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A novel control scheme for typical unstable processes with time delay

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***Abstract*—In this paper, a new two-degree-of-freedom control structure is proposed for controlling unstable processes with time delay. The setpoint tracking is of open-loop control and therefore is decoupled with the load disturbance rejection, for which the controller is analytically designed in terms of H_2 optimal performance specification. Meanwhile a conventional proportional(P) controller is additionally employed to stabilize the setpoint response. By proposing the desired complementary sensitivity transfer function form of the inner closed-loop for the load disturbance rejection, the closed-loop disturbance estimator is inversely derived. It is a dominant merit that the time domain system response specification of the proposed control structure can be quantitatively estimated by virtue of the analytical design procedures developed for the controllers. Illustrative simulation tests are included to demonstrate the superiority of the proposed method.**

1 Introduction

Unstable processes are well known difficult to control especially when there exists dominant time delay in the system responses. Many different approaches have been developed for unstable processes. The conventional proportional-integral(PI) or proportional-integral-derivative(PID) methods based on the unity feedback control structure have been well provided by Visioli, A.(2001), Datta, A.(1999), Weidong Zhang et al(1999), Ho, W.K. et al(1998), and De Paor, A.M. et al(1989). However the setpoint response is usually accompanied with excessive overshoot and large settling time in terms of the above methods. Hence some two-degree-of-freedom control methods of PID controller have been proposed to overcome the aforementioned deficiencies, such as Ya-Gang Wang et al(2002), Yongho, Lee. et al(2000), Park, J.H. et al(1998), and Shafiei, Z.(1997). Wen Tan et al(2003) proposed a modified internal model control(IMC) method for obtaining the no overshoot setpoint tracking, and Xue-Ping Yang and Qing-Guo Wang et al(2002) presented an IMC-Based control scheme and adopted the recursive least squares(RLS) algorithm to find the optimum controllers, by which it shows superiority over recent other PI or PID controller methods for both the setpoint response and the load disturbance response. Other methods derived from the Smith predictor control structure such as Majhi, S. et al(2000) and Kwak, H.J.(1999) have also obtained good system performance for typical first order unstable processes with time delay. It is a notable merit that there is no overshoot in the setpoint response for unstable processes in terms of either the modified IMC methods or the modified Smith predictor methods as above. Moreover the setpoint response tends to be faster compared with the aforementioned PI or PID methods based on the unity feedback control structure. Essentially, a common character of the aforementioned modified IMC methods and Smith predictor methods is employing the process model with dead-time compensator in their control structures, which significantly helps to achieve the above merits. Consequently, this paper goes along the idea of employing the process model and develops a new two-degree-of-freedom control structure, which is shown in Fig.1.

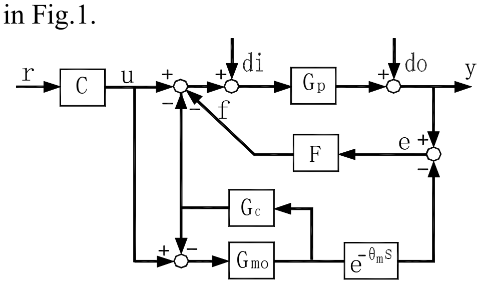

Fig.1 The proposed control structure

In Fig.1, G_{mo} is the delay-free part of the process model G_m, i.e. $G_m = G_{mo}e^{-\theta_m s}$. There are three controllers in the proposed control scheme. The controller G_c is employed for stabilizing the setpoint response and is selected as a proportional(P) controller for convenience. Another controller C is used for the setpoint tracking. The controller F in the feedback channel of the inner closed-loop is designed for the load disturbance rejection and is therefore called as the disturbance estimator. Obviously the setpoint response is decoupled with the load disturbance response because of the open-loop control strategy for the setpoint tracking in the proposed control structure. In addition, the stabilizing controller G_c will not affect the setpoint tracking performance specification, which is indeed tuned singly by the controller C. Both of the setpoint tracking controller C and the disturbance estimator F are analytically designed in terms of H_2 optimal performance specification, which can lead the system response to achieve the integral-square-error(ISE) criterion. Another virtue of the proposed method is that both the controllers C and F can be monotonously tuned by a single parameter respectively to meet the actual process uncertainty. The controller design procedures are carefully provided in the following section.

