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Variable Structure Robust Fin Control for Ship Roll Stabilization with Actuator System

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***Abstract*— The ship roll stabilization by fin control system with actuator is considered in this paper. Assuming that there exist uncertain parameters and uncertain external perturbations in the ship roll model, a variable structure robust control algorithm for ship roll stabilization system is presented for a class of uncertain systems with the absence of matching assumption in which the uncertainty is not acted within channels implicit in the control inputs. A simulation example illustrating the method described is included in this paper. It is shown that it makes the designed system guarantee the performance of robustness with respect to the perturbations and uncertainties.**

I. INTRODUCTION

Ship excessive roll motion induced by wave disturbances would make the crew feel uncomfortable and may also cause damage to the cargoes and equipment on board, such that the stabilization of ship roll motion has been a goal that people always strive to achieve. The fin stabilizer, which is a hull stability equipment for reduction of ship rolling by using the generating lift of the fins extended to the both sides of a ship, was invented 60 years ago and began to be equipped on the ship and showed good performance [1]. As we all know, a fin stabilizer is a kind of active stabilization system, the performance of which is effected greatly by its control methodology. To achieve better performance, its advanced control scheme has received considerable attention. From 1970s, some advanced control schemes are put into practices, such as optimal control [2], fuzzy logic control [3], self-organizing fuzzy control [4], adaptive LQ control [5], H_∞ control [6], internal model control [7] and etc. However, there exists nonlinearity, parametric uncertainty and environmental disturbances in the ship roll nonlinear system from the changing sea conditions. To handle those problems, the author has ever proposed several robust adaptive fuzzy control schemes [8],[9]. Therefore, developing the control scheme with large robustness is of much interest in the research field of fin roll stabilization systems.

In fin roll stabilization system, the controller's output drives the controller components through the device which is called as a process actuator, as shown in Fig. 1. The actuator is composed of electrical-hydraulic system in the ship roll fin stabilization system. Generally speaking, actual fin angle has a time-delay compared with order fin angle, such that step response is unlikely. Generally, the actuator can be described by use of 1-order inertia dynamical system given by

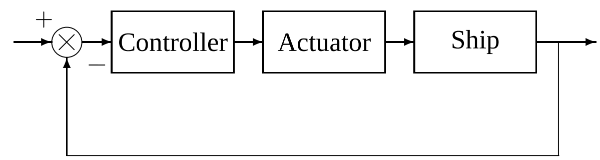

Fig. 1. Ship roll stabilization system with actuator

$$T_E\dot{\alpha}+\alpha = K_E\alpha_C \tag{1}$$

where α_C is order fin angle (i.e. controller's output), α is actual fin angle, T_E is the actuator's time constant, K_E is input gain.

As for the actuator, the smaller time constant T_E is, the less its influence on closed-loop system is. But, the magnitude of T_E is inversely proportional to actuator's driving power. Hence, the decrease of T_E will definitely increase the driving power, which is proportional to the cost of device and its volume. So the actuator's T_E is unlikely to be arbitrarily small and have a certain value.

Generally speaking, when the actuator's T_E is greatly less than the controlled-process's time constant, the actuator's influence may not be considered in the control system design. But when both are similar, neglecting the fin actuator's T_E existence may not only affect the closed-loop's performance, but also destroy its stability. In ship's fin roll stabilization system design, actuator's T_E was regarded as small value and could be negligible in the past. But, after our research, T_E can not be neglected, because in T_E's effective range, the fin stabilizer by use of conventional methodology may have no capacity of roll stabilization, even aggravate ship's roll amplitude and cause instability. Those will be illustrated through simulation example in this paper. If we take the fin actuator into the design consideration, the active anti-rolling fin stabilizer will be actually a mismatching uncertain system, because the initial control system's parametric uncertainty and external disturbance no longer take place in control channel. In this paper, by use of the robust switching-function variable structure control theory of uncertain linear system [10], a variable structure robust controller is designed, based on fin roll stabilizer's mismatching uncertain model. Simulation research shows

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that the adopted controller has closed-loop's stability unaffected by the actuator's T_E, and the performance is also improved.

This paper is organized as follows. In section II, we will design variable structure control for mismatching uncertain system. Section III contains the description of mismatching uncertain system model of ship fin stabilization with actuator. In section IV, a systematic procedure for the synthesis of variable structure robust controller for ship's fin stabilization system with actuator is developed. In section V, we demonstrate how the adaptive robust fuzzy control scheme can be applied to the controller design for ship roll stabilization and a container ship is used as an example for simulation. The simulation results are described and compared. The final section contains conclusions.

