

Adaptive Fuzzy Sliding Control for a Three-Link Passive Robotic Manipulator

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Abstract

An adaptive fuzzy sliding control (AFSC) scheme is proposed to control a passive robotic manipulator. The motivation for the design of the adaptive fuzzy sliding controller is to eliminate the chattering and the requirement of pre-knowledge on the bounds of the errors associated with the conventional sliding control. The stability and convergence of the adaptive fuzzy sliding controller is proven both theoretically and practically by simulations. A three-link passive manipulator model with two unactuated joints is derived to be used in the simulations. Simulation results demonstrate that the proposed system is robust against structured and unstructured uncertainties.

1. Introduction

An underactuated or passive manipulator is one in which some of the joints are not actuated. This could be the result of failure of some of the actuators, for example, a robot in space that has a dysfunctional joint actuator or that of a poor design. In many cases where the underactuated mechanism is due to failure of the joint, it is hard and expensive to repair or replace the failed actuator. A passive manipulator could also be constructed by choice to reduce energy consumption, weight and cost. The control of such underactuated manipulators has been gaining a lot of attention in the recent time. Most of the work done in this area is on manipulators mounted with some kind of braking mechanism on the passive joints. Research on purely passive systems with no braking mechanism is still in an infantile stage.

Usually, the number of degrees of freedom (dof) of a manipulator is the same as the number of actuators. However, for passive manipulators, the number of degrees of freedom is more than the number of actuators. The absence of actuators for some of the joints introduces nonholonomic constraints in the system. Control of passive manipulators poses a considerable challenge due to their highly non-linear and coupled structure. However, there have been many successful works in controlling passive manipulators using various control strategies [1][2][3][4]. [1] is the first work to propose the method of controlling

passive manipulators equipped with brakes by using the dynamic coupling between the joints. The condition for controllability of manipulators with passive joints, a control algorithm and optimal control strategy are presented in [2]. [3] presents the sliding control of a three-link manipulator with two passive joints and also presents a control algorithm. The control of a two-link passive manipulator with sliding control is discussed in [4].

Sliding control is a robust control scheme whose advantages include less computation and problem order reduction. However, the design of a sliding control requires knowledge on the bounds of the system disturbances and uncertainties. Another major problem with sliding control is the presence of chattering in the control input, which has a destabilizing effect of the system. Often, to solve this problem, smooth function control has been used. While this type of control alleviates the chattering problem in sliding control, it doesn't guarantee convergence of the output to the desired value [5][6].

Fuzzy control is recently becoming a popular control scheme, especially for complicated non-linear systems including robotic systems and electrical systems. The main reason for its popularity is its model free approach and ability to incorporate human knowledge to effectively control the systems in concern. In the field of robotics as well as others, there has been considerable research with successful results on the application of fuzzy control [6][7][8][9][10][11][12]. While fuzzy control is intuitive, its design is often based on trial and error and is not always optimal. It is also hard to theoretically prove the stability of the system with a fuzzy control. To tackle these issues, fuzzy control has been combined with other systems such as neural networks. The control of a robotic manipulator with a neural fuzzy controller is discussed in [7]. Another solution is to make fuzzy control adaptive by modeling the fuzzy system as an adaptor regressor model with a definition of an update algorithm based on the Lyapunov approach [8] [9]. An adaptive fuzzy control scheme is proposed in [10] for compensating the nonlinear gravity component of the manipulator dynamics. There has also been work done in combining adaptive fuzzy control with conventional minimum variance control such as sliding control [11]. These types of systems have the combined benefits of sliding control and adaptive fuzzy control. By making the fuzzy sliding control adaptive, the system in

concern is updated according to changing parameters such as the weight of the payload and friction. Such adaptive fuzzy control schemes are universal approximators, capable of modeling any continuous system within reasonable accuracy [8][13][14]. For these reasons, adaptive fuzzy sliding control is emerging as a popular control scheme for nonlinear systems. Sun et al [15][16] apply a fuzzy system to approximate the system dynamics of a robotic manipulator. There is a complicated term in their control input to provide the robust control. The discontinuous term $\text{sgn}(s)$ still exists in the control input. Xu et al [17] apply Takagi-Sugeno type fuzzy systems to estimate the system dynamics only. As is asserted in [8], this type of fuzzy systems may not provide a natural framework to represent human knowledge.