2 Controller design

2.1 Stabilizing controller G_c

From Fig.1, it is easy to figure out the setpoint response transfer function in form of

$$H_r(s) = \frac{CG_p}{1+G_cG_{mo}} \cdot \frac{1+FG_{mo}e^{-\theta_m s}}{1+FG_p} \qquad (1)$$

In nominal case, that is, G_m is the perfect model of the process G_p, i.e. $G_m=G_p$, the setpoint response transfer function can be simplified as

$$H_r(s) = \frac{CG_p}{1+G_cG_{mo}} \qquad (2)$$

Generally in industrial and chemical practice, the actual unstable processes can be identified as the first order form of

$$G_p(s) = \frac{ke^{-\theta s}}{\tau s - 1} \qquad (3)$$

Take the stabilizing controller G_c as a conventional P controller for simplicity, i.e $G_c = k_c$. Hence the characteristic equation of the setpoint response transfer function in Eq.(2) is

$$\tau s + k_c k - 1 = 0$$

Note that the controller C is designed to be stable in the following section 2.2 and therefore is omitted to consider for the stability issue discussed here. By employing the Routh-Hurwitz Judgement for system stability, we know that it keeps stable when take $k_c > 1/k$. It should be noted that the controller G_c may be chosen as a conventional PID controller to stabilize the setpoint response, which however tends to make the tuning of the control parameters much complicated and therefore is not recommended.

2.2 Setpoint tracking controller C

We adopt the H_2 optimal performance specification $\min \| e \|_2^2$ to design the setpoint tracking controller C. Here it conforms to the system performance specification $\min \| W(s)(1-H_r(s)) \|_2^2$, where $W(s)$ is the setpoint input weight function. Usually the setpoint input is of step change in practice and accordingly it can be selected as $1/s$.

By employing the n/n order all-pass Pade approximation for the pure time delay $e^{-\theta s}$ of the process model in Eq.(3), we obtain

$$G_p(s) = \frac{kQ_{nn}(-\theta s)}{(\tau s - 1)Q_{nn}(\theta s)}$$

where

$$Q_{nn}(\theta s) = \sum_{j=0}^{n} \frac{(2n-j)!n!}{(2n)!j!(n-j)!}(\theta s)^j$$

and n is an integer large enough to ensure that the introduced approximation error is negligible compared with the actual process uncertainty. By using Eqs.(2) and (3), it follows that

$$\|W(s)(1-H_r(s))\|_2^2 = \left\| \frac{1}{s}\left(1 - \frac{kC(s)Q_{nn}(-\theta s)}{(\tau s + k_c k - 1)Q_{nn}(\theta s)}\right) \right\|_2^2$$

$$= \left\| \frac{Q_{nn}(\theta s)}{sQ_{nn}(-\theta s)} - \frac{kC(s)}{s(\tau s + k_c k - 1)} \right\|_2^2$$

Note that $Q_{nn}(0) = 1$ and all zeros of $Q_{nn}(-\theta s)$ are in the right half plane(RHP). Utilizing the orthogonality property of H_2 norm , we have

$$\|W(s)(1-H_r(s))\|_2^2 = \left\| \frac{Q_{nn}(\theta s) - Q_{nn}(-\theta s)}{sQ_{nn}(-\theta s)} \right\|_2^2 + \left\| \frac{\tau s + k_c k - 1 - kC(s)}{s(\tau s + k_c k - 1)} \right\|_2^2$$

Minimizing the right side, i.e. let its second term be equal to zero, we obtain the ideal optimal controller

$$C_{im}(s) = \frac{\tau s + k_c k - 1}{k}$$

However it is not proper and can not be physically realized in practice. Hence a low-pass filter

$$J_c(s) = \frac{1}{\lambda_c s + 1}$$

is introduced to copy it out and the practical optimal controller is in form of

$$C(s) = \frac{\tau s + k_c k - 1}{k(\lambda_c s + 1)} \qquad (4)$$

where λ_c is the tuning parameter and when it tends to zero, the controller C recovers the optimality.