II. VARIABLE STRUCTURE ROBUST CONTROL OF MISMATCHING UNCERTAIN SYSTEMS

A. Description of Mismatching Uncertain Systems

Consider the following system

$$\begin{cases} \dot{X}(t) = [A+\Delta A(\sigma(t))]X(t) + [B+\Delta B(\sigma(t))]u(t) \\ \qquad +Cv(t) \\ X(t_0) = X_0 \end{cases} \tag{2}$$

where $t \in R^+$, $X(t) \in R^n$ is state vector, $u(t) \in R^m$ is input vector, $\sigma(t) \in \Omega \subset R^p$ is the uncertainty vector of model's parameter, $v(t) \in R^l$ is input noise vector with unit amplitude. A and B are known system matrix and control matrix, $\Delta A(\sigma(t))$ is the uncertainty matrix, $\Delta B(\sigma(t))$ is input uncertainty matrix, either of which depend on the uncertainty parameters' continuous function matrix.

Throughout the paper, the following assumptions are made at first.

Assumption 1: Uncertain parameter $\sigma(t) \in \Omega \subset R^p$, uncertain input noise $v(t) \in \Upsilon \subset R^l$, Ω and Υ are compact sets that lie in R^p and R^p, respectively. All of them are Lebesgue measurable.

Assumption 2: Nominal system (A,B) satisfies the system's controllability condition.

We introduce an orthogonal transformation matrix $\tilde{T} \in R^{n\times n}$, and partition (2). Furthermore, $\tilde{T}$ satisfies the following conditions

$$\tilde{T}B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{3}$$

where $B_2 \in R^{m\times m}$.

We define

$$z = \tilde{T}X = \begin{bmatrix} T \\ T^1 \end{bmatrix} X = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{4}$$

where $z_1 \in R^{n-m}$, $z_2 \in R^m$, and T, T^1 are transformation matrix with appropriate dimension. Then the system (2) can be transformed into

$$\begin{cases} \dot{z}_1 = A_{11}z_1 + A_{12}z_2 + \Delta A_{11}z_1 + \Delta A_{12}z_2 + C_1v(t) \\ \dot{z}_2 = A_{21}z_1 + A_{22}z_2 + B_2u + \Delta A_{21}z_1 + \Delta A_{22}z_2 \\ \qquad +\Delta B_2u + C_2v(t) \end{cases} \tag{5}$$

where $\tilde{T}A\tilde{T}^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $\tilde{T}\Delta B = \begin{bmatrix} 0 \\ \Delta B_2 \end{bmatrix}$,

$$\tilde{T}\Delta A\tilde{T}^T = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix}, \quad C_1 = TC, \quad C_2 = T^1C.$$

Assumption 3: For (5), we introduce the matching condition, i.e. there exist two matrix functions $h_{11}(\cdot)$, $E_1(\cdot)$ and a constant matrix F_1 satisfying

$$\begin{cases} \Delta A_{11}(\sigma(t)) = A_{12}h_{11}(\sigma(t)) \\ \Delta A_{12}(\sigma(t)) = A_{12}E_1(\sigma(t)) \\ C_1 = A_{12}F_1 \end{cases}$$

For the second one of (5), B_2 is reversible, and can satisfy the matching condition, i.e. there are 3 matrix functions $h_{21}(\cdot)$,$h_{22}(\cdot)$,$E_2(\cdot)$ and a constant matrix F_2 with appropriate dimension satisfying

$$\begin{cases} \Delta A_{21}(s(t)) = B_2h_{21}(\sigma(t)) \\ \Delta A_{22}(s(t)) = B_2h_{22}(\sigma(t)) \\ \Delta B_2 = B_2E_2(\sigma(t)) \\ C_2 = B_2F_2 \end{cases}$$

where $h_{21}(\cdot) = B_2^{-1}\Delta A_{21}(\cdot)$, $E_2(\cdot) = B_2^{-1}\Delta B_2(\cdot)$, $F_2 = B_2^{-1}C_2$. Then the system (5) can be converted into

