Jerk derivative feedforward control for motion systems

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Abstract— **This work discusses reference trajectory relevant model based feedforward design. For motion systems which contain at least one rigid body mode and which are subject to reference trajectories with mostly low frequency energy, the proposed feedforward controller improves tracking performance significantly. The feedforward controller may be of much lower order than the plant. The proposed feedforward controller is introduced using a model of an industrial XY-table as an application example.**

I. INTRODUCTION

In many today's motion systems, performance requirements include short motion times and small settling times. Typical examples are pick and place machines, hard disk drives, XY-tables and many robots. To meet these requirements typically, a combination of a feedback and feedforward controller is used in a so called two degree of freedom (2DOF) control architecture, see Figure 1, where
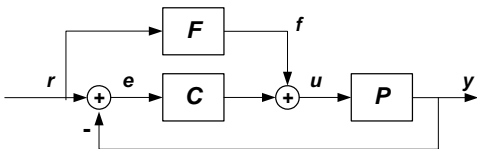
Fig. 1. 2DOF control architecture.

P, C and F represent the plant, feedback controller and feedforward controller, respectively. Signals are denoted by lower case: the reference trajectory r, servo error e, plant input u, plant output y and feedforward function f. The feedback controller guards stability and improves disturbance rejection [9], while the feedforward controller is designed to improve tracking performance. This work will consider the design of a feedforward controller in order to reduce tracking errors during motion (a typical industrial example is displayed in Figure 2).

A. Reference profiles

In industrial practice, commonly used reference profiles describe varying scanning and point-to-point motions. These profiles are often designed as piecewise finite order polynomials. Finite order polynomials typically contain

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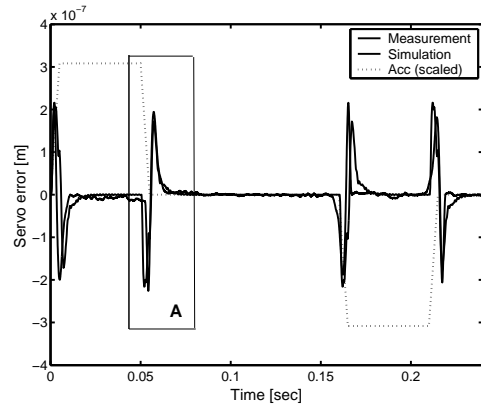
Fig. 2. Servo errors during point-to-point motion of a XY-positioning table using rigid body feedforward and PD controller. Measured servo error (solid thick), simulated servo error (solid thin) and scaled acceleration of reference trajectory (dashed). The area A is studied in later simulations.

constant motion phases (velocity, acceleration, jerk, derivative of jerk, etc.), [1],[5]. A property of these profiles is that they contain mostly low frequency energy, as will be illustrated in section II. The profiles are generally designed such that the resonance dynamics of the plant are not excited. This paper focuses on the case where this is done properly. Would this not be the case, input shaping techniques may be applied, as demonstrated in e.g. [2],[4],[5],[7],[8]. In this paper it is further assumed that the fourth derivative of the position trajectory exists. As will be shown later, this requirement is often satisfied in practice due to discrete time implementation aspects.

B. Plant dynamics

In industrial systems, effort is put in designing mechanics with very high resonance frequencies, with the goal to extend the rigid body-like behavior over a frequency range as large as possible. A typical motion system can therefore be seen as a plant which contains one or more dominant rigid body modes and several resonance modes at higher frequencies (due to limited mechanical stiffness), see also Figure 3. A transfer function model of a SISO motion system can therefore be given as follows:

$$P(s) = \frac{1}{m_t s^2} + \sum_{i=1}^{N} \frac{k_i}{m_t(s^2 + 2\zeta_i \omega_i s + \omega_i^2)}, \quad (1)$$

where m_t is the total mass of the system and N the number of resonance modes with ω_i, ζ_i and k_i the mode

resonance frequency, damping and mode gain $k_i \in \{-1, 1\}$ respectively. Note that by changing the sign of k_i, the modes can show up as resonances or anti-resonances in a bode diagram. When $i = 1$, a two-mass-spring system arises, as presented in [13]. The model does not include damping or friction to the world which is assumed to be negligible or compensated for otherwise.

