A BMI BASED DESIGN FOR ROBUST PID CONTROLLERS WITH TWO-DEGREES-OF-FREEDOM STRUCTURE

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***Abstract*— This paper provides a design method for two-degrees-of-freedom I-PD (2DOF-I-PD) controllers including switched PD compensator based on bilinear matrix inequalities (BMIs). Two design specifications based on $\mathcal{H}_2$ norm are formulated in BMIs, and PID parameters can be exactly obtained by solving the BMI problems via branch and bound algorithms. A set of PD compensators can be obtained simultaneously using proposing design method. The most effective parameter is selected out of the set of PD compensator based on the switching criterion which obtained from estimated system conditions using the statistic algorithms. Numerical examples are also shown.**

I. INTRODUCTION

PID control algorithms [1], [2], [3], [4] play a critical role in 80-90 percent of chemical process systems [5]. They are widely used because of their simple structures which consist of only three parameters, that is, proportional parameter, integral parameter, and derivative parameter. It is, however, difficult to tune those parameters practically since the process dynamics often change due to changes in operating conditions or various disturbances. We have to design controllers such that they have both robustness for changes in conditions of the systems and good tracking properties. PID controllers with one-degree-of-freedom can not have robustness and good tracking properties since they are contrary properties. In order to design the controller with robustness and good tracking properties, this paper deals with 2DOF-I-PD control systems, which have a I-PD control system and a PD compensator. Authors have already proposed a design method for two-degrees-of-freedom PID control systems [6], which provides a PID controller with robustness and good tracking properties. This paper applies the proposed method to I-PD control systems.

The design of many conventional control systems has resulted in an optimization problem, which can be solved by numerical computation based on powerful computer support. One of the most useful tools is bilinear matrix inequality (BMI), which is a flexible framework for analysis and synthesis of control systems. Although checking the solvability of BMI problems is NP hard [7], it is not hard to obtain an exact solution of a BMI problem via branch and bound algorithms if it has a few parameters. Fortunately, a design problem of PID controller has only three parameters, so that we can design PID controller based on BMI.

This paper formulates the design problem of I-PD controllers with two-degrees-of-freedom as a BMI problem. The aim of the control design is to make the control system has both robustness and good tracking properties. In order to reduce the conservativeness of the control system, this paper deal with PD compensator which has switching structure. This switching structure is constructed from a system estimator using recursive least squares algorithms, the switching criterion based on stationary gain of the estimated system and a set of pre-specified PD parameters corresponding to the switching criterion.

This paper is organized as follows. The system description, problem formulations and the design method of 2DOF-I-PD controller based on BMI are given in Section II. In Section III, for more effective PD compensator, a switching structure based on the statistic algorithm is constructed. Section IV provides branch and bound algorithms in order to obtain an exact solution of BMI problems. Finally, numerical simulation examples are presented in Section V. This paper refers to the reference [6].

II. CONTROLLER DESIGN BASED ON BMI

A. System description

Consider a system described by the following continuous-time model:

$$G(s) = \frac{K_0}{1+Ts} e^{-Ls}, \tag{1}$$

where K_0 expresses the system gain, T is the time-constant and L refers to the delay. By using the first order Padé approximation of the delay, the system is approximate

$$G(s) \cong \frac{K_0}{1+Ts} \cdot \frac{1-\frac{Ls}{2}}{1+\frac{Ls}{2}}.$$

By using the sampling time period T_s, the continu time model (2) is transformed to the following discrete- model:

$$A(z^{-1})y(t) = z^{-1}B(z^{-1})u(t) + \frac{1}{\Delta}\xi(t),$$

where

$$\begin{aligned} A(z^{-1}) =& \; 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) =& \; b_0 + b_1 z^{-1} \end{aligned} \quad , \tag{4}$$

and $u(t)$, $y(t)$ and $\xi(t)$ denote the control input signal, the corresponding output signal and the stochastic noise, respectively. The operator z^{-1} denotes a backward shift, that is, $z^{-1}y(t) = y(t-1)$, and Δ denotes the differencing operator defined as $1 - z^{-1}$. This paper deals with the discrete-time model (3) as the controlled object instead of the continuous-model (1).