$$\begin{cases} \dot{z}_1 = A_{11}z_1 + A_{12}\left[(I+E_1)z_2 + \eta_1\right] \\ \dot{z}_2 = A_{21}z_1 + A_{22}z_2 + B_2\left[(I+E_2)u + \eta_2\right] \end{cases} \tag{6}$$

where

$$\eta_1(z_1(t),z_2(t),\sigma(t),v(t)) = h_{11}(\sigma(t))z_1(t) \\ + F_1v(t) + h_{22}(\sigma(t))z_2(t) + F_2v(t),$$

$$\eta_2(z_1(t),z_2(t),\sigma(t),v(t)) = h_{21}(\sigma(t))z_1(t) \\ + h_{22}(\sigma(t))z_2(t) + E_2u(t) + F_2v(t).$$

To meet the requirements of the robust control design, we introduce the following assumption on the uncertainty's bound.

Assumption 4:

$$\begin{cases} \|\eta_1(z_1(t),z_2(t),\sigma(t),v(t))\| \le k_{11}\|z_1(t)\| \\ \qquad +k_{12}\|z_2(t)\| + k_{13} \\ \|\eta_2(z_1(t),z_2(t),\sigma(t),v(t))\| \le k_{21}\|z_1(t)\| \\ \qquad +k_{22}\|z_2(t)\| + k_u\|u(t)\| + k_{23} \end{cases} \tag{7}$$

where $k_{11} = \max_{\sigma\in\Omega}\|h_{11}(\sigma(t))\|$, $k_{12} = \max_{\sigma\in\Omega}\|h_{12}(\sigma(t))\| < 1$, $k_{13} = \max_{v\in\Upsilon}\|F_1v(t)\|$, $k_{21} = \max_{\sigma\in\Omega}\|h_{21}(\sigma(t))\|$, $k_{22} = \max_{\sigma\in\Omega}\|h_{22}(\sigma(t))\|$, $k_{23} = \max_{v\in\Upsilon}\|F_2v(t)\|$ and $k_u = \max_{\sigma\in\Omega}\|E_2(\sigma(t))\| < 1$.

Here, $\|\Xi\|$ employs Euclidean modulus if Ξ is a vector, and $\|\Xi\|$ employs matrix modulus if Ξ is a matrix. The employed modulus is $\|\Xi\| = [\lambda_M(\Xi^T\Xi)]^{\frac{1}{2}}$, where $\lambda_M(\cdot)(\lambda_m(\cdot))$ is the matrix's maximal or minimal eigenvalue.

B. Design of Variable Structure Robust Controller

This paper focuses on regulator problem, which is to design a bounded control scheme $u(t)$ driving the state $X(t)$ to transfer from the initial state X_0 to zero state, i.e. $\lim_{t\to\infty} X(t)=0$. According to the above orthogonal transformation, the problem is converted into that new state variable $z(t)$ moves from the initial state z_0 to zero, i.e. $\lim_{t\to\infty} z(t)=0$. Based on system (6), a variable structure robust controller can be designed, which follows two steps:

(1)choose sliding hyperplane function;

(2)drive the state to move to the hyperplane, keep them staying on the hyperplane, and move asymptotically to equilibrium along the plane.

1) Design of the robust sliding hyperplane function:: As (A,B) is controllable, (A_{11},A_{12}), the matrices after orthogonal transformation is also controllable [10]. Define that $\rho_1 \in R$, $Q=Q^T \in R^{n\times n}$, and $Q\geq 0$, then there exists a real symmetric positive definite matrix P_1 satisfying the algebraic Riccati equation

$$A_{11}^T P_1 + P_1 A_{11} - \rho_1 P_1 A_{12} A_{12}^T P_1 + Q_1 = 0 \tag{8}$$

where ρ_1 and Q are specified by designer.

Choose switching function as

$$S(z_1(t),z_2(t),t) = z_2(t) + \frac{1}{2}(\rho_1+\gamma_1)A_{12}^T P_1 z_1(t) \tag{9}$$

which is robust because the parameter γ_1 is a constant related to the bound of uncertain items. γ_1 is chosen by

$$\gamma_1 \geq (1-k_{12})^{-1} k_{12}\rho_1 + \delta_1 + \delta_2 + \overline{\gamma} \tag{10}$$

where $\overline{\gamma}$ is the controller's parameter specified by designer, and

