =:& \ -Q+\tau A_\tau^TA_\tau^TXA_\tau A_\tau\\ \Pi_{23} =:& \ \tau A_\tau^TA_\tau^TXA_\tau B.\end{aligned}$$

By Schur complement formula, the above matrix inequality is equivalent to the negativeness of the coefficient matrix of $\dot{V}(t)$, this shows that the system (1) is robustly delay-dependently stable.

We summarize the result as follows.

Theorem 1. For system (1), if there exist positive definite matrices P, X, Q and matrices $W \in \mathbf{R}^{n\times n}$ such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & -W^TA_\tau & \Sigma_{13} & \Sigma_{14} & \Sigma_{15}\\ -A_\tau^TW & -Q & 0 & \Sigma_{24} & 0\\ \Sigma_{13}^T & 0 & \Sigma_{33} & \Sigma_{34} & 0\\ \Sigma_{14}^T & \Sigma_{24} & \Sigma_{34}^T & -\tau X & 0\\ \Sigma_{15}^T & 0 & 0 & 0 & -\tau X\end{bmatrix} < 0, \quad (13)$$

where

$$\begin{aligned}\Sigma_{13} =:& \ C^TD+PB,\\ \Sigma_{14} =:& \ \tau A^TA_\tau^TX,\\ \Sigma_{24} =:& \ \tau A_\tau^TA_\tau^TX,\\ \Sigma_{33} =:& \ D^TD-I\\ \Sigma_{34} =:& \ \tau B^TA_\tau^TX,\\ \Sigma_{15} =:& \ \Pi_{15}\end{aligned}$$

and

$$\begin{aligned}\Sigma_{11} =:& \ P(A+A_\tau)+(A+A_\tau)^TP+C^TC\\ &+Q+W^TA_\tau+A_\tau^TW\end{aligned} \quad (14)$$

then the system is robustly delay-dependently stable w.r.t. Δ.

Proof. By Schur complement formula, (12) is equivalent to (13). This completes the proof. $\diamondsuit$

Remark 1. Taking $V =: \tau X$, Theorem 1 can be rewritten as:

Theorem 1'. For system (1), if there exist positive definite matrices P, X, Q and matrices $W \in \mathbf{R}^{n\times n}$ such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & -W^TA_\tau & \Sigma_{13} & A^TA_\tau^TV & \tau\Sigma_{15}\\ -A_\tau^TW & -Q & 0 & A_\tau^TA_\tau^TV & 0\\ \Sigma_{13}^T & 0 & D^TD-I & B^TA_\tau^TV & 0\\ VA_\tau A & VA_\tau A_\tau & VA_\tau B & -V & 0\\ \tau\Sigma_{15}^T & 0 & 0 & 0 & -V\end{bmatrix} < 0, \quad (15)$$

where Σ_{11} is defined in (14), then the system is robustly delay-dependently stable w.r.t. Δ. $\diamondsuit$

Remark 2. From Theorem 1', letting $B = C = D \equiv 0$, we obtain the following result (see [11]) without uncertainty.

Corollary 1. If there exist positive definite matrices P, X, Q and matrices $W \in \mathbf{R}^{n\times n}$ such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & -W^TA_\tau & A^TA_\tau^TV & \tau(W^T+P)\\ -A_\tau^TW & -Q & A_\tau^TA_\tau^TV & 0\\ VA_\tau A & VA_\tau A_\tau & -V & 0\\ \tau(W+P) & 0 & 0 & -V\end{bmatrix} < 0, \quad (16)$$

where Σ_{11} is defined in (14), then the system is delay-dependently stable. $\diamondsuit$

The condition in the corollary has also been obtained recently in [11] for delay-dependent stability, therefore, our result is a direct extension of Theorem 1 in [11].

Remark 3. To estimate the maximal admissible delayed-time τ such that the system is robustly delay-dependently stable, we first note that the problem can be formulated as

$$\max \tau > 0$$

s.t. there exist positive definite matrices P, X, Q and matrices $W \in \mathbf{R}^{n\times n}$ such that (13) holds. By Schur complement formula, (13) is equivalent to

$$\begin{bmatrix} \Sigma_{11} & -W^TA_\tau & \Sigma_{13} & A^TA_\tau^TV & \Sigma_{15}\\ -A_\tau^TW & -Q & 0 & A_\tau^TA_\tau^TV & 0\\ \Sigma_{13}^T & 0 & \Sigma_{33} & B^TA_\tau^TV & 0\\ VA_\tau A & VA_\tau A_\tau & VA_\tau B & -V & 0\\ \Sigma_{15}^T & 0 & 0 & 0 & -\frac{1}{\tau^2}X\end{bmatrix} < 0.$$