In this paper, an adaptive fuzzy sliding control is proposed and applied to a three-link passive manipulator. The motivation for the design of this controller is to eliminate the chattering and the requirement of pre-knowledge on bounds of error associated with the conventional sliding control. This approach is natural considering the complicated model of the passive manipulator, the presence of disturbances and the possibility of changes in the system parameters. The idea is to model the disturbances according to rules based on human knowledge about the relation between the disturbances and some measurable states of the system. These rules are constructed by observations in different kinds of control schemes such as computed torque control or conventional sliding control. Once the value of the disturbance is calculated, it is fed back to the system to cancel the actual disturbance.

2. Mathematical Model and Procedure to Control a Three-Link Passive Robotic Manipulator

The passive robotic manipulator in concern has three planar links of lengths l_1 , l_2 and l_3 , respectively. The first joint is equipped with an actuator, which is the only driving mechanism for all the three links. The second and the third joints are equipped with brakes of masses m_1 and m_2 , respectively. These brakes help to lock the corresponding links into position. An end-effector of mass m_3 is attached to the end of the third link.

The dynamics equation of the three-link passive robotic manipulator is

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ 0 \\ 0 \end{bmatrix} \quad (2.1)$$

since the torques of the second and third joints, τ_2 and τ_3 , are 0 when they are unactuated. $[c_1 \ c_2 \ c_3]^T$ represents the sum of the Coriolis/centripetal vector and the gravity vector. \ddot{q}_1, \ddot{q}_2 and \ddot{q}_3 represent the acceleration of the first, second and third link, respectively. Since there is only one

actuator in the above manipulator, it is possible to control utmost one unactuated link at a time [2].

To control the passive link 3, the passive joint 2 should be locked at its current position, i.e.,

$$\dot{q}_2 = \ddot{q}_2 = 0 \quad (2.2)$$

The relationship between the acceleration of link 3 and the torque supplied at joint 1, with joint 2 locked into position, through mechanical coupling can be derived as follows. Substituting (2.2) into (2.1) yields

$$\ddot{q}_1 = \frac{m_{33}}{m_{31}} \ddot{q}_3 - \frac{c_3}{m_{31}} \quad (2.3)$$

Substituting (2.3) into (2.1) yields

$$\ddot{q}_3 = \left(m_{13} - \frac{m_{33}}{m_{31}} m_{11} \right)^{-1} \left(\tau_1 - c_1 + \frac{m_{11}}{m_{31}} c_3 \right) \quad (2.4)$$

(2.4) gives the relation between the acceleration of link 3 and the torque provided by the actuator at link 1. Using (2.4), link 3 can be controlled.

To control the passive link 2, the passive joint 3 should be locked at its current position, i.e.,

$$\dot{q}_3 = \ddot{q}_3 = 0 \quad (2.5)$$

Substituting (2.5) into (2.1) yields

$$\ddot{q}_1 = \frac{m_{22}}{m_{21}} \ddot{q}_2 - \frac{c_2}{m_{21}} \quad (2.6)$$

Substituting (2.6) into (2.1) yields

$$\ddot{q}_2 = \left(m_{12} - \frac{m_{11} m_{22}}{m_{21}} \right)^{-1} \left(\tau_1 - c_1 + \frac{m_{11}}{m_{21}} c_2 \right) \quad (2.7)$$

Using (2.7), link 2 can be controlled by the torque provided by the actuator at link 1.

To control link 1, joints 2 and 3 should be locked at their current positions, thereby giving

$$\dot{q}_2 = \ddot{q}_2 = \dot{q}_3 = \ddot{q}_3 = 0 \quad (2.8)$$

Substituting (2.8) into (2.1) yields

$$\ddot{q}_1 = \frac{\tau_1 - c_1}{m_{11}} \quad (2.9)$$

Using (2.9), link 1 can be controlled.

3. Adaptive Fuzzy Sliding Control for a Passive Robotic Manipulator

In this section, an adaptive fuzzy sliding control scheme, which gets triggered whenever the plant deviates from the sliding surface pushing the system towards the sliding surface, is proposed. As is known, a fuzzy system can be made to be adaptive to maintain consistent performance in situations where there is large uncertainty or unknown variations in the plant parameters.

The dynamics equation of the three-link passive manipulator discussed in (2.1) is rewritten as

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = - \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}^{-1} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}^{-1} \begin{bmatrix} \tau_1 \\ 0 \\ 0 \end{bmatrix} \quad (3.1)$$

which is of the form

$$\ddot{x} = f(\bar{x}) + b(\bar{x})u \quad (3.2)$$

where

$$f(\bar{x}) = - \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}^{-1} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

and

$$b(\bar{x}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}^{-1}$$

where $q \in \mathbb{R}^n$ represents the joint positions of the system and $u \in \mathbb{R}$ is the control input. Thus, the above system is a 2nd-order system and the sliding surface $S = \dot{e} + e$ where $e = x - x_d$.