$$\delta_1 \geq k_{11}^2 \big/ (\lambda_{\min}(Q_1) + \lambda_{\max}(P_1))$$

$$\delta_2 \geq (1+k_{12})^2 \Big/ (1+\lambda_{\max}(P_2))$$

2) Design of variable structure robust controller:: Differentiating (9) with respect to time t and substituting it into (6), we get

$$\dot{S} = \Phi S + \Omega z_1 + B_2[u + \tilde{\eta}_2] \tag{11}$$

where $K=\frac{1}{2}(\rho_1+\gamma_1)A_{12}^T P_1$, $\Sigma = A_{11}-A_{12}K$, $\Phi = A_{22}+KA_{12}$, $\Omega = A_{21}-A_{22}K+K\Sigma$ and $\tilde{\eta}_2 = B_2^{-1}K\eta_1+\eta_2$.

Choose control law as follows

$$u = u_L + u_N. \tag{12}$$

which is combined with linear part and nonlinear part.

Ryan and Corless [11] method is used to meet the reachable condition, which chooses a matrix $\Phi^*(m\times m)$ having eigenvalues in the left half plane. Especially, when we define $\{\mu_i : Re(\mu_i)<0, i=1,2,\cdots,m\}$, we have $\Phi^* = \mathrm{diag}\{\mu_i : i=1,2,\cdots,m\}$.

Choose the linear part u_L as

$$u_L = -B_2^{-1}[\Omega z_1 + (\Phi-\Phi^*)S] \tag{13}$$

then (11) will be

$$\dot{S} = \Phi^* S + B_2[u_N + \tilde{\eta}_2] \tag{14}$$

As Φ^* is of the eigenvalues in the left half plane, there exists a systematic positive definite matrix P_2 satisfying the Lyapunov equation

$$P_2\Phi^* + \Phi^{*T}P_2 + I_{m\times m} = 0 \tag{15}$$

Hence, the nonlinear part u_N in (12) is

$$u_N = -\rho(1-k_u)^{-1} B_2^{-1} P_2 S \big/ (\|P_2 S\| + \varepsilon) \tag{16}$$

where $\varepsilon > 0$ is specified by designer, and

$$\rho \geq \left(kk_{11} + k^2 k_{12} + k_{21} + kk_{22} + k_u k_{L1}\right)\|z_1\| + (k_{22}+kk_{12}+k_u k_{L2})\|S\| + (k_{23}+kk_{13}),$$

$$k = \left\|B_2^{-1}K\right\| = \frac{1}{2}(\rho_1+\gamma_1)\left\|B_2^{-1}\right\|\left\|A_{12}^T P_1\right\|,$$

$$k_{L1} = \left\|B_2^{-1}\Omega\right\| k_{L2} = \left\|B_2^{-1}(\Phi-\Phi^*)\right\|.$$

Theorem 1: For the mismatching uncertain linear system (2), under the condition of orthogonal transformation matrix $\tilde{T} \in R^{n\times n}$ and meeting Assumption 1∼4, system's sliding mode hyperplane (9) is globally reachable and a motion on the hyperplane is globally ultimately bounded, and we choose (12) as the variable structure robust control law, then the system (2) with mismatching uncertainty is globally practically stabilizable.

Proof: See [10].

Remark 1: The closed-loop system which is globally practically stable means that, its track will be uniformly ultimately bounded and converge to closed-ball region $B(r)$ containing equilibrium point. The radius r of which is specified by the controller's parameters and can be arbitrarily small by reasonable choice of the parameters $\overline{\gamma}$ and ε.

III. DESCRIPTION OF MISMATCHING UNCERTAIN SYSTEM MODEL OF SHIP FIN

In this paper, uncertainties of parameters and environmental disturbances are taken into consideration in ship roll stabilization mathematical model, which are originated from the variation of ship speed and stability height. The following model also includes the influence of actuator implementation device [12].

$$\begin{cases} (I_{xx}+J_{xx})\ddot{\theta} + 2(N_{\theta 0}+\Delta N_\theta)\dot{\theta} + W(h_0+\Delta h)\theta \\ \quad = F_\alpha + F_W \sin\omega_e t \\ (T_{E0}+\Delta T_E)\dot{\alpha} + \alpha = (K_{E0}+\Delta K_E)u \end{cases} \tag{17}$$

where $\theta, \dot{\theta}, \ddot{\theta}$ denote the ship roll angle, angular rate, and angular acceleration, respectively. $F_\alpha = -\rho(V_0+\Delta V)^2 A_F C_{L\alpha} l_F \alpha$, and ρ is the density of sea water. $N_{\theta 0}$ denotes ship linear damping coefficient wihch is initial parameter value. h_0 is the initial transverse metacentric height and V_0 is initial ship velocity. ΔN_θ, Δh, ΔV are the variation of the parameters resulted by ship loaded conditions, external environment and other reasons.