Note that there is open-loop control from the setpoint input to the system output, and now the setpoint tracking controller C is designed to be stable and correspondingly the nominal setpoint response transfer function is in form of

$$H_r(s)=\frac{1}{\lambda_c s+1}e^{-\theta s} \tag{5}$$

which is obviously proper and stable. Certainly the setpoint response of the control system keeps stable. By inverse Laplace transform, we obtain

$$y_r(t)=\begin{cases}0 & t\le\theta\\ 1-e^{-(t-\theta)/\lambda_c} & t>\theta\end{cases} \tag{6}$$

It shows that there is no overshoot in the nominal setpoint response and the time domain performance specification can be quantitatively met by tuning the single parameter λ_c. For instance, define the rise time t_r of the setpoint response be the period that the system output reaches 90 percent of the setpoint value, we figure out $t_r = 2.3026\lambda_c + \theta$ from Eq.(6). It can be seen that when λ_c is tuned to be smaller, the nominal setpoint tracking performance becomes faster while the output energy of the setpoint tracking controller requires larger, and consequently more aggressive dynamic behavior of the setpoint response occurs in the presence of the actual process uncertainty; when λ_c is tuned to be larger, the nominal setpoint response turns out to be slower while the output energy of the setpoint tracking controller requires smaller, and consequently less aggressive dynamic behavior of the setpoint response appears in the presence of the actual process uncertainty. A large quantity of simulation shows that the rule of thumb for tuning the control parameter λ_c is within the range of $0.5\theta-3.0\theta$, where θ is the unstable process pure time delay. Generally it is recommended to fix λ_c around the process time delay value to make the best compromise between the nominal performance of the setpoint response and the output energy of the setpoint tracking controller C. In addition, here it can be identified that the stabilizing controller G_c really does not affect the setpoint tracking performance specification.

2.3 Disturbance estimator F

From Fig.1 we obtain the load disturbance transfer functions

$$H_{di}(s)=\frac{y_{di}}{di}=\frac{G_p(s)}{1+F(s)G_p(s)} \tag{7}$$

$$H_{do}(s)=\frac{y_{do}}{do}=\frac{1}{1+F(s)G_p(s)} \tag{8}$$

Hence the complementary sensitivity function of the inner closed-loop for the load disturbance rejection is in form of

$$T_d(s)=\frac{f}{di}=\frac{F(s)G_p(s)}{1+F(s)G_p(s)} \tag{9}$$

In ideal case, the desired complementary sensitivity function should be $T_d(s)=e^{-\theta s}$. That is, when the load disturbance di in Fig.1 is added to the process input, the disturbance estimator F should detect the resultant system output error just after the process pure time delay θ and produce an inversely equivalent signal f to offset the load disturbance. However there actually exist two asymptotic tracking constraints as follows.

$$\lim_{s\to\tau} H_{do}(s)=0 \tag{10}$$

$$\lim_{s\to 0} H_{do}(s)=0 \tag{11}$$

where τ is the RHP pole of G_p. In practice, Eqs.(10) and (11) have to be satisfied for the load disturbance rejection. This means some constraints have to be subordinated to the desired T_d to keep the closed-loop internal stability. Inspired by the robust IMC control theory(Morari, M., 1989), we propose the practical optimal T_d in terms of the H_2 performance objective, i.e.

$$T_d(s)=\frac{as+1}{(\lambda_f s+1)^2}e^{-\theta s} \tag{12}$$

where λ_f is the control tuning parameter, and a is determined by Eq.(10). Substitute Eq.(12) into Eq.(10), there is

$\lim_{s\to 1/\tau} H_{do}(s)=\lim_{s\to 1/\tau}(1-T_d(s))=0$, i.e. $\lim_{s\to 1/\tau}[1-\frac{as+1}{(\lambda_f s+1)^2}e^{-\theta s}]=0$