Consider the system represented in (3.2) along with disturbances. The disturbances could be a combination of structured and unstructured uncertainties. Structured or parametric uncertainties represent the inaccuracies in the terms in the system model like payload mass, while unstructured uncertainties or unmodeled dynamics represent inaccuracies in the estimation of the system order. Such a system can be represented by the following differential equation

$$\ddot{x} = \hat{f}(\bar{x}) + \Delta f(\bar{x}) + \hat{b}(\bar{x})[u(t) + h(\bar{x})] \quad (3.3)$$

where $\ddot{x} = \frac{d^2x}{dt^2}$, $\hat{f}(\bar{x})$ and $\hat{b}(\bar{x})$ are the nominal values of

$f(\bar{x})$ and $b(\bar{x})$, respectively. $\Delta f(\bar{x})$ represents the uncertainties in $f(\bar{x})$, $u(t)$ represents the input and $h(\bar{x})$ represents the disturbances and unmodeled dynamics of the input. Assume that there exists $z(\bar{x})$ such that

$$\Delta f(\bar{x}) = \hat{b}(\bar{x})z(\bar{x}) \quad (3.4)$$

Then we can rewrite (3.3) as

$$\ddot{x} = \hat{f}(\bar{x}) + \hat{b}(\bar{x})u(t) + d(\bar{x}) \quad (3.5)$$

where

$$d(\bar{x}) = \hat{b}(\bar{x})[z(\bar{x}) + h(\bar{x})]$$

According to [8], for a given $\varepsilon \geq 0$, there exists a fuzzy logic system $d_{fz}(\bar{x})$ such that

$$\sup_{\bar{x} \in U} |d(\bar{x}) - d_{fz}(\bar{x})| \leq \varepsilon \quad (3.6)$$

where ε is known as the minimum approximation error [3]. If sufficient number of rules are used to construct $u(t)$, ε is

proven to be small based on the universal approximation theorem in [12] and [13]. In the following derivations it is assumed that its value is specified. Due to (3.6), there exists a fuzzy logic system for the given S

$$\begin{aligned} d_{fz}(\bar{x}) &= \sum_{i=1}^M \theta_i \phi_i(\bar{x}) \\ &= \theta^T \phi(\bar{x}) \end{aligned} \quad (3.7)$$

such that

$$\begin{aligned} d(\bar{x}) &= \sum_{i=1}^M \theta_i \phi_i(\bar{x}) + D(\bar{x}) \\ &= \theta^T \phi(\bar{x}) + D(\bar{x}) \end{aligned} \quad (3.8)$$

where $|D(\bar{x})| < \varepsilon$. $\theta^T = [\theta_1 \dots \theta_M]$ is a parameter vector and $\phi^T = [\phi_1 \dots \phi_M]$ is a regressor vector.

A controller u_{re} , which does not have any chattering and drives the system trajectories to the reaching condition described by

$$\frac{1}{2} \frac{d}{dt} S^2 = S^T \dot{S} \leq -\eta |S|$$

, needs to be designed.

In order to achieve the above sliding condition we propose a control scheme which combines the conventional sliding control with a fuzzy system as follows

$$u_{re} = \hat{b}(\bar{x})^{-1} (-\hat{f}(\bar{x}) + \ddot{x}_d - \dot{e} - \hat{d}_{fz}(\bar{x}) - k \text{sgn}(S)) \quad (3.9)$$

where

$$\begin{aligned} \hat{d}_{fz}(\bar{x}) &= \sum_{i=1}^M \hat{\theta}_i \phi_i(\bar{x}) \\ &= \hat{\theta}^T \phi(\bar{x}) \end{aligned} \quad (3.10)$$

which is the fuzzy estimate of $d_{fz}(\bar{x})$ in (3.7), is obtained

$$\text{by considering } \frac{\prod_{i=1}^n \mu_{A_i^l(x_i)}}{\sum_{l=1}^M \prod_{i=1}^n \mu_{A_i^l(x_i)}} \text{ as the regressor } \phi_i(\bar{x}) \text{ and}$$