F_W is the external wave amplitude and ω_e is the externa wave frequency met with by ship when sailing at sea. ΔT_E and ΔK_E are the variations of time constant and input gain of the actuator. $u=\alpha_c$ is the controller output.

Choose state variables $X_1=\theta$, $X_2=\dot{\theta}$ and $X_3=\alpha$, and separate the uncertain items from certain items.

$$\dot{X}(t)=[A+\Delta A(\sigma)]X(t)+[B+\Delta B(\sigma)]u(t)+Cv(t) \quad (18)$$

where $A=\begin{bmatrix} 0 & 1 & 0 \\ -a_1 & -a_2 & a_3 \\ 0 & 0 & -a_4 \end{bmatrix}$, $B=\begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$,

$$\Delta A(\sigma)=\begin{bmatrix} 0 & 0 & 0 \\ \Delta a_1 & \Delta a_2 & \Delta a_3 \\ 0 & 0 & \Delta a_4 \end{bmatrix}, \quad \Delta B(\sigma)=\begin{bmatrix} 0 \\ 0 \\ \Delta b \end{bmatrix},$$

$$C=\begin{bmatrix} 0 \\ c_1 \\ 0 \end{bmatrix}, \quad a_1=\frac{Wh_0}{I_{xx}+J_{xx}}, \quad a_2=\frac{2N_{\theta 0}}{I_{xx}+J_{xx}},$$

$$a_3=-\frac{1}{(I_{xx}+J_{xx})}\rho V_0^2 A_F l_F C_{L\alpha},$$

$$a_4=\frac{1}{T_{E0}}, \quad b=\frac{K_{E0}}{T_{E0}}, \quad \Delta a_1=\frac{W\Delta h}{I_{xx}+J_{xx}},$$

$$\Delta a_2=a_2\left\{\left(\sqrt{1+\frac{\Delta h}{h_0}}-1\right)+\frac{\Delta\mu_\theta}{\mu_{\theta 0}} +\frac{\Delta\mu_\theta}{\mu_{\theta 0}}\left(\sqrt{1+\frac{\Delta h}{h_0}}-1\right)\right\},$$

$$\Delta a_3=a_3\left(2\frac{\Delta V}{V_0}+\left(\frac{\Delta V}{V_0}\right)^2\right), \quad \Delta a_4=\frac{\Delta T_E}{T_{E0}(T_{E0}+\Delta T_E)},$$

$$\Delta b=\frac{T_{E0}\Delta K_E-K_{E0}\Delta T_E}{T_{E0}(T_{E0}+\Delta T_E)}, \quad c_1=\frac{F_W}{I_{xx}+J_{xx}}v(t)=\sin\omega_e t,$$

$$\mu_{\theta 0}=\mu_\theta(0)\left(1+3.3V_0\big/\sqrt{gL}\right),$$

$$\Delta\mu_\theta=\mu_\theta(0)\left(3.3\Delta V\big/\sqrt{gL}\right).$$

Equation (18) shows that uncertain matrices $\Delta A(\sigma)$ and C cannot be expressed linearly by use of matrix B. Therefore, that is a standard unmatching uncertain system.

IV. DESIGN VARIABLE STRUCTURE ROBUST CONTROLLER FOR SHIP'S FIN STABILIZATION SYSTEM

To meet the design of variable structure robust controller, (18) can be written in the form of the following partition matrix

$$\begin{cases} \dot{z}_1=A_{11}z_1+A_{12}z_2+\Delta A_{11}z_1+\Delta A_{12}z_2+C_1w(t) \\ \dot{z}_2=A_{21}z_1+A_{22}z_2+B_2u+\Delta A_{21}z_1+\Delta A_{22}z_2 \\ \quad +\Delta B_2u+C_2w(t) \end{cases} \quad (19)$$

where $z_1=\begin{bmatrix} X_1 & X_2 \end{bmatrix}^T$, $z_2=X_3$,

$$A_{11}=\begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix}, \quad A_{12}=\begin{bmatrix} 0 \\ a_3 \end{bmatrix},$$