Next, consider the control system represented by the I-PD controller with two-degrees-of-freedom in Fig.1, where $r(t)$ and $e(t)$ refer to the reference signal and the control error, respectively. $C_1(z^{-1})$ and $C_2(z^{-1})$ denote the I-PD controller and the PD compensator, respectively. And they are given by

$$C_2(z^{-1}) = k_\alpha + \Delta k_\beta, \tag{5}$$

and

$$\begin{aligned} u(t) = -k_c y(t) + \frac{k_i}{\Delta} e(t) - \Delta k_d y(t) \\ + k_\alpha r(t) + \Delta k_\beta r(t). \end{aligned} \tag{6}$$

The 2DOF-I-PD controller in (5) and (6) includes five parameters: proportional gains k_c and k_α, integral gain k_i and derivative gains k_d and k_β. The one-degree-of-freedom I-PD controller $C_1(z^{-1})$ is required to satisfy the design specification for the system perturbation and the stochastic noise by using fixed I-PD parameters which are obtained from the BMI solution discussed in Section IV. And the PD compensator $C_2(z^{-1})$ which has a set of pre-specified PD parameters corresponding to the divided small perturbations, is required to satisfy the good tracking property by using switching structure based on the estimator discussed in Section III.

B. Problem formulation

This paper deals with the $\mathcal{H}_2$ norms which represent the integral squared errors (ISE) of the control system. They can evaluate the two design specifications which require the robustness for the control system and the tracking property for the reference signal. Moreover these evaluation measures result in the optimization problem which is represented by matrix inequalities.

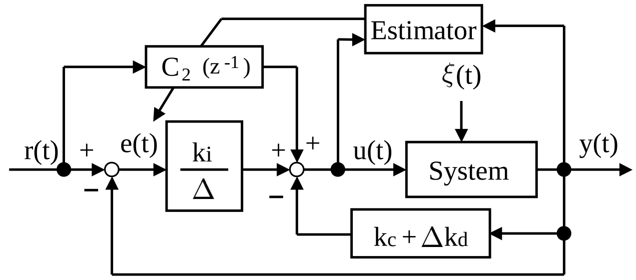

Fig. 1. Closed-loop system with two-degrees-of-freedom.

First, we consider the error transfer function of the control system in Fig.1. In order to evaluate the tracking property for the step reference signal, $E_r(z^{-1})$ is defined as the transfer function from $r(t)$ to $e(t)$. Since a step input is given by $r(z^{-1}) = 1/(1-z^{-1})$ and $\xi(z^{-1}) = 0$, $E_r(z^{-1})$ can be expressed as

$$E_r(z^{-1}) = \frac{A(z^{-1}) - z^{-1}B(z^{-1})C_2(z^{-1})}{\Delta A(z^{-1}) + z^{-1}B(z^{-1})\Delta C_1(z^{-1})} \tag{7}$$

. Similarly, in order to evaluate the influence of the stochastic noise $\xi(t)$, $E_d(z^{-1})$ is defined as the transfer function from $\xi(t)$ to $e(t)$. We assume that $\xi(z^{-1})$ is a white noise which is represented by $\xi(z^{-1}) = 1$ and $r(z^{-1}) = 0$, then $E_d(z^{-1})$ can be expressed as follows.

$$E_d(z^{-1}) = \frac{-1}{\Delta A(z^{-1}) + z^{-1}B(z^{-1})\Delta C_1(z^{-1})} \tag{8}$$

The ISE is described as

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} E\,(j\omega)E(-j\omega)\; d\omega \tag{9}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\,E(j\omega)\,|^2 \; d\omega \; < \; \gamma, \tag{10}$$

where $E = E_r$ or E_d and γ is positive constant. Because the $\mathcal{H}_2$-norm of $E(z^{-1})$ is defined as

$$\|\; E \;\|_2 = \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} |\,E(j\omega)\,|^2 \; d\omega\right)^{\frac{1}{2}}, \tag{11}$$

the performance measure based on ISE results in the following two inequalities.

$$\|\; E_r \;\|_2 < \sqrt{\gamma_r} \tag{12}$$

$$\|\; E_d \;\|_2 < \sqrt{\gamma_d} \tag{13}$$

The purpose of this paper is to minimize γ_r in (12) for a given $\sqrt{\gamma_d}$.