\bar{y}^l as the parameter $\hat{\theta}_i$. This is because, as is well known to fuzzy researchers, a fuzzy system consisting of a singleton fuzzifier, a product-operation inference with algebraic product t-norm and a center-average defuzzifier yields a crisp output [8]

$$y^* = \frac{\sum_{l=1}^M \bar{y}^l \left(\prod_{i=1}^n \mu_{A_i^l(x_i)} \right)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{A_i^l(x_i)} \right)} \quad (3.11)$$

where $\mu_{A_i^l}$ is the fuzzy set in the output space and

$y = \bar{y}^l$ is the center of $\mu_{A_i^l}$. Also, we specify the update

rule of $\hat{\theta}_i$ as

$$\dot{\hat{\theta}}_i = \lambda S^T \phi(\bar{x}) \quad (3.12)$$

with $\lambda > 0$, and $k = \varepsilon + \eta$ where ε is the minimum approximation error, which is a very small value and is assumed to be known.

The overall system consisting of the adaptive fuzzy sliding controller and the passive robotic manipulator is given in Figure 1.

4. Stability Proof for Adaptive Fuzzy Sliding Control

In this section, the asymptotic stability of the system is proved by LaSalle's theorem with the use of a Lyapunov function.

LaSalle's Theorem: Given the autonomous nonlinear system

$$\ddot{x} = f(\bar{x}), \quad \bar{x}(0) = x_0 \quad (4.1)$$

with the equilibrium at the origin, then:

Asymptotic Stability: Suppose that a Lyapunov function $V(\bar{x})$ has been found such that for $\bar{x} \in N \subset R^n$, $V(\bar{x}) > 0$ and $\dot{V}(\bar{x}) \leq 0$, then the origin is asymptotically stable if and only if $\dot{V}(\bar{x}) = 0$ only at $\bar{x} = 0$.

Consider a Lyapunov function

$$V = \frac{1}{2} S^T S + \frac{1}{2\lambda} \tilde{\theta}^T \tilde{\theta} \quad (4.2)$$

where $\tilde{\theta} = \hat{\theta} - \theta$ is the difference between the nominal parameter vector $\hat{\theta}$ in (3.10) and the actual parameter vector θ in (3.7). The derivative of (4.2) is given by

$$\dot{V} = S^T \dot{S} + \frac{1}{\lambda} \tilde{\theta}^T \dot{\tilde{\theta}} \quad (4.3)$$

since $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$. A necessary condition for the system state trajectory to remain on the surface S is

$$\dot{S} = \dot{e} + \dot{e} = f(\bar{x}) + b(\bar{x})u - \ddot{x}_d + \dot{e} = 0 \quad (4.4)$$

Applying (3.5) to the equation of \dot{S} in (4.4) and putting the \dot{S} into (4.3), we have

$$\dot{V} = S^T \left[\hat{f}(\bar{x}) + \hat{b}(\bar{x})u + d(\bar{x}) - \ddot{x}_d + \dot{e} \right] + \frac{1}{\lambda} \tilde{\theta}^T \dot{\tilde{\theta}}. \quad (4.5)$$

Applying the input function in (3.9) to (4.5) and simplifying the result leads to

$$\dot{V} = S^T (d(\bar{x}) - \hat{d}_{fz}(\bar{x}) - k \operatorname{sgn}(S)) + \frac{1}{\lambda} \tilde{\theta}^T \dot{\tilde{\theta}} \quad (4.6)$$

Applying (3.8) and (3.10) to (4.6) leads to

$$\dot{V} = S^T \left[-\tilde{\theta}^T \phi(\bar{x}) + D(\bar{x}) - k \operatorname{sgn}(S) \right] + \frac{1}{\lambda} \tilde{\theta}^T \dot{\tilde{\theta}}. \quad (4.7)$$

Applying the parameter update rule in (3.12) to (4.7) leads to

$$\dot{V} = S^T (D(\bar{x}) - k \operatorname{sgn}(S)) \leq |S| (|D(\bar{x})| - k) \quad (4.8)$$

Due to the fact $k = \varepsilon + \eta$, we have

$$\dot{V} \leq -\eta |S| \quad (4.9)$$

Hence, it is seen that \dot{V} is negative semi-definite, becoming zero only when $S = 0$. From the definition of S , it is seen that $S = 0$ implies that $e = 0$ and $\dot{e} = 0$. In [6] it

is proven that given the results of the Lyapunov analysis above, $\lim_{t \rightarrow \infty} S = \lim_{t \rightarrow \infty} (e + \dot{e}) = 0$. Thus, the overall system is asymptotically stable.

