Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Using Anti-Windup Loops for Enlarging the Stability Region of Time-Delay Systems subject to Input Saturation

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Abstract— **This paper focus on the study and the characterization of stability regions for linear systems with delayed states and subject to input saturation through anti-windup strategies. In particular, the synthesis of anti-windup gains in order to guarantee the stability of the closed-loop system for a region of admissible initial states as large as possible is addressed. Based on the modelling of the closed-loop system, resulting from the controller plus the anti-windup loop, as a linear time-delay system with a deadzone nonlinearity, stability conditions in an LMI form are stated, for both the delay independent and delay dependent contexts, by using quadratic functionals and a new sector condition. LMI-based optimization schemes for computing the anti-windup gains that lead to the maximization of the size of the region of stability associated to the closed-loop system are then proposed. The application of the technique and the trade-off between the size of the delay and the region of stability are illustrated by means of a numerical example.**

I. INTRODUCTION

In the last few years, the study of systems presenting time-delays has received a special attention in the control systems literature (see [1] for an interesting overview). This interest comes from the fact that time-delays appear in many kinds of control systems (e.g. chemical, mechanical and communication systems) and their presence can be source of performance degradation and instability. In this sense, we can find in the literature many works giving conditions for ensuring stability as well as performance and robustness requirements, considering or not the delay dependence. Concerning the delay independent results, the stability is ensured no matter the size of the delay [2], [3], [4]. On the other hand, in the delay dependent results, the size of the delay is directly taken into account and this fact can lead, especially when the time-delays are small, to less conservative results [5], [6].

Since physical actuators cannot deliver unlimited signals to the controlled plants, the problem of control saturation and its impact on the stability and the performance of the closed-loop system has also received a lot of attention in the last years. The studies on the analysis and controller design problems for linear systems with input saturation have followed two main approaches. In the first one, the effects of the saturation are directly taken into account in the design of the control law. We can identify methods dealing with the stabilization of the closed-loop system in global, semi-global and local contexts (see among others [7], [8], [9]). The second approach assumes that a controller was previously designed, in order to guarantee some performances. The effects of the saturation on the stability and the performance of the closed-loop system are then considered *a posteriori*. The anti-windup technique fits in this last approach as it consists in introducing control modifications in order to recover, as much as possible, the performance induced by a previous design carried out on the basis of the unsaturated system (see, for example, [10], [11], [12], [13]). It should be pointed out that several results on the anti-windup problem are concerned with achieving global stability properties [11], [14]. Since global results cannot be achieved for open-loop unstable linear systems in the presence of actuator saturation, local results have to be developed. In this context, a key issue is the determination of domains of stability for the closed-loop system, i.e., sets of admissible initial states for which the asymptotic convergence of the corresponding trajectories to the origin is ensured. On the other hand, it should be highlighted that these global stability results do not consider the case of systems with time-delays.

Considering that many practical systems present both time-delays and saturating inputs, from the considerations above, it becomes important to study the stability issues regarding this kind of systems. In this sense, we can identify in the literature some results for systems with delays in the state, proposed mainly in the context of the stabilization via state feedback. In [15], a globally stabilizing state observer based controller is proposed. In [16], [17] and [18], conditions of stability or stabilization are proposed with state feedback and sampled state feedback. However, in these papers, the set of admissible initial conditions for which the asymptotic stability is ensured in the presence of control saturation is not mentioned or explicitly defined. In [19], it was underlined the importance of describing a set of admissible initial conditions associated to the stabilizing control law. On the other hand, considering an anti-windup approach we can cite [20]. In that paper, it is proposed a dynamic anti-windup method for linear systems with control input delays and output measurement delays that ensures bounded input - bounded state stability. It should be highlighted that this method cannot be applied to open-loop unstable systems.

The objective of this paper is the study and the characterization of regions of stability for linear systems with delayed states and subject to input saturation through anti-

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windup strategies. Differently of the most anti-windup techniques cited above, where the synthesis of the anti-windup loop is introduced with the objective of minimizing the performance degradation, we are particularly interested in the synthesis of anti-windup gains in order to guarantee the stability of the closed-loop system for a region of admissible initial states as large as possible. With this aim we propose results both in the delay independent and dependent contexts. These results are derived from a Lyapunov-Krasovskii approach combined with a new and original sector condition that encompasses the classical one found in the literature (see for instance [21] and [22]), and, furthermore, allows to obtain the conditions directly in an LMI form. The search of the anti-windup gains that lead to the maximization of the size of the region of stability associated to the closed-loop system can be carried out from an LMI-based optimization problem.

Notations. $A_{(i)}$ denotes the ith row of matrix A. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. I_m denotes the m-order identity matrix. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote respectively the maximal and minimal eigenvalues of matrix P. $\mathcal{C}_\tau = \mathcal{C}([-\tau, 0], \Re^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\Re^n$ with the topology of uniform convergence. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. $\| \phi \|_c = \sup_{-\tau \le t \le 0} \| \phi(t) \|$ stands for the norm of a function $\phi \in \mathcal{C}_\tau$. When the delay is finite then "sup" can be replaced by "max". $\mathcal{C}_\tau^v$ is the set defined by $\mathcal{C}_\tau^v = \{\phi \in \mathcal{C}_\tau \; ; \; \| \phi \|_c < v, \; v > 0\}$.