In this paper, the error systems (7) and (8) are realized in the controllable canonical form as the following equations.

$$E_r(z^{-1}) = \quad C_{er}(zI - A_{er})^{-1}B_{er} + D_{er} \tag{14}$$

$$E_d(z^{-1}) = \quad C_{ed}(zI - A_{ed})^{-1}B_{ed} + D_{ed} \tag{15}$$

where A_i, B_i, C_i and D_i ($i = er$ or ed) are given by the following matrices:

$$\begin{aligned}
A_i &= A_{e0} + k_c A_{e1} + k_i A_{e2} + k_d A_{e3}, \\
B_i &= [\ 0 \quad 0 \quad 0 \quad 1\ ]^T, \\
C_{er} &= C_{er0} + k_c C_{er1} + k_i C_{er2} + k_d C_{er3} \\
&\qquad + k_\alpha C_{er4} + k_\beta C_{er5}, \\
C_{ed} &= C_{ed0} + k_c C_{ed1} + k_i C_{ed2} + k_d C_{ed3}, \\
D_{er} &= 1, \\
D_{ed} &= -1,
\end{aligned} \tag{16}$$

where $A_i = A_{er} = A_{ed}$, $B_i = B_{er} = B_{ed}$ and

$$A_{e0} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_2 & a_1 - a_2 & 1 - a_1 \end{bmatrix},$$

$$A_{e1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_1 & b_0 - b_1 & -b_0 \end{bmatrix},$$

$$A_{e2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -b_1 & -b_0 \end{bmatrix},$$

$$A_{e3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -b_1 & 2b_1 - b_0 & 2b_0 - b_1 & -b_0 \end{bmatrix},$$

$$\begin{aligned}
C_{er0} &= [\ 0 \quad a_2 \quad a_1 \quad 1\ ], \\
C_{er1} &= [\ 0 \quad b_1 \quad b_0 \quad 0\ ], \\
C_{er2} &= [\ 0 \quad 0 \quad -b_1 \quad -b_0\ ], \\
C_{er3} &= [\ -b_1 \quad b_1 - b_0 \quad b_0 \quad 0\ ], \\
C_{er4} &= [\ 0 \quad 0 \quad -b_1 \quad -b_0\ ], \\
C_{er5} &= [\ 0 \quad b_1 \quad -b_1 + b_0 \quad -b_0\ ], \\
C_{ed0} &= [\ 0 \quad -a_2 \quad a_2 - a_1 \quad a_1 - 1\ ], \\
C_{ed1} &= [\ 0 \quad -b_1 \quad b_1 - b_0 \quad b_0\ ], \\
C_{ed2} &= [\ 0 \quad 0 \quad b_1 \quad b_0\ ], \\
C_{ed3} &= [\ b_1 \quad b_0 - 2b_1 \quad b_1 - 2b_0 \quad b_0\ ].
\end{aligned}$$

Since the continuous system (1) is perturbed, four system parameters in realized systems have perturbations. In order to treat with the system perturbations, we define a parameter vector θ and the set Ω of perturbations as the following equations.

$$\theta := [\ a_1 \quad a_2 \quad b_0 \quad b_1\ ]^T \tag{17}$$

$$\Omega := \left\{ \theta = \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \end{bmatrix} \in \Re^4 : \begin{array}{c} a_{1min} \le a_1 \le a_{1max} \\ a_{2min} \le a_2 \le a_{2max} \\ b_{0min} \le b_0 \le b_{0max} \\ b_{1min} \le b_1 \le b_{1max} \end{array} \right\} \tag{18}$$

This paper deals with the following $\mathcal{H}_2$ problem which formulates the robustness for the control system.