Following simple calculations we obtain

$$a=\tau[(\frac{\lambda_f}{\tau}+1)^2 e^{\frac{\theta}{\tau}}-1] \tag{13}$$

Hence by using Eq.(9), we obtain the H_2 optimal disturbance estimator F in form of

$$F_{im}(s)=\frac{T_d(s)}{1-T_d(s)}\cdot\frac{1}{G_p(s)}$$

i.e.

$$F_{im}(s)=\frac{(\tau s-1)(as+1)}{k[(\lambda_f s+1)^2-(as+1)e^{-\theta s}]} \tag{14}$$

However it can be seen that there exists RHP zero-pole cancelling at $s=1/\tau$ in Eq.(14) which may cause the disturbance estimator to work unstably and can not be removed directly. Therefore the mathematical Maclaurin expansion formula is here utilized to copy out the H_2 optimal disturbance estimator F_{im}. Let $F_{im}(s)=M(s)/s$, we obtain

$$F_{im}(s)=\frac{1}{s}[M(0)+M'(0)s+\frac{M''(0)}{2!}s^2+\cdots+\frac{M^{(i)}(0)}{i!}s^i+\cdots] \quad (15)$$

Obviously the first three terms of the above expansion is exactly a standard PID controller in form of

$$F(s)=k_f+\frac{1}{T_I s}+T_D s \quad (16)$$

where $k_f=M'(0), T_I=1/M(0)$ and $T_D=M''(0)/2$. It is the practically proposed disturbance estimator in form of PID. It should be mentioned that the pure derivative term in Eq.(16) can be physically implemented by cascading with a first order low-pass filter in which the time constant can be selected as $(0.01\sim 0.1)T_D$.

Remark. According to the well known Small-Gain Theorem(Doyle, J.C., 1992), the closed-loop for the load disturbance rejection is robustly stable iff

$$\| \Delta_m T_d \|_\infty < 1 \quad (17)$$

where Δ_m defines the process multiplicative uncertainty. Substitute Eqs.(12-13) into Eq.(17), we obtain the tuning constraint for the control parameter λ_f to ensure the closed-loop robust stability, i.e.

$$\left\| \frac{\tau[(\lambda_f/\tau+1)^2 e^{\theta/\tau}-1]s+1}{(\lambda_f s+1)^2} \right\|_\infty < \frac{1}{\|\Delta_m(s)\|_\infty}$$

For example, as for the process gain uncertainty $\Delta_m=\Delta k/k$, the robust stability constraint to λ_f is

$$\frac{\lambda_f^2\omega^2+1}{\sqrt{\omega^2\tau^2[(\lambda_f/\tau+1)^2 e^{\theta/\tau}-1]^2+1}} > \frac{\Delta k}{k}, \quad \forall\, \omega>0$$

As for the process time delay uncertainty $\Delta\theta$, which may be converted to the multiplicative uncertainty $\Delta_m=e^{-\Delta\theta s}-1$, the robust stability constraint to λ_f is

$$\frac{\sqrt{\omega^2\tau^2[(\lambda_f/\tau+1)^2 e^{\theta/\tau}-1]^2+1}}{\lambda_f^2\omega^2+1} < \frac{1}{|e^{-j\Delta\theta\omega}-1|}, \quad \forall\, \omega>0$$

As for the process uncertainty of both gain and time delay, the similar robust stability constraint to λ_f can be derived as above.

Moreover there is another constraint between the robust stability and nominal performance of the load disturbance rejection loop for tuning the control paramete λ_f in terms of the robust control theory(Morari, M., 1989), i.e.

$$|\Delta_m T_d(s)| + |W(s) H_{do}(s)| < 1$$

where $W(s)$ is the sensitivity weight function and usually is selected as $1/s$ in terms of the frequently-encountered step load disturbance in industrial and chemical practice. It indicates that tuning the control parameter λ_f has to meet the trade-off between the robust stability and the nominal performance of the closed-loop for the load disturbance rejection. That is, decreasing λ_f improves the load disturbance rejection performance of the closed-loop but deteriorates its robust stability in the presence of the actual process uncertainty. On the other hand, increasing λ_f tends to strengthen the closed-loop robust stability but decays its disturbance rejection performance.