II. Problem Statement

Consider the linear continuous-time delay system:

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + A_d x(t-\tau) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \quad (1)$$

with the initial conditions

$$x(t_0 + \theta) = \phi_x(\theta), \forall \theta \in [-\tau, 0], \; t_0 \in \Re_+, \phi_x \in \mathcal{C}_\tau^v \quad (2)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, the input and the measured output vectors, respectively. Matrices A, A_d, B and C are real constant matrices of appropriate dimensions. We suppose also that the input vector u is subject to amplitude limitations defined as follows:

$$|u_{(i)}| \le u_{0(i)}, \quad u_{0(i)} > 0, \; i = 1, ..., m \quad (3)$$

Considering system (1), assume therefore that an n_c-order dynamic output stabilizing compensator

$$\begin{array}{l} \dot{\eta}(t) = A_c \eta(t) + B_c y(t) \\ y_c(t) = C_c \eta(t) + D_c y(t) \end{array} \quad (4)$$

where $\eta(t) \in \Re^{n_c}$ is the controller state, $u_c(t) = y(t) \in \Re^p$ is the controller input and $y_c(t) \in \Re^m$ is the controller output, has been designed in order to guarantee some performance requirements and the stability of the closed-loop system in the absence of the control saturation. Matrices A_c, B_c, C_c et D_c are of appropriate dimensions. In consequence of the control bounds, the control signal to be injected in the system is a saturated one, that is,

$$u(t) = sat(y_c(t)) = sat(C_c \eta(t) + D_c C x(t)) \quad (5)$$

where each component of $sat(y_c)$ is defined as

$$sat(y_c)_{(i)} = sat(y_{c(i)}) = sign(y_{c(i)}) \min(|y_{c(i)}|, u_{0(i)}) \quad (6)$$

In order to mitigate the undesirable effects of windup, caused by input saturation, an anti-windup term $E_c(sat(y_c(t)) - y_c(t))$, $E_c \in \Re^{n_c \times m}$, can be added to the controller [11]. Thus, considering the dynamic controller and this anti-windup strategy, the closed-loop system reads:

$$\begin{array}{l} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + B sat(y_c(t)) \\ y(t) = Cx(t) \\ \dot{\eta}(t) = A_c \eta(t) + B_c y(t) + E_c(sat(y_c(t)) - y_c(t)) \\ y_c(t) = C_c \eta(t) + D_c y(t) \end{array} \quad (7)$$

Define now an extended state vector $\xi(t) = [x(t)' \quad \eta(t)']' \in \Re^{n+n_c}$, and the following matrices: $\mathbb{A} = \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix}$, $\mathbb{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\mathbb{R} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix}$ and $\mathbb{K} = \begin{bmatrix} D_cC & C_c \end{bmatrix}$. Hence, the closed-loop system reads:

$$\dot{\xi}(t) = \mathbb{A}\xi(t) + \mathbb{A}_d \xi(t-\tau) - (\mathbb{B} + \mathbb{R}E_c)\psi(\mathbb{K}\xi(t)) \quad (8)$$

with $\psi(\mathbb{K}\xi(t)) = y_c(t) - sat(y_c(t)) = \mathbb{K}\xi(t) - sat(\mathbb{K}\xi(t))$ Note that, $\psi(\mathbb{K}\xi)$ corresponds to a decentralized deadzone nonlinearity, i.e., $\psi(\mathbb{K}\xi(t))_{(i)} = \psi(\mathbb{K}_{(i)}\xi(t)) = \mathbb{K}_{(i)}\xi(t) - sat(\mathbb{K}_{(i)}\xi(t))$.

The augmented system (8) admits an augmented initial condition

$$\xi(t_0 + \theta) = \phi_\xi(\theta) = \begin{bmatrix} x(t_0 + \theta) \\ \eta(t_0 + \theta) \end{bmatrix} = \begin{bmatrix} \phi_x(\theta) \\ \phi_\eta(\theta) \end{bmatrix},$$

$\forall \theta \in [-\tau, 0]$, where $\phi_\xi(\theta)$ satisfies $\| \phi_\xi \|_c \le v$, $v > 0$.

System (8) will be said globally asymptotically stable if for any initial condition satisfying $\|\phi_\xi\|_c \le v$ with any finite v, the trajectories of system (8) converge asymptotically to the origin [15], [23]. Similar to the case of delay-free ($\tau = 0$), the determination of a global stabilizing controller is possible only when some stability assumptions are verified by the open-loop system ($u(t) = 0$) [8]. When this hypothesis is not verified, it is only possible to achieve local stabilization. In fact, in the generic case, given a stabilizing matrix $\mathbb{K}$, we associate a *basin of attraction* to the equilibrium point $\xi_e = 0$ of system (8). The basin of attraction corresponds to all initial conditions $\phi_\xi(\theta) \in \mathcal{C}_\tau$ such that the corresponding trajectories of system (8) converge asymptotically to the origin. Since the determination of the exact basin of attraction is practically impossible, a problem of interest is to ensure the asymptotic stability for a set $B(\delta) = \{\phi_\xi \in \mathcal{C}_\tau; \| \phi_\xi \|_c^2 \le \delta\}$ of admissible initial conditions $\phi_\xi(\theta)$ [19]. Of course, the set $B(\delta)$ is included in the basin of attraction. Throughout the paper we will refer a set $B(\delta)$ as *a region of stability* for system (8). The problem we aim to solve throughout this paper can then be summarized as follows.