Letting $\delta = \tau^{-2}$, we can transform the problem into the following generalized eigenvalue problem (see [14]) for more details):

$$\min \delta > 0$$

s.t. there exist positive definite matrices P, X, Q and matrices $W \in \mathbf{R}^{n\times n}$ satisfying

$$\begin{bmatrix} \Sigma_{11} & -W^TA_\tau & \Sigma_{13} & A^TA_\tau^TV & \Sigma_{15}\\ -A_\tau^TW & -Q & 0 & A_\tau^TA_\tau^TV & 0\\ \Sigma_{13}^T & 0 & \Sigma_{33} & B^TA_\tau^TV & 0\\ VA_\tau A & VA_\tau A_\tau & VA_\tau B & -V & 0\\ \Sigma_{15}^T & 0 & 0 & 0 & -\delta X\end{bmatrix} < 0.$$

This problem can be solved numerically by using LMI toolbox in [16]. $\diamondsuit$

IV. Robust mixed delay (delay-dependent/delay-independent) stability

In the sequel, we will give a direct extension of the obtained result in the last section. Now we consider the following uncertain system with multi-delays:

$$\begin{cases}\dot{x}(t) = Ax(t)+A_dx(t-d)+A_\tau x(t-\tau)+Bp(t)\\ q(t)=Cx(t)+Dp(t)\\ p(t)=\Delta q(t), \qquad \|\Delta\|\le 1, \quad \overline{\sigma}(D)<1\\ x(t_0+s)=\psi(s),\ s\in[-\bar{\tau},\ 0],\end{cases} \quad (17)$$

where $\bar{\tau} = \max\{\tau, d\}$. For system (17), we can take candidate Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t). \quad (18)$$

where $V_1(t) - V_4(t)$ are given by (4)–(7) and

$$V_5(t) = \int_{t-d}^{t} x^T(\xi) R x(\xi) d\xi,\ R > 0\,.$$

Similar to the proof of Theorem 1, we can obtain the following result.

Theorem 2. For system (17), if there exist positive definite matrices P, V, Q, R and matrices $W \in \mathbf{R}^{n\times n}$ such that the following LMI holds:

$$\begin{bmatrix} \Phi_{11} & -W^T A_\tau & 0 & \Sigma_{13} & \Phi_{15} & \tau\Sigma_{15} \\ -A_\tau^T W & -Q & 0 & 0 & \Phi_{25} & 0 \\ 0 & 0 & -R & 0 & \Phi_{35} & 0 \\ \Sigma_{13}^T & 0 & 0 & \Sigma_{33} & \Phi_{45} & 0 \\ \Phi_{15}^T & \Phi_{25}^T & \Phi_{35}^T & \Phi_{45}^T & -V & 0 \\ \tau\Sigma_{15}^T & 0 & 0 & 0 & 0 & -V \end{bmatrix} < 0, \quad (19)$$

where

$$\begin{aligned} \Phi_{11} =: &\ P(A + A_\tau) + (A + A_\tau)^T P + C^T C \\ &+ Q + R + W^T A_\tau + A_\tau^T W, \\ \Phi_{15} =: &\ A^T A_\tau^T V, \\ \Phi_{25} =: &\ A_\tau^T A_\tau^T V, \\ \Phi_{15} =: &\ A_d^T A_\tau^T V, \\ \Phi_{15} =: &\ B^T A_\tau^T V, \end{aligned}$$

then the system is robustly (w.r.t. Δ) delay-dependently stable w.r.t. the delay τ and delay-independently stable w.r.t. the delay d. ♢

Remark 4. Similar to Remark 3, we can estimate the maximal delay for mixed delay stability. ♢

Remark 5. For the simple delay system (i.e., $A_d \equiv 0$), we can establish the same result of the delay-dependent stability as Theorem 1. If $A_\tau \equiv 0$, then Theorem 2 gives a criterion on delay-independent stability of system (17). ♢

V. Numerical examples

Example 1. Consider the example from [11], that is, the system (1) with the following parameters:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

and $A_d = 0, B = 0, C = 0, D = 0$. Since the matrix $A + A_\tau$ is stable and $A - A_\tau$ is not, the time-delay system is stable dependent on the size of the delay τ as noted in [1]. Any delay-independent stability criterion fails to verify asymptotical stability. In [11], the author showed a comparative result for the maximal delay bound according to different method in the references. In fact, the maximal delay can be taken as larger than 4.3588. By Corollary 1, using the LMI-Matlab Toolbox in [16], we find that (16) is feasible for $\tau = 15$. We obtain a solution to (16) for $\tau = 15$ as follows:

$$P = 10^4 \begin{bmatrix} 2.8130 & -0.7091 \\ -0.7091 & 3.9714 \end{bmatrix},$$

$$Q = 10^4 \begin{bmatrix} 6.7799 & 0.7290 \\ 0.7290 & 6.7739 \end{bmatrix},$$

$$V = 10^4 \begin{bmatrix} 2.2160 & -1.1795 \\ -1.1795 & 1.2785 \end{bmatrix},$$

$$W = 10^3 \begin{bmatrix} -9.6070 & 2.1342 \\ 2.1342 & -8.6951 \end{bmatrix}.$$

In fact, solving the optimization problem formulated in Remark 3, we can obtain $\tau_{\max} = 1.2517e + 07$ such that the system is stable for all $\tau : 0 < \tau \le \tau_{\max}$. This greatly enlarge the admissible delay bound.