C. Feedforward design

A common model based feedforward design approach is to make the feedforward controller F equal to the inverse of the plant, thereby directly minimizing the transfer between servo error $e(s)$ and reference trajectory $r(s)$, derived from Figure 1;

$$\frac{e(s)}{r(s)} = \frac{1 - P(s)F(s)}{1 + P(s)C(s)} \qquad (2)$$

Model inversion is not always feasible due to non-minimum phase behavior of the plant. A popular method to overcome these difficulties is the application of a zero phase error tracking controller (ZPETC) [10] or extensions to this scheme [3],[11],[12]. As many motion system contain dominant rigid body behavior, a straightforward approach is rigid body inversion by means of *acceleration feedforward*. In this case the feedforward controller equals a double differentiator times the modelled mass $\hat{m}_t$ of the plant;

$$F(s) = \hat{m}_t s^2 \qquad (3)$$

leading to a feedforward function which is equal to the scaled acceleration of the reference trajectory (hence justifying the method's name). Its simplicity and effectiveness made acceleration feedforward widely applicable in industry.

D. Problem statement

Limitations of acceleration feedforward are experienced in industrial applications. Servo errors during jerk phases in a motion remain, which have typically low frequency behavior and thereby imply an increase of settling time[1], deteriorating performance of motion systems, see Figure 2. These residual servo errors are highly reproducible [6] and have a strong dependence on the acceleration of the motion.

In this work, a feedforward controller which extends the commonly accepted acceleration feedforward with an additional term, is proposed to reduce these residual servo errors. It will be shown that at least a fourth order feedforward controller is needed to compensate for low frequency residual servo errors. The proposed feedforward strategy is therefore named *jerk derivative feedforward*.

[1]The 'settling time' is the time interval *after* acceleration or deceleration after which the servo error is required to be within certain bounds.

E. Outline

The next section starts with an analysis of the residual servo errors when using acceleration feedforward control. In the third section the jerk derivative feedforward controller is described. In section IV, implementation aspects of the proposed method will be discussed. Finally a short discussion wil follow and the work will close with conclusions. Throughout this work, a model of an industrial XY-table is used as a motivating example.

II. OPEN LOOP SERVO ERROR DURING MOTION

To find the origin of residual servo errors during motion, the transfer between the reference trajectory ($r(s)$) and the open loop servo error ($e_o(s)$) is studied;

$$e_o(s) = r(s) - P(s)F(s)r(s) \qquad (4)$$

If the plant is represented by Equation 1 and an acceleration feedforward controller $F(s) = m_t s^2$ is used, the following relation between the reference acceleration and the open loop servo errors results;

$$\frac{e_o(s)}{\ddot{r}(s)} = \sum_{i=1}^{N} \frac{k_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}. \qquad (5)$$

When it is assumed that the natural frequencies of the plant lay much higher then the spectral content of the reference profile (see Figure 3), Equation 5 may be approximated by;

$$\frac{e_o(s)}{\ddot{r}(s)}\Big|_{s\to 0} = \frac{\sum_{i=1}^{N} k_i \prod_{j\in\{1,\dots,N|j\neq i\}} \omega_j^2}{\prod_{i=1}^{N} \omega_i^2}. \qquad (6)$$

The open loop servo error during motion therefore has the shape of the acceleration of the reference, scaled by the right side of Equation 6. Hence, for a plant with a high natural frequency (ω_i), low frequency open loop servo errors remain during acceleration which can not be compensated for using acceleration feedforward.

The closed loop servo error $e(s)$ during motion can be found by filtering the open loop servo error $e_o(s)$ with the sensitivity function $S = (1 + P(s)C(s))^{-1}$, see Equation 2. The sensitivity function has at least a $+2$ slope in lower frequencies due to the rigid body mode of the plant. The closed loop servo error is therefore proportional to the double derivative of the open loop servo error. This results in peaks during non-zero jerk phases of a motion, see Figure 2. In order to illustrate the insight presented in this section, an example from industry is shown below.

A. Motivating Example

We consider a motion system with frequency transfer function given in Figure 3. This model represents the dynamics in one axis of an industrial XY-table. The plant has a dominant rigid body behavior, its first natural frequency lays at approximately $600Hz$. The system is
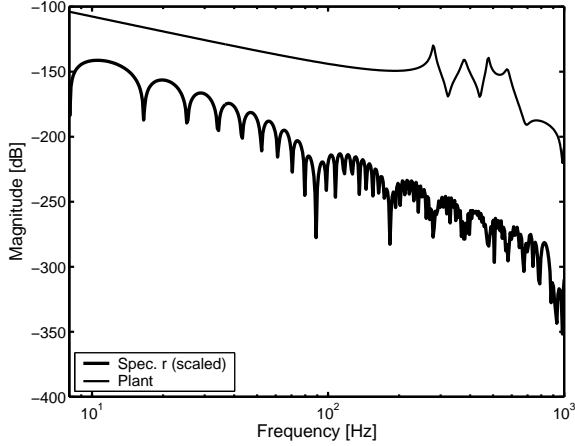
Fig. 3. Spectrum of reference trajectory (scaled) and frequency response of the plant.