However all the aforementioned constraints for tuning the control parameter λ_f can not be solved analytically. Hence numerical simulations are motivated to ascertain the rule of thumb for tuning the control parameter λ_f. Define perturbation peak be the closed-loop output peak when a unit step load disturbance di is added to the process input and the step input r is cut off in Fig.1. By employing the practical PID form of the disturbance estimator F in Eq.(16), simulation results on the definition are provided in Fig.2.

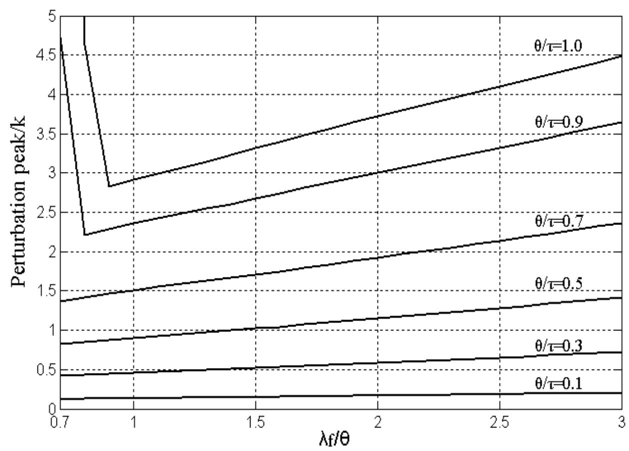

Fig.2 Relationship between the perturbation peak and λ_f/θ

Therefore it is suggested by simulations to tune the control parameter λ_f in the range of $0.8\theta-2.0\theta$. Generally it is recommended to fix λ_f around the process time delay θ value to achieve the best trade-off between the robust stability and nominal performance of the closed-loop for the load disturbance rejection.

3 Simulation tests

Consider the widely studied unstable process such as in recent Wen Tan et al(2003).

$$G_p(s) = \frac{e^{-0.4s}}{s-1}$$

In Wen method, which had already shown its superiority over many other previous approaches, the control parameters were $k_0 = 2$, $\lambda = 0.4$, $K_c = 2.079$, $T_c = 0.156$. Meanwhile the modified Smith predictor method—Majhi et al, 2000 is also employed here for comparison, in which the control parameters are taken as $k_p = 1$, $T_i = 0.4$, $T_f = -0.3$, $K_f = 2$, $K_d = 1.5811$ in terms of the tuning formulas. In the proposed method, take $k_c = 2$, $\lambda_c = \lambda_f = 1.0\theta = 0.4$. By employing the design formula Eqs.(4), (14) and (16) we obtain the setpoint tracking controller and disturbance estimator respectively in form of

$$C(s) = \frac{s+1}{0.4s+1}, \quad F(s) = 2.8972 + \frac{1}{0.724s} + 0.469s$$

A unit step setpoint input is added at t=0 and an inverse unit step load disturbance is added to the process input at t=5. Simulation results are shown in Fig.3.

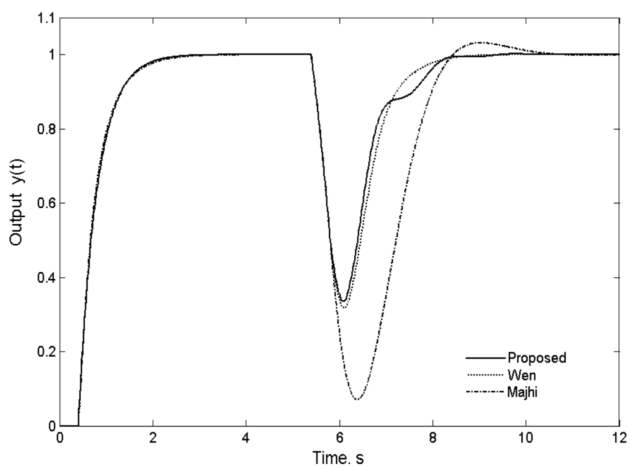

Fig.3 Nominal system responses for the unstable process

It can be seen from Fig.3 that the proposed method shows better load disturbance rejection performance compared with the other two methods in terms of the same setpoint tracking specification. The ISE specifications for the load disturbance response in terms of the three methods are listed in Tab.1.