Problem 1: Minimize γ_r such that $\| E_r \|_2 < \sqrt{\gamma_r}$ and $\| E_d \|_2 < \sqrt{\gamma_d}$ for all $\theta \in \Omega$. ■

According to the reference [8], the ISE criterions which are represented by $\mathcal{H}_2$ norm in (12) equal to the following matrix inequality,

$$\Phi = \begin{bmatrix} \phi_{e0}(P^{-1}, \mathbf{k}_1) & 0 & 0 \\ 0 & \phi_{ed}(P^{-1}, \mathbf{k}_1, \gamma_d) & 0 \\ 0 & 0 & \phi_{er}(P^{-1}, \mathbf{k}_2, \gamma_r) \end{bmatrix} \succ 0 \quad , \tag{19}$$

where '$\Phi \succ 0$' denotes that Φ is a positive definite matrix,

$$\phi_{e0}(P^{-1}, \mathbf{k}_1) = \begin{bmatrix} P^{-1} & P^{-1}B_{ed} & P^{-1}A_{ed} \\ B_{ed}^T P^{-T} & 1 & 0 \\ A_{ed}^T P^{-T} & 0 & P^{-1} \end{bmatrix}, \tag{20}$$

$$\phi_{ed}(P^{-1}, \mathbf{k}_1, \gamma_d) = \begin{bmatrix} \gamma_d & D_{ed} & C_{ed} \\ D_{ed}^T & 1 & 0 \\ C_{ed}^T & 0 & P^{-1} \end{bmatrix}, \tag{21}$$

and

$$\phi_{er}(P^{-1}, \mathbf{k}_2, \gamma_r) = \begin{bmatrix} \gamma_r & D_{er} & C_{er} \\ D_{er}^T & 1 & 0 \\ C_{er}^T & 0 & P^{-1} \end{bmatrix}. \tag{22}$$

Furthermore, $\mathbf{k}_1 = [k_c \ k_i \ k_d]$ and $\mathbf{k}_2 = [k_c \ k_i \ k_d \ k_\alpha \ k_\beta]$ are parameter vectors of the controller and P is a positive symmetric matrix.

By using (19), the following problem which is equivalent to the Problem 1 can be formulated.

Problem 2: For given constant $\tilde{\gamma}_d$ and hyper-rectangle Q_D in $\Re^{19}$,

$$\begin{array}{lll}
\text{Minimize} & \gamma_r & (23) \\
\text{subject to} & \gamma_d < \tilde{\gamma}_d & (23-\mathrm{a}) \\
 & \left(\mathbf{k}_1, P^{-1}\right) \in Q_D & (23-\mathrm{b}) \\
 & \Phi \succ 0 \quad \text{for all} \quad \theta \in \Omega. & (23-\mathrm{c})
\end{array}$$

■

Although it is hard to solve BMi problems, we can obtain the exact solution of BMi problem (23) in practical time via the branch and bound algorithms [9] because it has only few parameters.

III. SWITCHING PD COMPENSATOR BASED ON STATISTIC ALGORITHM

As we discussed so far in this paper, it became obvious that 2DOF-PID controller can be designed by using the numerical optimization problems uniformly. However, there fatefully exists the conservativeness of the proposed controller due to the robust structure itself. To reduce such conservativeness, the design method of a set of PD compensators corresponding to small divided perturbations is considered. Moreover we assume that the system condition of the controlled object can be roughly estimated by using the existing adaptive methods or the newly estimate ways.

First, a small divided perturbation is defined as follows.

$$\Lambda^{(i)} := \left\{ \theta \in \Re^4 : \begin{array}{c} a_{1min}^{(i)} \le a_1 \le a_{1max}^{(i)} \\ a_{2min}^{(i)} \le a_2 \le a_{2max}^{(i)} \\ b_{0min}^{(i)} \le b_0 \le b_{0max}^{(i)} \\ b_{1min}^{(i)} \le b_1 \le b_{1max}^{(i)} \end{array} \right\} \quad (24)$$

Then, PD compensator $C_2(z^{-1})$ is switched based on the following detection rule:

$$C_2(z^{-1}) = \begin{cases} C_2^{(1)}(z^{-1}) & \text{if } \theta \in \Lambda^{(1)}, \\ C_2^{(2)}(z^{-1}) & \text{if } \theta \in \Lambda^{(2)}, \\ \vdots & \vdots \\ C_2^{(p)}(z^{-1}) & \text{if } \theta \in \Lambda^{(p)}, \end{cases} \quad (25)$$

where $C_2^{(j)}(z^{-1})$ $(j = 1 \cdots p)$ are PD compensators optimized for each sector. The design problem of the PD compensator $C_2^{(i)}(z^{-1})$ corresponding to the pre-specified small-ranged system perturbation is given as the following problem.