$$\Delta A_{11}=\begin{bmatrix} 0 & 0 \\ \Delta a_1 & \Delta a_2 \end{bmatrix}, \quad \Delta A_{12}=\begin{bmatrix} 0 \\ \Delta a_3 \end{bmatrix},$$

$$C_1=\begin{bmatrix} 0 \\ c_1 \end{bmatrix}, \quad A_{21}=\begin{bmatrix} 0 & 0 \end{bmatrix}, \quad A_{22}=[-a_4],$$

$$\Delta A_{21}=\begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \Delta A_{22}=[\Delta a_4],$$

$$B_2=[b], \quad \Delta B_2=[\Delta b], \quad C_2=[0].$$

According to the assumption, the uncertain items are written into matching form

$$\Delta A_{11}=A_{12}h_{11}=A_{12}\begin{bmatrix} \Delta a_1/a_3 & \Delta a_2/a_3 \end{bmatrix},$$

$$h_{11}=\begin{bmatrix} \Delta a_1/a_3 & \Delta a_2/a_3 \end{bmatrix},$$

$$\Delta A_{12}=A_{12}h_{12}=A_{12}\left[\Delta a_3/a_3\right],$$

$$h_{12}=\Delta a_3/a_3, \quad C_1=A_{12}F_1=A_{12}\left[c_1/a_3\right],$$

$$F_1=c_1/a_3, \quad \Delta A_{21}=B_2h_{21}, \quad h_{21}=\begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$\Delta A_{22}=B_2h_{22}=B_2\left[\Delta a_4/b\right], \quad h_{22}=\Delta a_4/b,$$

$$\Delta B_2=B_2E=B_2\left[\Delta b/b\right], \quad E=\Delta b/b$$

$$C_2=B_2F_2=B_2\left[c_2/b\right], \quad F_2=0.$$

Choose $\rho_1=1/\lambda_\alpha$, $Q_1=\mathrm{diag}\begin{bmatrix} \lambda_\theta & \lambda_p \end{bmatrix}$, where λ_α, λ_θ and λ_p are the system's parameters and usually are explicitly given. Then solve algebraic Riccati equation (8) and we get

$$P_1=\begin{bmatrix} k_{11}^1 & k_{12}^1 \\ k_{21}^1 & k_{22}^1 \end{bmatrix}$$

where $k_{11}^1=a_1k_{22}^1+a_2k_{21}^1+k_{21}^1k_{22}^1a_3^2/\lambda_\alpha$

$$k_{21}^1=k_{12}^1=2\frac{-a_1+\sqrt{a_1^2+a_3\lambda_\theta/\lambda_\alpha}}{a_3^2/\lambda_\alpha}$$

$$k_{22}^1=2\frac{-a_2+\sqrt{a_2^2-2a_1+k}}{a_3^2/\lambda_\alpha}$$

$$k=2\sqrt{a_1^2+a_3^2\lambda_\theta/\lambda_p}+a_3^2\lambda_p/\lambda_\alpha$$

Uncertain bound of system's parameters k_{11}, k_{12}, k_{21}, k_{22}, k_u, and environmental disturbances k_{13}, k_{23} are given by

$$\begin{cases} k_{11}=\max\limits_{\Delta V,\Delta h}\left\{ \begin{matrix} |\Delta a_1/a_3| & |\Delta a_2/a_3| \end{matrix} \right\} \\ k_{12}=\max\limits_{\Delta V,\Delta h}\left(|\Delta a_3/a_3|\right) \\ k_{13}=\max\limits_{\Delta V,\Delta h,v(t)}\left(|c_1/a_3|\right) \end{cases} \quad (20)$$

$$\begin{cases} k_{21}=k_{23}=0 \\ k_{22}=\max\limits_{\Delta T_E}\left(|\Delta a_4/b|\right) \\ k_u=\max\limits_{\Delta T_E,\Delta K_E}\left(|\Delta b/b|\right) \end{cases} \quad (21)$$