Fig. 4. Ideal additive feedforward controller F^* and approximation δs^2.

TABLE I

REFERENCE TRAJECTORY PARAMETERS.

Parameter	x_{max}	v_{max}	a_{max}	j_{max}	Ts
Value	$8cm$	$0.5m/s$	$10m/s^2$	$2000m/s^3$	$0.25ms$

subject to reference trajectories with parameters listed in Table I. The (scaled) acceleration trajectory in time is depicted in Figure 2 by the dotted line. The spectrum of the reference position trajectory is shown in Figure 3. If acceleration feedforward control is used, residual dynamics are described by Equation 5. The maximum open loop servo error during motion may be estimated using Equation 6. As the maximum acceleration equals $10m/s^2$, the maximum open loop error during motion is approximately $500nm$. The open loop servo error has the shape of the acceleration of the reference trajectory, see Figure 5.

Filtering the open loop servo error with the sensitivity obtained by using a PD feedback controller ($0dB$ crossing at $80Hz$, $6dB$ S peak) gives the closed loop servo error shown in Figure 2 with the enlarged area A in Figure 5.

From this example, it appears that the assumptions made earlier are justified in this application. Non-rigid body behavior causes servo errors during motion which cannot be compensated for using acceleration feedforward control. The next section will propose an extension of acceleration feedforward control in order to increase tracking performance.

III. JERK DERIVATIVE FEEDFORWARD

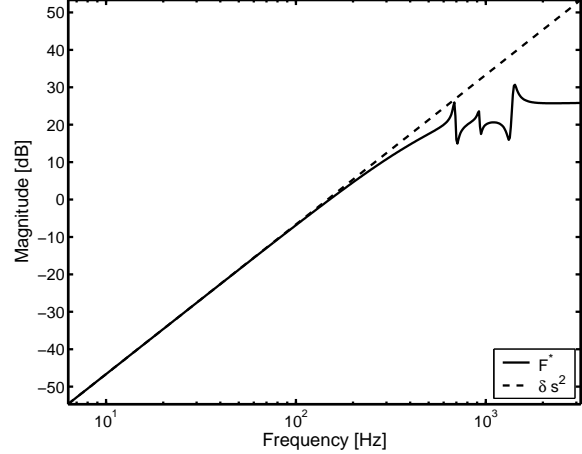In order to reduce the open loop servo error during motion, a new feedforward controller is proposed. The new feedforward controller is assumed to be the original acceleration feedforward controller with an additional term F^*;

$$F = \hat{m}_t s^2 + F^* s^2. \tag{7}$$

A hint on how the term F^* can be designed can be found by putting the open loop servo error during motion (4) equal to zero;

$$F^* = P^{-1} \frac{1}{s^2} - m_t. \tag{8}$$

For this example plant, the frequency response of this 'ideal' feedforward controller, is shown in Figure 4. Since we are interested in the low-frequency contribution of the additional feedforward only, F^* can be approximated by a gain δ times a double differentiator, see Figure 4. So that

$$F^*|_{s \to 0} = \delta s^2, \tag{9}$$

then δ can be derived from Equation 1 and Equation 8;

$$\delta = \frac{-m_t \sum_{i=1}^{N} k_i \prod_{j \in \{1,...,N|j \neq i\}} \omega_j^2}{\prod_{i=1}^{N} \omega_i^2}. \tag{10}$$

Hence, the proposed feedforward controller becomes;

$$F = m_t s^2 + \delta s^4. \tag{11}$$

The second part of this controller is proportional to the fourth derivative or, in other words, the derivative of the jerk of the setpoint. Note that δ is a real scalar, which makes online tuning feasible. The example below will illustrate an application of the jerk derivative feedforward controller.