Example 2. Consider system (1) with the following parameters: A, A_τ are the same as Example 1 and

$$B = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

By Theorem 1' and Remark 3, we obtain the maximal delay $\tau_{\max} = 0.2559$ and a solution to (15) for $\tau = 0.2559$ as follows:

$$P = \begin{bmatrix} 369.5786 & 0.1496 \\ 0.1496 & 1.8021 \end{bmatrix},$$

$$Q = 10^7 \begin{bmatrix} 2.4046 & 0.0059 \\ 0.0059 & 1.0139 \end{bmatrix},$$

$$V = \begin{bmatrix} 62.5000 & -0.5920 \\ -0.5920 & 0.9563 \end{bmatrix},$$

$$W = 10^7 \begin{bmatrix} 2.3987 & -1.0080 \\ -1.0080 & 1.0139 \end{bmatrix}.$$

Therefore, the uncertain system is robustly delay-dependently stable for arbitrary $\bar{\sigma}(\Delta) \le 1$ by Theorem 1.

VI. Conclusions

We have addressed the stability problems for a class of uncertain systems with delay. A sufficient condition of robust delay-dependent stability for this class of systems was established in terms of linear matrix inequalities. By the result, an extended result on mixed delay stability for systems with multi-delays was further obtained. We also considered the issue of estimating the maximum admissible delay for stability and formulated it as a generalized eigenvalue problem which can be solved numerically with the efficient LMI Tool Box. Finally, two numerical examples showed less conservatism and the feasibility of our results.

References

[1] Dugard, L., and Verriest E. I. *Stability and Control of time-delay systems*, New York: Springer-Verlag, 1997.

[2] Chen, J., and H. A. Latchman, "Asymptotic stability independent of delays: Simple necessary and sufficient conditions," *Proceedings of American control conference*, Baltimore, USA, pp. 1027–1031, 1994.

[3] Chen, J., D. Xu, and B. Shafai, "On sufficient conditions for stability independent of delays," *IEEE Trans. Automat. Contr.*, vol. 40 (9), pp. 1640–1645, Sep. 1995.

[4] Choi, H. H., and M. J. Chung, "An LMI approach to H_∞ controller design for linear time-delay systems," *Automatica*, 33 (4): 737–739, 1997.

[5] Lee, J.H., S.W. Kim, and W. H. Kwon, "Memoryless H_∞ controllers for state delayed systems," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 159–162, Jan. 1994.

[6] Li, X., and C. E. Souza, "Criteria for robust stability of uncertain linear systems with time-varying state delays," *13th IFAC World Confr.*, San Franciso, CA, vol. 1, pp. 137–142, 1996.

[7] Kolmanovskii, V., S.-I.Niculescu, and J.P. Richard, "On the Lyapunov-Krasovskii functionals for stability analysis of linear delay systems," *Internat. J. Control*, 72: 374–384, 1999.

[8] J. H. Su, "Further results on the robust stability of linear systems with a single time delay," *Syst. Contr. Lett.*, vol. 23, no. 5, pp. 375–379, 1994.

[9] Gu, K., and S.-I. Niculescu, "Additional eigenvalues in transformed time-delay systems," *IEEE Transactions on Automatic Control*, 45 (3): 572–576, 2000.

[10] Li X., and C. deSouza, "Delay-dependent robust stability and stabilization of uncertain linear delay systems," *IEEE Transactions on Automatic Control*, 42 (8): 1144–1148, 1997.

[11] Park, P., "A delay dependent stability criterion for systems with uncertain time-invariant delays," *IEEE Transactions on Automatic Control*, 44 (5): 876–878, 1999.

[12] Verriest, E. I., "Robust stability of differential-delay systems," *Zeitschrift fur Angewandte Mathematikund Mechanik* pp. S1107–S1108, 1998.

[13] S.S. Wang, B.S. Chen, and T.P. Lin, Robust stability of uncertain time-delay systems, *International Journal of Control* vol. 46, pp. 963–976, 1987.

[14] Boyd, S., L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, SIAM, Philadelphia, PA 1994.

[15] Zhou, K., J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, New Jersey: Prentice-Hall 1996.

[16] Gahinet G., A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox for use with Matlab*. The Mathworks Inc, 1995.