Robust slide mode hyperplane is

$$S=z_3+\begin{bmatrix} k_1 & k_2 \end{bmatrix}z_1=x_3+k_1x_1+k_2x_2 \quad (22)$$

where

$$k_1 = \gamma_{E1}^{-1}\left(1+\lambda_\alpha k_{11}^2/\lambda_{\min}(Q_1)+\lambda_\alpha(1+k_{12})^2 + \lambda_\alpha\overline{\gamma}\right)\left(-a_1+\sqrt{a_1^2+a_3\lambda_\theta/\lambda_\alpha}\right)\Big/a_3,$$

$$k_2 = \gamma_{E1}^{-1}\left(1+\lambda_\alpha k_{11}^2/\lambda_{\min}(Q_1)+\lambda_\alpha(1+k_{12})^2 + \lambda_\alpha\overline{\gamma}\right)(-a_2+\beta)/a_3,$$

$$\beta = \sqrt{a_2^2-2a_1+2\sqrt{a_1^2+a_3^2\lambda_\theta/\lambda_p}+a_3^2\lambda_p/\lambda_\alpha}.$$

If $\Phi^* = -1$, $P_2 = 1/2$ can be obtained from (15). Hence, variable structure robust controller for ship's fin stabilization system is

$$u = -b^{-1}\left[\phi_1 x_1+\phi_2 x_2+(k_2a_3-a_4-1)S + \rho(1-k_u)^{-1}S/(|S|+\varepsilon)\right] \quad (23)$$

where $\phi_1 = k_1(a_4-a_1-a_3k_1)$,

$$\phi_2 = k_1 + k_2(a_4-a_2-a_3k_2),$$

$$\rho = a_1(|x_1|+|x_2|)+a_2|S|+a_3,$$

$$a_1 = kk_{11}+k^2k_{12}+kk_{22}+k_uk_{L1},$$

$$a_2 = k_{22}+kk_{12}+k_uk_{L2}a_3 = kk_{13},$$

$$k = (|k_1|+|k_2|)/|b|k_{L1} = (|\phi_1|+|\phi_2|)/|b|,$$

$$k_{L2} = |k_2a_3-a_4-1|/|b|.$$

V. SIMULATION EXAMPLE

The parameters in ship roll motion equation is obtained using the method in (17). Then the matrices in (18) is shown as

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -0.0106 & -0.1117 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 \\ -0.043 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$A_{22} = [0.5], \quad B_2 = [0.5],$$

$$C_1 = \begin{bmatrix} 0 & 0.006 \end{bmatrix}^T, \quad C_2 = 0.$$

According to the assumptions introduced for the simulation, the bounds of uncertain parameters in (21) and (22) are obtained, namely

$$\begin{cases} k_{11} = 2.6 \\ k_{12} = 0.66 \\ k_{13} = 0.135 \end{cases} \qquad \begin{cases} k_{21} = k_{23} = 0 \\ k_{22} = 0.45 \\ k_u = 0 \end{cases}$$

Choose $\rho_1 = 1/\lambda_\alpha$, $Q_1 = \mathrm{diag}\begin{bmatrix} \lambda_\theta & \lambda_p \end{bmatrix}$, where $\lambda_\alpha = 0.24$, $\lambda_\theta = 0.5$ and $\lambda_p = 55$. Then the sliding mode hyperplane is written as

$$S = x_3 - 109.42x_1 - 175.99x_2$$

From (23), the robust variable structure control law of ship's fin roll stabilization system is expressed as

$$u = -\left(81.96x_1+1697.6x_2+8.6658S + \rho(x,S)S/(|S|+0.15)\right)$$

where $\rho(x,S) = 17578(|x_1|+|x_2|)+339.53|S|+69.35$.

At the design speed and with the specified metacentric height, the time responses of ship's roll angle under the action of fin roll stabilizer and its control angle are shown in Fig. 2 and Fig. 3, respectively. As we can see, the proposed method achieves better performance of roll rejection, and the range of the fin's control angle is satisfactory.

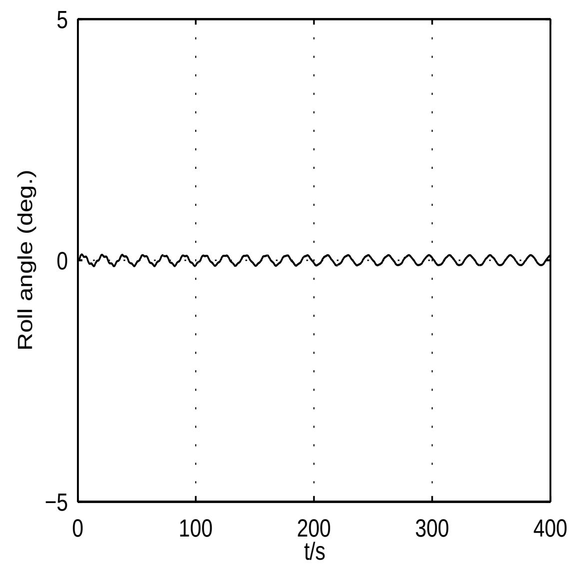

Fig. 2. Time response of ship's roll angle under the control of anti-rolling fin system