5. Simulations and Discussions

The adaptive fuzzy sliding control scheme proposed in Section 3 is applied to control the three-link passive manipulator with two unactuated joints, whose mathematical model is derived in Section 2. The performance and stability of the system are verified by simulations. The simulation results are validated against those obtained for a conventional sliding controller. The parameters of the length and mass of the links are

Length of link1 - 1.5 (m)

Length of link2 - 1.3 (m)

Length of link3 - 1.2 (m)

Mass of brake1 - 1 (Kg)

Mass of brake2 - .75 (Kg)

Mass of end-effector - .65 (Kg)

Both structured and unstructured uncertainties are applied to the system. The structured uncertainty consists of a payload change from 0.65Kg to 2.5Kg at time $t = 12$ sec. The unstructured uncertainty represents dynamic friction, random disturbance and possible unmodelled high frequency components of the dynamics. The plant model with parametric inaccuracies as given in (3.5) is

$$\ddot{x} = \hat{f}(\bar{x}) + \hat{b}(\bar{x})u(t) + d(\bar{x}) \quad (5.1)$$

By assuming the actual model parameters as

$$\hat{f}(\bar{x}) = (1 + 0.9 \sin(2\pi/3)) \hat{f}(\bar{x}) \quad (5.2)$$

$$\hat{b}(\bar{x}) = (1 + 0.2 \sin(2\pi/3)) \hat{b}(\bar{x}) \quad (5.3)$$

the disturbance $d(\bar{x})$ is given by

$$d(\bar{x}) = 0.9 \sin(2\pi/3) \hat{f}(\bar{x}) + 0.2 \sin(2\pi/3) \hat{b}(\bar{x})u(t), \quad (5.4)$$

which is changing with respect to time. In our simulations, the third link is expected to track a sinusoidal trajectory in the presence of these uncertainties. An initial error of 1 rad is assumed for the third link. A 4th-order Runge Kutta integration is used to simulate the three-link passive manipulator. The software is written in Matlab.

The adaptive fuzzy sliding control proposed in Section 3 is applied to control the three-link passive robotic manipulator. The simulation results are shown in Figure 2 - Figure 5. The parameter vector of the adaptive fuzzy sliding controller introduced in (3.8) representing the points where the output membership functions attain their maximum values of 1 is assumed to have the initial value

$\hat{\theta}^T = [-0.3003, -0.3721, -0.0887, 1.3105, 7.2130]$, which is updated during the course of the simulation by using the update rule (3.12) with the update constant $\lambda = 0.07$.

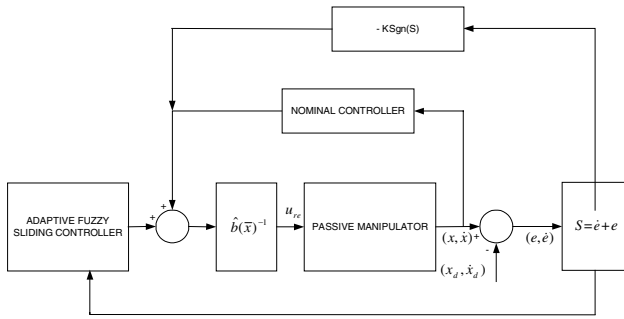


Figure 1 Overall system architecture

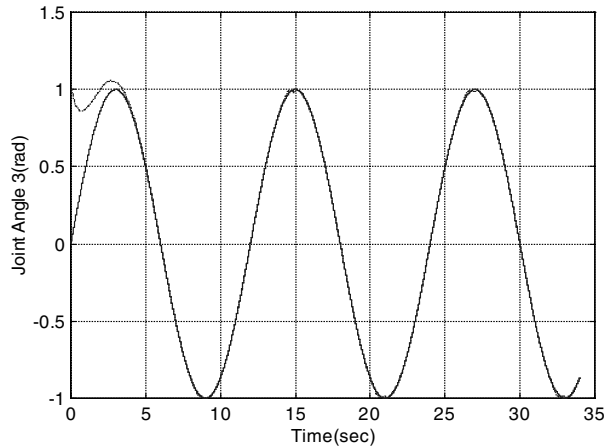


Figure 2 Dynamic tracking of joint 3 by AFSC with payload change at time=12sec and unstructured uncertainty