Problem 3: For given given k_c, k_i and k_d,

$$\begin{array}{ll} \text{Minimize} & \gamma_r \\ \text{subject to} & \Phi_{PD} \succ 0 \quad \text{for all} \quad \theta \in \Lambda^{(i)}. \end{array} \quad \begin{array}{r} (26) \\ (26-\text{a}) \end{array}$$

where, Φ_{PD} is defined as the following matrix.

$$\Phi_{PD} = \begin{bmatrix} \phi_{e0}(P^{-1}, \mathbf{k}_1) & 0 \\ 0 & \phi_{er}(P^{-1}, \mathbf{k}_2, \gamma_r) \end{bmatrix} \succ 0 \quad (27)$$

■

The matrix inequality (27) can be represented by LMI which includes P^{-1}, k_α and k_β as the variables. Therefore it is easy to obtain the optimal solution of the problem (26) because there exist polynomial algorithms based on the interior point method.

By using the switching structure as mentioned above, the most effective PD compensator which satisfies the good tracking property is selected out of the set of pre-specified PD parameters. The switching algorithm for the proposed 2DOF-I-PD controller is summarized as follows.

[Switching algorithm for 2DOF-I-PD controller]

[Step 1] Design the I-PD controller and the PD compensator by solving the BMI problem (23).
[Step 2] Design the set of PD parameters corresponding to the divided perturbations by solving the LMI problem (26).
[Step 3] Estimate the system conditions.
[Step 4] Choose the most effective PD parameter from the detection rule in (25).
[Step 5] Return to **[Step 3]**.

In **[Step 3]** and **[Step 4]**, the following detection algorithm is employed.

[Detection algorithm for switching PD compensators]

[Step 1] Design the center model $\lambda^{(j)}$ of the set $\Lambda^{(j)}$.
[Step 2] Input the control input $u(t)$ to the $\lambda^{(j)}$ and obtain the corresponding output $y_\lambda^{(j)}(t)$.
[Step 3] Calculate the summation $\sigma^{(j)}$ of $\mid y(t) - y_\lambda^{(j)}(t) \mid^2$ during the given period τ.
[Step 4] Find the model $\lambda^{(o)}$ such that minimizes $\sigma^{(j)}$. And assume $\lambda^{(o)}$ as the most likely control model.
[Step 5] Select the $C_2^{(o)}(z^{-1})$ corresponding to the set $\Lambda^{(o)}$.
[Step 6] Return to **[Step 2]**.

IV. BMI SOLUTION BY USING AN EXACT ALGORITHM

This section provides an exact algorithm for solving problem (23) based on branch and bound algorithms [9]. Branch and bound algorithms give us the lower bound Ψ_L and the upper bound Ψ_U satisfying $\Psi_L \le \inf \gamma_r \le \Psi_U$ and $(\Psi_U - \Psi_L)/\Psi_L \le \varepsilon$ for any $\varepsilon > 0$. The lower bounds are obtained using the SDP relaxation [10], [11].

Let us define the function $\Psi(\cdot)$, $\Psi_L(\cdot)$ and $\Psi_U(\cdot)$ as follows.