	Proposed	Wen	Majhi
Attenuation	0.3098	0.3429	0.966

Tab.1 ISE specifications for the load disturbance rejection

Now suppose that there exists 10% error for estimating the process time delay θ such as it is actually 10% larger. In this case, add a unit step setpoint input at t=0 and an inverse unit step load disturbance to the process input at t=7. The perturbed system responses are provided in Fig.4.

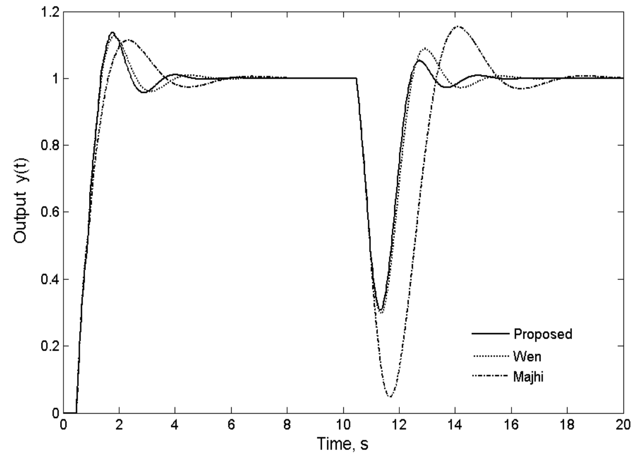

Fig.4 Perturbed system responses for the unstable process

Fig.4 shows that the proposed method keeps the control system good robust stability compared with the other two methods in the presence of the process time delay perturbation. It should be noted that by monotonously increasing the tuning parameter λ_f of the disturbance estimator in the proposed control structure, the control system robust stability becomes better at the cost of the slower load disturbance rejection performance. For illustration, we make simulation tests for the above perturbed unstable process, i.e. increasing the tuning parameters λ_f and λ_c for two cases:

1) $\lambda_c = 0.4, \lambda_f = 0.5$, i.e. $k_f = 2.634, T_I = 0.9566, T_D = 0.4058$

2) $\lambda_c = 0.8, \lambda_f = 0.6$, i.e. $k_f = 2.4394, T_I = 1.219, T_D = 0.3596$

According to the above simulation condition, the simulation results are provided in Fig.5.

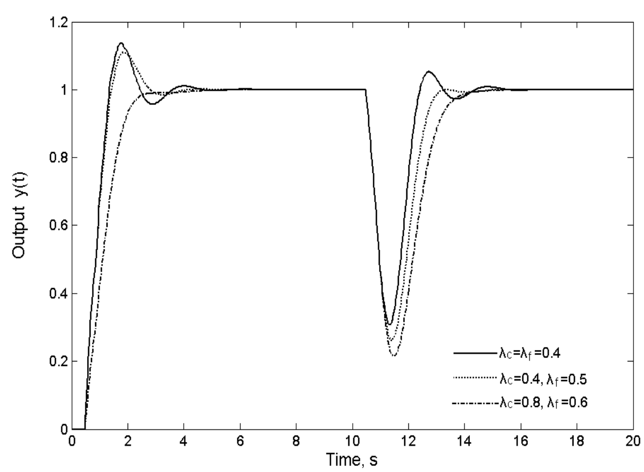

Fig.5 Perturbed system responses in terms of increasing the tuning parameters

Fig.5 shows that increasing the tuning parameter λ_f gradually calms down the load disturbance response oscillation. On the other hand, increasing the tuning parameter λ_c gradually calms down the setpoint response oscillation. Hence it is convenient to monotonously tune the control parameter λ_f to achieve the best trade-off between the nominal performance and robust stability of the closed-loop in the proposed control structure so as to satisfy the disturbance rejection requirement in practice. So is the case for tuning the parameter λ_c to achieve the best trade-off between the nominal performance and dynamic behavior of the setpoint response. Therefore it is convenient to tune λ_c and λ_f monotonously in the proposed control structure to meet the actual process uncertainty, which is really frequently-encountered in industrial and chemical practice.