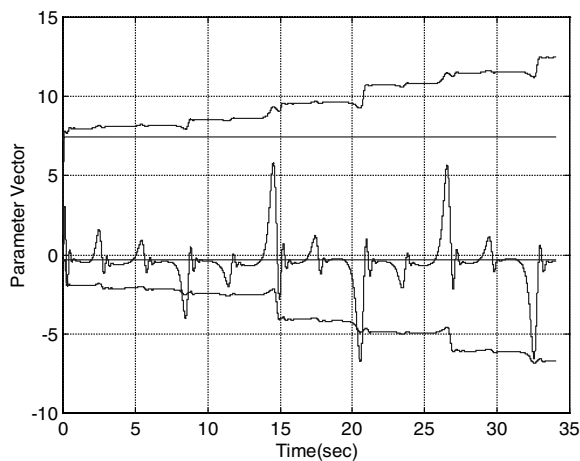


Figure 3 Parameter vector update for dynamic tracking of joint 3 by AFSC with payload change and unstructured uncertainty

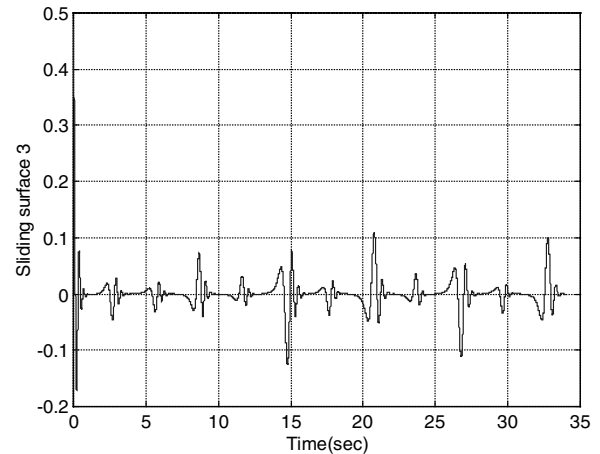


Figure 4 Sliding surface for dynamic tracking of joint 3 by AFSC with payload change and unstructured uncertainty

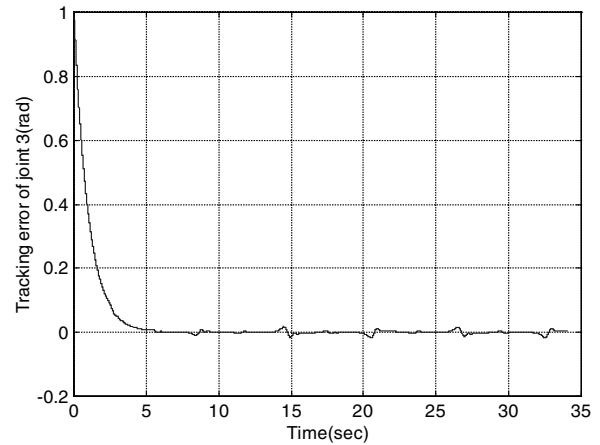


Figure 5 Tracking error for dynamic tracking of joint 3 by AFSC with payload change and unstructured uncertainty

During the simulation it is found that the system performs satisfactorily. It is observed, as mentioned in [6], that the order of parameter update is dependant on the value of S . For example, when the initial value of S is large, the center of membership function PositiveLarge gets updated first, and Positive and Zero get updated only when S is reduced. This may lead to slower convergence. However, this behavior can be modified to some extent by choosing a larger support for input membership functions Negative, Zero and Positive, i.e., modifying the membership functions to include larger ranges of S .

6. Conclusions

In this paper, an adaptive fuzzy sliding control scheme is proposed to control a passive robotic manipulator. The motivation for the design of the adaptive fuzzy sliding controller is to eliminate the chattering and the requirement of pre-knowledge on bounds of error associated with the

conventional sliding control. The stability and convergence of the adaptive fuzzy sliding controller is proven both theoretically and practically by simulations. A three-link passive manipulator with two unactuated joints is simulated. Simulation results demonstrate that the proposed system is robust against structured and unstructured uncertainties. Satisfactory tracking of the desired trajectory is achieved and the chattering associated with the conventional sliding control is eliminated. The contributions of this work include

1. Proposal of an adaptive fuzzy sliding control scheme to control a passive robotic manipulator. The proposed controller does not require an accurate model of the manipulator, which simplifies its implementation, and is capable of adapting itself to varying system parameters and disturbances.
2. Mathematical proof of the stability of the proposed control on a passive robotic manipulator by the Lyapunov method.
3. Practical validation of the stability and convergence of the overall system through simulation.

This work assumes that the passive joints are equipped with brakes. In many practical situations, however, the joints are not necessarily mounted with brakes. Controlling such passive manipulators is an interesting area of study.

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