$$\Psi(Q) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d,\, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \Phi \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \quad (28)$$

$$\Psi_L(Q) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d,\, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \hat{\Phi} \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \quad (29)$$

$$\Psi_U(Q, \mathbf{k}_1^*, \mathbf{k}_2^*) \equiv \inf_{\substack{\gamma_d < \tilde{\gamma}_d, \\ \Phi^* \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r, \quad (30)$$

where $\hat{\Phi}$ is the SDP relaxation of Φ obtained using the method in the papers [11], [12],

$$\begin{bmatrix} \mathbf{k}_1^* \\ \mathbf{k}_2^* \end{bmatrix} = \arg \inf_{\substack{\gamma_d < \tilde{\gamma}_d,\, [\mathbf{k}_1, \mathbf{k}_2]^T \in Q, \\ \hat{\Phi} \succ 0 \text{ for all } [a_1, a_2, b_0, b_1]^T \in \Omega}} \gamma_r,$$

and Φ^* is obtained by substituting $[\mathbf{k}_1^*, \mathbf{k}_2^*]^T$ into Φ in (19). Then $\Psi_L(Q) \leq \Psi(Q) \leq \Psi_U(Q)$ holds for any Q. We can obtain Ψ_L^* and Ψ_U^* such that $\Psi_L \leq \inf \gamma_r \leq \Psi_U$ holds for any ε using the following algorithm.

[Branch and Bound Algorithm]

[Step 1] Set $k \leftarrow 0, Q_0 \leftarrow Q_D, S0 \leftarrow \{Q_0\}, L_0 \leftarrow \Psi(Q_0), U_0 \leftarrow_U (Q_0)$.
[Step 2] Select $\bar{Q}$ from S_k such that $L_k = \Psi_L(\bar{Q})$. $S_{k+1} \leftarrow S_k \setminus \{\bar{Q}\}$.
[Step 3] Split $\bar{Q}$ along its longest edge into $\bar{Q}_1$ and $\bar{Q}_2$.
[Step 4] For $i = 1, 2$ if $\Psi_L(\bar{Q}_i) \leq U_k$ then $S_{k+1} \leftarrow S_{k+1} \cup \{\bar{Q}_i\}$.
[Step 5] $U_{k+1} \leftarrow \min_{Q \in S_{k+1}} \Psi_U(Q)$.
[Step 6] Pruning: $S_{k+1} \leftarrow S_{k+1} \setminus \{Q : \Psi_L(Q) > U_{k+1}\}$.
[Step 7] $L_{k+1} \leftarrow \min_{Q \in S_{k+1}} \Psi_L(Q)$.
[Step 8] if $(U_k - L_k)/L_k \leq \varepsilon$ then end else $k \leftarrow k+1$ and goto **[Step 2]**.

V. NUMERICAL EXAMPLES

In order to investigate the behavior of the proposed control scheme, numerical simulation examples are illustrated in this section.

Let us consider the continuous-time model given by the following equation.

$$G(s) = \frac{K_0}{1+Ts} e^{-Ls} \quad \text{where} \quad \begin{cases} 2.5 \leq K_0 \leq 3.5 \\ 15 \leq T \leq 16 \\ L = 3 \end{cases} \tag{31}$$

From (3) and (4), the system parameters of the discrete-time model which are transformed by using sampling time period $T_s = 1$ are obtained as follows:

$$\begin{array}{rl} A(z^{-1}) = & 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) = & b_0 + b_1 z^{-1} \end{array}, \tag{32}$$

where

$$\begin{cases} -1.4528 \leq a_1 \leq -1.4489 \\ 0.4803 \leq a_2 \leq 0.4823 \\ -0.1026 \leq b_0 \leq -0.0689 \\ 0.1426 \leq b_1 \leq 0.2124 \end{cases}. \tag{33}$$

By solving the BMI problem (23), parameters of the I-PD controller and the PD compensator are designed as follows:

$$\begin{array}{lll} k_c = 1.0801, & k_i = 0.07377, & k_d = 0.5062, \\ k_\alpha = 1.0110, & k_\beta = 0.4153, & \end{array} \tag{34}$$

where, $\mathcal{H}_2$ norms of the error transfer functions are obtained as follows.

$$\begin{array}{ll} \gamma_r & = 150.00 \\ \gamma_d & = 10.134 \end{array} \tag{35}$$

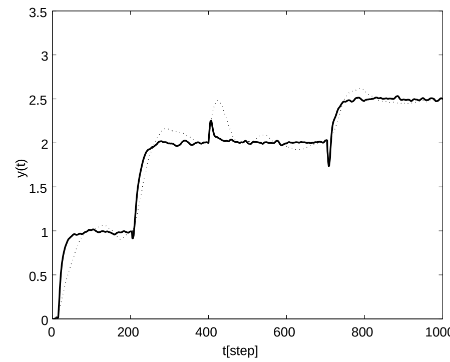

Fig. 2. Control results of the proposed 2DOF-I-PD controller and the existing 1DOF-I-PD controller.