A. Example using jerk derivative feedforward

The jerk derivative feedforward controller is applied to the example discussed earlier. The constant δ is derived from Equation 10 and used in the new feedforward controller. The reference profile with parameters given in Table I is applied. The resulting servo error during motion
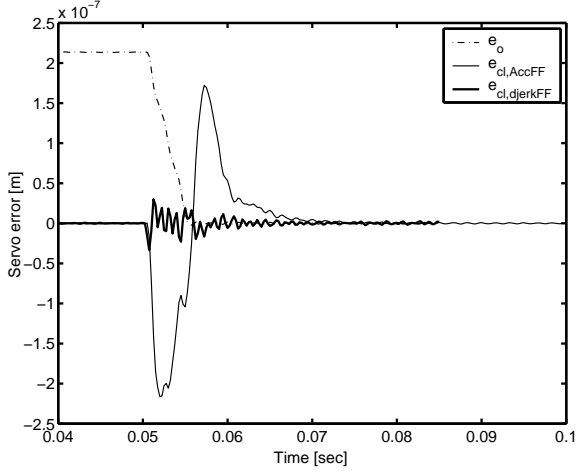
Fig. 5. Servo error during point to point motion, reaching constant velocity (Blok A, Figure 2). Dashed: (scaled) open loop servo error, Solid thin: closed loop servo error using acceleration feedforward, Solid thick: closed loop servo error using jerk derivative feedforward.
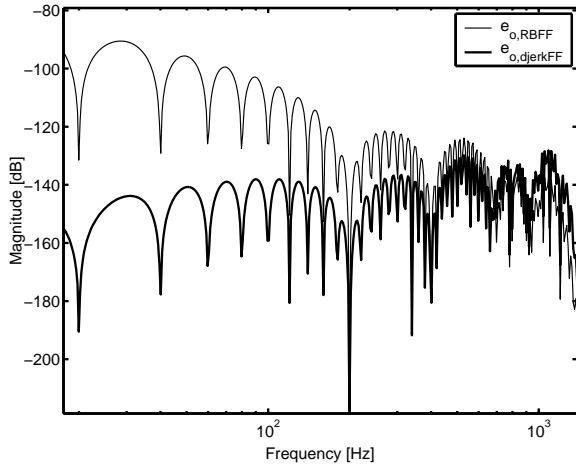

Fig. 6. Spectrum of open loop servo error using acceleration feedforward (solid thin) and jerk derivative feedforward (solid thick).

is shown in Figure 5. Clearly, the servo error is reduced significantly. Comparison of the spectra of the open loop servo errors of acceleration and jerk derivative control shows that jerk derivative control leads to a significant improvement of low frequency tracking, Figure 6.

Notice that δ/m_t is a low frequency approximation of Equation 5. For comparison, the frequency response of both terms is shown in Figure 7. Clearly, the jerk derivative feedforward controller does not follow the residual dynamics at higher frequencies. However, the low frequency contributions of *all* plant modes are compensated for exactly. Jerk derivative feedforward therefore has the freedom to compensate for low frequency
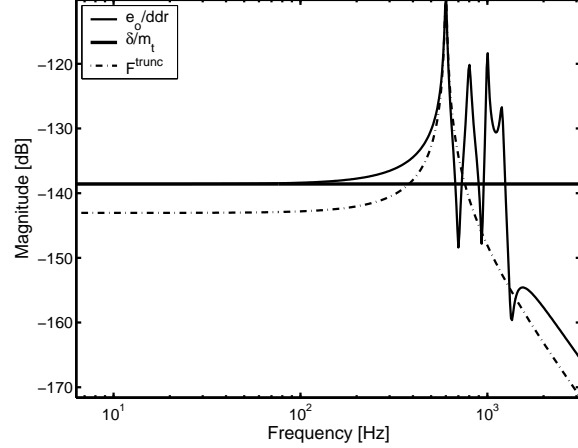

Fig. 7. Residual dynamics $e_o/\ddot{r}$, δ and low frequency effect of F^{trunc}.

residual dynamics of plants with orders much higher than the order of the feedforward controller ($= 4$).

IV. IMPLEMENTATION ASPECTS

For practical implementation of the jerk derivative feedforward, a discrete time equivalent of Equation 11 needs to be derived. Neglecting measurement delay, a possible implementation of the feedforward controller is presented in Figure 8, where q is the shift operator. The gray box
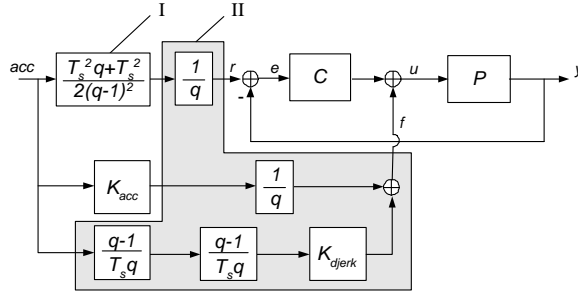

Fig. 8. Possible implementation of jerk derivative feedforward.