4 Conclusions

This paper have proposed a new two-degree-of-freedom control structure for controlling unstable processes with time delay. Both of the setpoint response and the load disturbance response can be independently tuned by the setpoint tracking controller and the disturbance estimator in the inner closed-loop for the load disturbance rejection respectively. Hence the setpoint response is decoupled from the load disturbance response. Moreover there exists quantitative tuning relationship between the nominal time domain system response and the control parameters λ_c and λ_f of the setpoint tracking controller and the disturbance estimator, and both the control parameters can be tuned monotonously to meet the actual process uncertainty, which are surely very attractive for operating the proposed control system in practice. By virtue of the analytical design procedures developed for the controllers, the proposed control method is absolutely transparent, and therefore can be conveniently utilized for various unstable processes. Final simulation tests effectively demonstrate the superiority of the proposed control methods.

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References

1 Datta, A., Ho, M.-T and Bhattacharyya, S.P. Structure and Synthesis of PID Controllers. *Springer, NY,* **1999**.

2 De paor, A.M. and O'Malley, M. Controllers of Ziegler-Nichols type for unstable processes. *Int. J. Control,* **1989**, *49*, 1273-1284.

3 Doyle, J.C. and Francis, B.A. et al. Feedback Control Theory. *Macmillan Publishing Company,* **1992.**

4 Ho, W.K. and Xu, A. PID tuning for unstable processes based on gain and phase- margin specifications. *IEE Proc.Control Theory Appl,* **1998**, *145(5)*, 392-396.

5 Kwak, H.J., Sung, S.W. and Lee, In-Beum et al. Modeified Smith predictor with a new structure for unstable processes. *Ind.Eng.Chem.Res,* **1999**, *38(2)*, 405-411.

6 Majhi, S. and Atherton, D. P. Obtaining controller parameters for a new Smith predictor using autotuning. *Automatica,* **2000**, *36*, 1651-1658.

7 Morari, M. and Zafiriou, E. Robust Process Control. *Prentice hall, Englewood Cliffs, NY,* **1989.**

8 Park, J.H., Sung, S.W. and Lee, I.B. An enhanced PID control strategy for unstable processes. *Automatica,* **1998**, *34(6)*, 751-756.

9 Shafiei, Z. and Shenton, A.T. Frequency-domain design of PID controllers for stable and unstable systems with time delay. *Automatica,* **1997**, *33*, 2223-2232.

10 Visioli, A. Optimal tuning of PID controllers for integral and unstable processes. *IEE Proc.Control Theory Appl,* **2001**, *148(2)*, 180-184.

11 Wen, Tan, Horacio J. Marquez. and Tongwen, Chen. IMC design for unstable processes with time delays. *Journal of Process Control,* **2003**, *13*, 203-213.

12 Xue-Ping Yang, Qing-Guo Wang and Hang, C.C. IMC-Based control system design for unstable processes. *Ind.Eng.Chem.Res,* **2002**, *41(17)*, 4288-4294.

13 Ya-Gang Wang and Wen-Jian Cai. Advanced Proportional-Integral-Derivative tuning for integrating and unstable processes with gain and phase margin specifications . *Ind.Eng.Chem.Res,* **2002**, *41(12)*, 2910-2914.

14 Yongho, Lee. and Jeongseok, Lee. PID controllers tuning for integrating and unstable processes with time delay. *Chemical Engineering Science,* **2000**, *55*, 3481-3493.

15 Zhang, W.D. and Xu, X.M. et al. Analytical design for open loop unstable processes with time delay. *Proc. American Control Conference,* **1999**, *6*, 1822-1826.