The set of the small-divided perturbation is defined as below:

$$\begin{array}{l} \Lambda^{(1)} := \left\{ \theta \in \mathcal{R}^4 : \begin{array}{c} 2.5 \leq K_c \leq 3.0 \\ 15 \leq T \leq 16 \\ L = 3 \end{array} \right\}, \\ \Lambda^{(2)} := \left\{ \theta \in \mathcal{R}^4 : \begin{array}{c} 3.0 \leq K_c \leq 3.5 \\ 15 \leq T \leq 16 \\ L = 3 \end{array} \right\}. \end{array} \tag{36}$$

Then, we designed the pre-specified PD compensators by solving the LMI problems (26), and they are obtained as follows.

$$\begin{array}{ll} C_2^{(1)}(z^{-1}) = 1.0801 + 0.5062\Delta & if\ \theta \in \Lambda^{(1)} \\ & (\gamma_r = 7.0422) \\ C_2^{(2)}(z^{-1}) = 1.0274 + 0.5062\Delta & if\ \theta \in \Lambda^{(2)} \\ & (\gamma_r = 6.4317) \end{array} \tag{37}$$

In the above detection algorithm, the center models $\lambda^{(1)}$ of the system models (36) is set based on $K_0 = 2.75$, $T = 15.5$, and $L = 3$, and $\lambda^{(2)}$ is designed based on $K_0 = 3.25$, $T = 15.5$, and $L = 3$. The evaluation period τ equals to 10[step].

The system parameters of the controlled object are given as follows.

$$G(s) = \begin{cases} \dfrac{2.3}{1+16s} e^{-3s} & 0 \leq t \leq 400[\text{step}] \\ \\ \dfrac{3.2}{1+15s} e^{-3s} & 400 \leq t \leq 401[\text{step}] \end{cases} \tag{38}$$

The reference signal is given as the step inputs. The stochastic noise $\xi(t)$ is given as a normal distribution with $\mathcal{N}(0, 0.01^2)$.

Fig.2 shows the control results by using the obtained 2DOF-I-PD parameters in (34) and the existing 1DOF-I-PD controller tuned by the procedure of the reference [13]. In Fig.2, the solid line denotes the control result using the proposed 2DOF-I-PD controller, and the dotted line denotes the control result using the existing 1DOF-I-PD controller

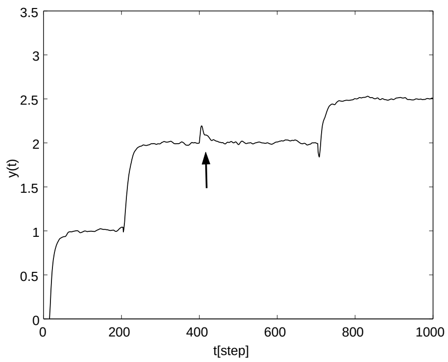

Fig. 3. 2DOF-I-PD control result using the switching algorithm.

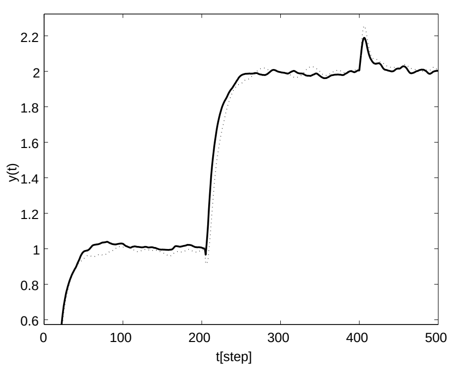

Fig. 4. Control result using the switched 2DOF-I-PD controller and non-switched 2DOF-I-PDcontroller.

($k_c = 0.0014$,$k_i = 9.5465$, and $k_d = 0.0034$). Although the proposed controller does not have a switching structure in this result, we can see that the system has the better tracking properties than the existing controller.