includes the blocks that are added to a conventional acceleration feedforward solution. The conventional acceleration feedforward consists of an acceleration feedforward gain $K_{acc}(= m_t)$ and the discrete time equivalent of a double integrator (block I) which computes the position reference from the acceleration signal. The jerk-derivative feedforward path basically consists of a series connection of two differentiating filters and a jerk derivative feedforward gain $K_{djerk}(= \delta)$. The differentiating filters are, by means of delay, implemented in a causal fashion. To compensate for the combined delay of the jerk derivative path, delay operators have to be added to the position and acceleration feedforward path. This is done to match the phase of the setpoint position with the phase of the corresponding

terms in the feedforward controller. Finally, note that in the discrete time implementation of the proposed controller the requirement for the reference trajectory to be at least of fourth order can be relaxed.

V. DISCUSSION

The proposed jerk derivative feedforward controller proves to be capable of compensating low frequency tracking errors which arise due to flexibility in a motion system. The residual dynamics after using jerk derivative feedforward can be calculated following the same strategy as was used in section II. For the example used in this work, these residual dynamics have a magnitude $-280dB$ in low frequency regions. And hence, higher order (higher than 4) feedforward controllers will not lead to a substantial improvement in the low frequency region. In the high frequency region, higher order feedforward design may lead to a better approximation of the residual dynamics. This becomes particularly interesting when resonance frequencies of a motion system are strongly excited by the reference profile. However, as stated in the introduction to this paper, this is in practice often prevented a priori through an appropriate design of the reference trajectory.

The additive form of the jerk derivative feedforward controller has many practical advantages. After tuning the acceleration feedforward, δ may be tuned to cancel residual servo errors during jerk phase. As δ is a constant parameter, the possibility arises to tune the jerk derivative feedforward controller online, monitoring the servo error in the time domain. This is similar to the way in which acceleration feedforward controllers are tuned in current industrial practice.

As was mentioned in the introduction, model based feedforward design is often based on obtaining an approximate inverse of the plant. Often, from a practical point of view, the order of the feedforward controller is constrained. A possible compromising design choice could be to use the inverse of a truncated plant model as a feedforward controller, F^{trunc}. If the order is constrained to be 4, this feedforward controller will have the same order as the jerk derivative feedforward controller. However, F^{trunc} only compensates for the low frequency contribution of this particular inverted mode, see Figure 7, while the jerk derivative feedforward controller compensates for all low frequency modal contributions. Jerk derivative feedforward will therefore have superior performance for references with mostly low frequency energy. In high frequency regions, F^{trunc} approximates the resonance of the inverted mode, while jerk derivative feedforward shows a poor approximation of the resonance behavior, Figure 9. Hence, it is expected that in this region, F^{trunc} will outperform jerk derivative feedforward control.
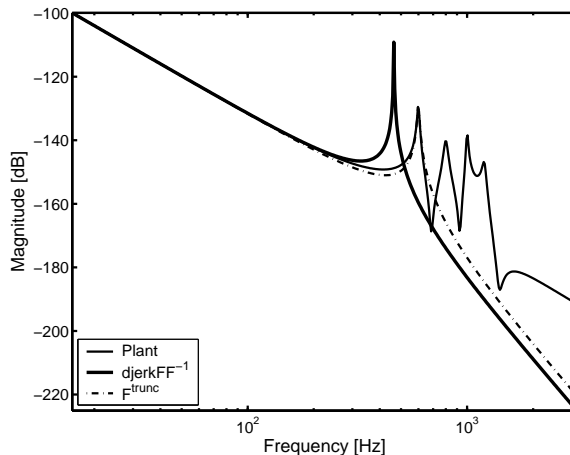

Fig. 9. Plant and inverse of feedforward controllers. Plant (solid thin), Mode inversion based feedforward controller (dashed), derivative of jerk feedforward controller (solid thick).

VI. CONCLUSIONS

For a plant with a rigid body mode and high frequency dynamics, which is subject to reference profiles with mostly low frequency energy, application of only rigid body feedforward may lead to performance limiting residual servo errors during motion. These residual servo errors result from low frequency modal contributions which cannot be compensated for using rigid body feedforward control. A new, setpoint relevant, model based feedforward controller is proposed which uses the derivative of jerk as an additional parameter. Jerk derivative feedforward control compensates for low frequency residual dynamics of plant with (possibly) much higher order than the feedforward controller. Jerk derivative feedforward can be tuned online, similar to common industrial acceleration feedforward tuning procedures.

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