Design of Robust Fault Detection and Isolation Observers for Singular Time Delay Systems

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Abstract— **This paper considers Robust Fault Detection and Isolation Observer (RDO) for uncertain singular time delay system with faults. A sufficient condition for the existence of RDO and an efficient design algorithm are obtained. The design of RDOs associated with various splitting of the faults are also discussed.**

Index Terms — **Fault detection and isolation(FDI), robustness, delay systems, singular systems.**

I. INTRODUCTION

This paper focuses on the problem of the Robust Fault Detection and Isolation Observer (RDO) design for singular time delay systems with structural uncertainties. All the observers are designed such their residual vectors are invariant to the non-fault faults and each observer is further specially designed such that its residual vector is invariant to a particular group of faults while sensitive to the rest. The approach presented is similar to and more mathematically simple than those developed recently in [1] and [2] which rely heavily on the Kronecker canonical decomposition.

II. PROBLEM FORMULATION

Consider uncertain singular time delay system with faults

$$\left\{\begin{array}{rcl} E\dot{x}(t) & = & (A+\Delta A)x(t)+(A_\tau+\Delta A_\tau)x(t-\tau) \\ & & +(B+\Delta B)u(t)+Dd(t)+\sum\limits_{i=1}^{k}L_i m_i(t) \\ y(t) & = & Cx(t) \end{array}\right. \tag{1}$$

where $x(t)\in R^n$, $u(t)\in R^r$, and $y(t)\in R^m$ are the state, control input and output. $u(t)$ and $y(t)$ can be measured through sensors, $\tau > 0$ is the known constant. $Dd(t)$ denotes the effects of the non-faults on the system, $d(t)\in R^{r_d}$ is the unknown input vector. $L_i m_i(t), i=1,2,\cdots,k$, is the ith fault, where L_i is a known matrix called the signature matrix of the ith fault and $m_i(t)\in R^{r_{k_i}}, i=1,2,\cdots,k$, is an unknown function vector called the mode of the ith fault. When fault i occurs, $m_i(t)\neq 0$; when fault i does not occur, $m_i(t)=0$. $A, A_\tau, B, D, C, L_i, i=1,2,\cdots,k$, are constant matrices with appropriate dimensions.

We shall assume the following for system (1)

(A1) The structural uncertainties $\Delta A, \Delta A_\tau, \Delta B$ are of the form as follows

$$\left[\begin{array}{ccc}\Delta A & \Delta A_\tau & \Delta B\end{array}\right] = G\left[\begin{array}{ccc}F_1(\sigma) & F_\tau(\sigma) & F_2(\sigma)\end{array}\right] \tag{2}$$

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where $\sigma\in\Theta, \Theta$ is a compact subset of R and $F_1(\sigma), F_\tau(\sigma)$ and $F_2(\sigma)$ are the continuous functions of σ.

(A2) The linear independence among all $m_i(t), i = 1,2,\cdots,k$, is assumed, i. e., there does not exist non-zero constant vector α, such that

$$\alpha^T\left[\begin{array}{cccc} m_1(t) & m_2(t) & \cdots & m_k(t)\end{array}\right]^T \equiv 0. \tag{3}$$

Denote $(f)_i$ as the ith element of vector f and $\Re(X)$ as the linear space spanned by the columns of matrix X. Then the objective is, for system (1), to design RDO of the following structure in [3]

$$\left\{\begin{array}{rcl} \dot{\omega}(t) & = & H_1\omega(t)+H_2\omega(t-\tau)+H_3 y(t) \\ & & +H_4 y(t-\tau)+H_5 u(t) \\ r(t) & = & \omega(t)+H_6 y(t)\end{array}\right. \tag{4}$$

associated with a given k-dimensional eigenvector f:

$$(f)_i = \left\{\begin{array}{ll} 1, & i\in w \\ 0, & i\in K-w\end{array}\right. \tag{5}$$

such that $m_i(t)=0, \forall i\in w \Longleftrightarrow r(t)\to 0, t\to\infty$. $r(t)$ is called residual vector. Where $K=\{1,2,\cdots,k\}, K, w$ are the index sets, $w\subset K$. The faults in the index set w belong to detected faults and those in the index set $K-w$ belong to isolated faults. A k-dimensional vector f is called a realizable eigenvector when it is associated with a RDO.