Next, the control result by using the statistic algorithm are demonstrated. Fig.3 shows the result. In Fig.3, the arrow denotes the switching point from $C_2^{(1)}(z^{-1})$ to $C_2^{(2)}(z^{-1})$. To make the effectiveness of the switching structure clearer, the non-switched 2DOF-I-PD controller in Fig. 2 and switched 2DOF-I-PD controller in Fig.3 are compared. Fig.4 shows the figureof these control results form 0 [step] to 500 [step] with expansion. In Fig.4, the dotted line denotes the control result using non-switched 2DOF-I-PD controller, and the solid line denotes the control result using switched 2DOF-I-PD controller.

We can see that the influence by the stochastic noise can be reduced, and that the switched 2DOF-I-PD control system can track the reference signal better than the system of the non-switched 2DOF-I-PD controller. When the system perturbed in 401 [step], the most effective PD compensator is quickly selected out.

VI. CONCLUSIONS

In this paper, a BMI based design scheme for switched I-PD controllers with two-degrees-of-freedom has been proposed. According to the proposed scheme, two design specification based on $\mathcal{H}_2$ norm are formulated in BMIs, and I-PD parameters can be exactly obtained by solving the BMI problems via branch and bound algorithms. In order to reduce the conservativeness of the control system, the proposed PD compensators have switching structure based on the statistic algorithm. Numerical examples have shown the effectiveness of the proposed method.

VII. REFERENCES

[1] K.L. Chien, J.A. Hrones, and J.B. Reswick. On the Automatic Control of Generalized Passive Systems. *Trans. ASME*, Vol. 74, pp. 175–185, 1972.

[2] P.T. Nims. Some Design Criteria for Automatic Controls. *Trams. AIEE*, Vol. 70, pp. 606–611, 1950.

[3] Y. Takahashi, C.S. Chan, and D.M. Auslander. Parametereinstellung bei Linearen DDC-Algorithmen, Regelungstechinik und Proze β-Datenverarbeitung. *Verlag R. Oldenbourg*, Vol. 19, No. 6, pp. 237–284, 1971.

[4] J.G. Ziegler and N.B. Nichols. Optimum Settings for Automatic Controllers. *Trans. ASME*, Vol. 64, No. 8, pp. 759–768, 1942.

[5] N. Suda. *PID control*. No. 6 in System, Control and Information Library. Asakura, 1992. (in Japanese).

[6] J. Aoyama, K. Konishi, T. Yamamoto, and T. Hinamoto. A BMI-Based Design of Switched PID Controllers. *Proc. of the ADCHEM*, pp. 992–997, 2004.

[7] O. Toker and H. Özbay. On the NP-hardness of solving Bilinear Matrix Inequalities and simultaneous stabilization with static output feedback. *Proc. of ACC*, pp. 2525–2526, 1995.

[8] K. Zbou, J. C. Doyle, and K. Glover. *Robust and optimal control*. Prentice Hall, 1996.

[9] K. C. Goh, M. G. Safonov, and G. P. Papavassilopoulos. A Global Optimization Approach for the BMI Problem. *Proc. of the 33rd Conf. of Decision and Control*, pp. 2009–2014, 1994.

[10] M. Kojima and L. Tunçel. Cones of matrices and succesive convex realxations of nonconvex sets. Research Reports Series B: Operations Research, Department of Mathematical and Computing Sciences, Tokyo Institute of Technology, 1999.

[11] T. Fujita and M. Kojima. Semidefinite relaxation for nonconvex programs. *J. of Global Optimization*, Vol. 10, pp. 367–380, 1997.

[12] H. Fujioka and K. Hoshijima. Bounds for the bmi eigenvalues problem. *Trans. of SICE*, Vol. 33, No. 7, pp. 616–621, 1997.

[13] T. Yamamoto and M. Kaneda. A Design of Self-Tuning PID Controllers Based on the Generallized Minimum Variance Control Law. *Trans. on ISCIE*, Vol. 11, No. 1, pp. 1–9, 1998. (in Japanese).