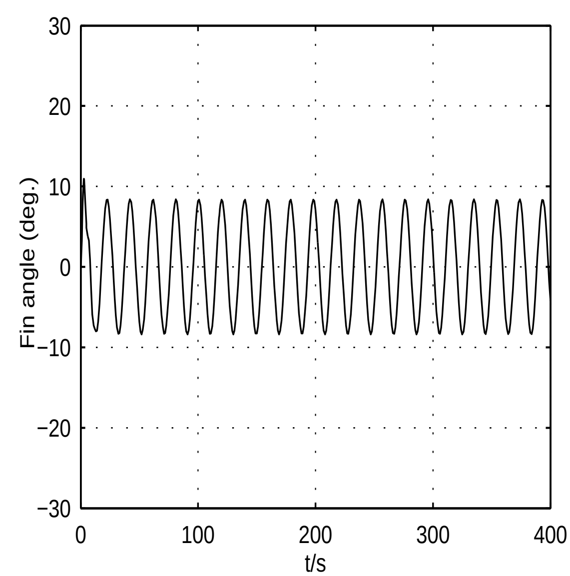

Fig. 3. Time response of fin's control angle

TABLE I

COMPARING THE PERFORMANCE OF CONTROLLER PROPOSED IN THIS PAPER WITH LQG AND CONTINUOUS ROBUST CONTROL LAWS

	Average amplitude(degree) of rolling angle	Average amplitude(degree) fin's control angle	Time constant T_E(sec.) of the fin's actuator when the system becomes unstable
LQG control law	0.62	11	1.9
Continuous robust control law	0.1	11	1.8
Variable structure robust control law	0.12	11.5	The system is still stable even when $T_E \geq 4$

To manifest the effect of the actuator's time constant on the closed-loop system's performance, according to the conventional design, another two kinds of control algorithms are also discussed under the negligence of the T_E's effect.

(1)Conventional LQG control law

Based on the quadratic performance criterion, the algorithm is designed, and takes no account of parametric uncertainty and external disturbances.

(2)Continuous robust control law

The law takes into consideration parametric uncertainty and external disturbances, and is designed according to the method shown in [13].

The simulation results of the adopted control law, which consider both the time constant T_E and uncertainty, is compared with the above two algorithms, as shown in Table I. In Table I's first two columns, we choose initial speed $V_0 = 7.71$ m / sec, and $T_E = 1$. In column 3, the minimal T_E , that is, the threshold value from stability to instability, is found out, via increasing T_E step by step.

From Table I, we can find that the performance of the 3 kinds of control laws are similar, when $T_E = 1$. Simulation research of the LQG controller and the continuous robust controller shows that, the system become unstable when $T_E \approx 2$. The system's instability means that, fin roll stabilizer not only cannot reject rolling but also increase its amplitude, which is quite dangerous. We think that $T_E \approx 2$ is a general time constant, because the smaller the time constant T_E is, the larger the actuator's power and volume needs to be, so as to be unacceptable in practice except military vessel. Hence, the use of the aforementioned LQG controller and continuous robust controller is dangerous sometimes, which should attract our attention. The variable structure robust control law, proposed in this paper, can guarantee that the system is still stable, even when $T_E \geq 4$. $T_E = 4$ is a very mild requirement to shipbuilding industry. Therefore, the variable structure robust control algorithm is of practical applicability in the future.

VI. CONCLUSIONS

The variable structure robust control law of fin roll stabilization system is designed with consideration of the actuator's effect, and show salient robustness and practical applicability in the future. However, the influence of the system's nonlinearties are still rather large, especially in strong waves and with large rolling angle motion, as is not discussed in this paper. If we see fin roll stabilizer as a nonlinear uncertain system, and takes account of the effect of its actuator's time constant, it will be a typical generalized matching uncertain system. The variable structure robust control design of this kind of fin roll stabilization system can utilize the methodology proposed in [13] by author. This paper doesn't tackle this problem, and further research in this respect will be continued.

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