III. DESIGN OF RDO

Lemma 1: The linear time delay system $\dot{x}(t)=Ax(t)+A_\tau x(t-\tau)$ is asymptotically stable, if there exist matrices $P>0, Q>0$ satisfying

$$\left[\begin{array}{cc} A^T P+PA+Q & A_\tau^T P \\ PA_\tau & -Q\end{array}\right] < 0. \tag{6}$$

Then we can get a sufficient condition about the existence of RDO for system (1) and a design algorithm.

Theorem 1: Given an eigenvector f, which is defined in (5), under the assumptions (A1) and (A2), system (1) exists a RDO of the form (4) if there exists a matrix T which is subject to the following structural conditions.

$$H_1TE+H_3C-TA=0 \tag{7.1}$$
$$H_2TE+H_4C-TA_\tau=0 \tag{7.2}$$
$$TE+H_6C=0 \tag{7.3}$$
$$TG=0 \tag{7.4}$$
$$TD=0 \tag{7.5}$$
$$TL_i=0, \forall i\in K-w \tag{7.6}$$
$$H_5=TB \tag{7.7}$$
$$system\ \dot{\zeta}(t)=H_1\zeta(t)+H_2\zeta(t-\tau)\ is\ stable \tag{7.8}$$
$$TL_i\neq 0, \forall i\in w \tag{7.9}$$

Algorithm 1:

Step 1. Define T_1 as a matrix of full column rank which satisfies

$$\Re(T_1) = \Re(D) + \Re(G) + \sum_{i \in K-w} \Re(L_i). \qquad (8)$$

Equations (7.4)-(7.6) are equivalent to

$$TT_1 = 0. \qquad (9)$$

If T_1 is nonsingular, the algorithm stops with no RDO of form (4) designed. Otherwise, let $RT_1 = 0$ with R being the left zero divisor of T_1 (R is of full row rank and has maximal rank). Then there exists a matrix S such that $T = SR$. Go to step 2.

Step 2. From (7.3) we have $TE = -H_6C$. Substituting it and $T = SR$ into (7.1) and (7.2) yields

$$M_1C - SRA = 0 \qquad (10)$$

$$M_2C - SRA_\tau = 0 \qquad (11)$$

with $M_1 = H_3 - H_1H_6$ and $M_2 = H_4 - H_2H_6$. The combination of (10), (11) and (7.3) gets equation

$$\begin{bmatrix} M_1 & M_2 & S & H_6 \end{bmatrix} \Pi = 0 \qquad (12)$$

where $\Pi = \begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ -RA & -RA_\tau & RE \\ 0 & 0 & C \end{bmatrix}$. So equations (7.1)-(7.6) are reduced to finding matrices M_1, M_2, S and H_6 satisfying (12). Choose $\begin{bmatrix} M_1 & M_2 & S & H_6 \end{bmatrix}$ as the left zero divisor of Π. If $S = 0$, the algorithm stops with no RDO of form (4) designed. Otherwise, go to step 3.

Step 3. Check whether $T = SR$ satisfies (7.9). If not, the algorithm stops with no RDO of form (4) designed; otherwise, RDO exists and go to step 4.

Step 4. Choose matrices H_1 and H_2 satisfying (7.8). In fact, from Lemma 1, we can choose them as follows. First, select matrices $P > 0$ and $Q > 0$ according to the performance requirement on RDO. Then solve LMI $\begin{bmatrix} H_1^TP + PH_1 + Q & H_2^TP \\ PH_2 & -Q \end{bmatrix} < 0$ for H_1 and H_2. Obviously, the matrices H_1 and H_2 satisfying (7.8) are guaranteed to be found.

Step 5. From $H_3 = M_1 + H_1H_6, H_4 = M_2 + H_2H_6$ and (7.7) we get matrices H_3, H_4 and H_5.

Remark 1: From step 2, we know that the solution $\begin{bmatrix} M_1 & M_2 & S & H_6 \end{bmatrix}$ is the left zero divisor of Π. So the RDO designed using Algorithm 1 is the minimal order RDO of form (4) for system (1). Moreover, this will not lose solutions, which can be seen from the following. Suppose $\begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 & \tilde{S} & \tilde{H}_6 \end{bmatrix}$ is an arbitrary solution of (12). Then, there exists a matrix Y such that $\begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 & \tilde{S} & \tilde{H}_6 \end{bmatrix} = Y \begin{bmatrix} M_1 & M_2 & S & H_6 \end{bmatrix}$. So $\tilde{S} = YS$ and $\tilde{T} = \tilde{S}R = YSR = YT$. So $S = 0$ implies $\tilde{S} = 0$ and if there exists $i_0 \in w$ such that $TL_{i_0} = 0$, we also have $\tilde{T}L_{i_0} = 0$.

Remark 2: Algorithm 1 is a complete algorithm solving the nine conditions shown in Theorem 1. That is, as long as there exist matrices $H_1, H_2, H_3, H_4, H_5, H_6$ and T satisfying (7.1)-(7.9), Algorithm 1 can guarantee that a RDO of form (4) will be designed.

Remark 3: Consider system (1) satisfying assumptions (A1) and (A2). For a given eigenvector defined in (5), design a RDO of form (4) using Algorithm 1. If algorithm stops in step 3, that is, condition (7.9) is not satisfied, but instead

$$\begin{array}{rcll} TL_i & = & 0, & \forall i \in w-u \\ TL_i & \neq & 0, & \forall i \in u \end{array} \qquad (13)$$

then: a. any eigenvector f_1:

$$(f_1)_i = \begin{cases} 1, & i \in v_1 \\ 0, & i \in K - v_1 \end{cases} \qquad (14)$$

with $v_1 \subseteq K, v_1 \cap (w-u) \neq \phi$, which satisfies

$$\Re(D) + \Re(G) + \sum_{i \in K-w} \Re(L_i) = \Re(D) + \Re(G) + \sum_{i \in K-v_1} \Re(L_i) \qquad (15)$$

is not realizable either with Algorithm 1.

b. any eigenvector f_2:

$$(f_2)_i = \begin{cases} 1, & i \in v_2 \\ 0, & i \in K - v_2 \end{cases} \qquad (16)$$

with $u \subset v_2 \subseteq w$ is not realizable either with Algorithm 1.

c. any eigenvector f_3:

$$(f_3)_i = \begin{cases} 1, & i \in u \\ 0, & i \in K - u \end{cases} \qquad (17)$$

is a realizable eigenvector associated with the RDO $(H_1, H_2, H_3, H_4, H_5, H_6)$.

IV. CONCLUSIONS

In this paper, the RDO design problem for singular time delay system is discussed. Theorem 1 has given a sufficient condition about the existence of RDO described by (4) for system (1). Moreover, a detailed and efficient design procedure is presented in Algorithm 1, which is a complete algorithm solving the conditions shown in Theorem 1. We also pointed out that the design result for one eigenvector, either a failure or a success, may provide information about the realizability of other eigenvectors so that the whole design procedure can be simplified.

REFERENCES

[1] Z. G. Yuan, C. C. Vansteenkiste and C. Y. Wen, “Improving the Observer-based FDI Design for Efficient Fault Isolation,” *Int. J. Control*, vol. 68, no. 1, 1997, pp. 197-218.

[2] X. R. Chen, Z. L. Cheng and J. D. Zhu, “Design of Fault Detection and Isolation Observers for Singular Systems,” *in Proc. 15th IFAC World Congress, Barcelona, Spain*, 2002.

[3] M. Darouach, “Linear Functional Observers for Systems with Delays in State Variables,” *IEEE Trans. Automat. Contr.*, vol. 46, no. 3, 2001, pp. 491-496.