Problem 1: Determine the anti-windup gain matrix E_c and a scalar δ, as large as possible, such that the asymptotic stability of system (8) is ensured for all initial conditions $\phi_\xi(\theta) \in B(\delta) = \{\phi_\xi \in \mathcal{C}_\tau ; \| \phi_\xi \|_c^2 \leq \delta\}$, $\forall \theta \in [-\tau, 0]$.

Since $B(\delta)$ can be viewed as an estimate of the basin of attraction of the system (8), the implicitly idea behind Problem 1 is to enlarge this basin over the choice of the anti-windup gain matrix E_c.

It should be pointed out that depending on the stability property of matrices $\mathbb{A}$, $\mathbb{A}+\mathbb{A}_d$ and $\mathbb{A}-\mathbb{A}_d$, we should study the closed-loop stability of system (8) in a delay-independent or delay-dependent context. Indeed, a necessary condition for the delay-independent stability is to have $\mathbb{A}$, $\mathbb{A}+\mathbb{A}_d$ Hurwitz and $\mathbb{A}-\mathbb{A}_d$ not strictly unstable (i.e., $\mathbb{A}-\mathbb{A}_d$ have not eigenvalues with positive real part) [24]. Hence, throughout the paper, we address Problem 1 both in the delay dependent and independent contexts. The measures and the criteria allowing to optimize the size of the region of stability of the closed-loop system, the tradeoff between this one and the admissible upper bound on the delay will be discussed in each case.

III. PRELIMINARIES

Consider a matrix $F \in \Re^{m \times (n+nc)}$ and define the set

$$\mathcal{S} \stackrel{\triangle}{=} \{\xi \in \Re^{n+n_c} \; ; \; |\mathbb{K}_{(i)} - F_{(i)}|\xi \leq u_{0(i)}, \; i=1,...,m\} \tag{9}$$

Lemma 1: Consider the function $\psi(\mathbb{K}\xi)$ defined in (II). If $\xi \in \mathcal{S}$ then the relation

$$\psi(\mathbb{K}\xi)'T[\psi(\mathbb{K}\xi) - F\xi] \leq 0 \tag{10}$$

is verified for any matrix $T \in \Re^{m \times m}$ diagonal and positive definite.

Proof: Consider the three cases below.

(a): $-u_{0(i)} \leq \mathbb{K}_{(i)}\xi \leq u_{0(i)}$. In this case, by definition, $\psi(\mathbb{K}_{(i)}\xi) = 0$ and then

$$\psi(\mathbb{K}_{(i)}\xi)T_{(i,i)}[\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi] = 0$$

(b): $\mathbb{K}_{(i)}\xi > u_{0(i)}$. In this case, $\psi(\mathbb{K}_{(i)}\xi) = \mathbb{K}_{(i)}\xi - u_{0(i)}$. If $\xi \in \mathcal{S}$ it follows that $\mathbb{K}_{(i)}\xi - F_{(i)}\xi \leq u_{0(i)}$. Hence, it follows that $\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi = \mathbb{K}_{(i)}\xi - u_{0(i)} - F_{(i)}\xi \leq 0$ and, since in this case $\psi(\mathbb{K}_{(i)}\xi) > 0$, one gets

$$\psi(\mathbb{K}_{(i)}\xi)T_{(i,i)}[\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi] \leq 0, \quad \forall T_{(i,i)} > 0$$

(c): $\mathbb{K}_{(i)}\xi < -u_{0(i)}$. In this case, $\psi(\mathbb{K}_{(i)}\xi) = \mathbb{K}_{(i)}\xi + u_{0(i)}$. If $\xi \in \mathcal{S}$ it follows that $\mathbb{K}_{(i)}\xi - F_{(i)}\xi \geq -u_{0(i)}$. Hence, it follows that $\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi = \mathbb{K}_{(i)}\xi + u_{0(i)} - F_{(i)}\xi \geq 0$ and, since in this case $\psi(\mathbb{K}_{(i)}\xi) < 0$, one gets

$$\psi(\mathbb{K}_{(i)}\xi)T_{(i,i)}[\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi] \leq 0, \quad \forall T_{(i,i)} > 0$$

From the 3 cases above, once $\xi \in \mathcal{S}$ we can conclude that $\psi(\mathbb{K}_{(i)}\xi)T_{(i,i)}[\psi(\mathbb{K}_{(i)}\xi) - F_{(i)}\xi] \leq 0$, $\forall T_{(i,i)} > 0$, $\forall i = 1, \ldots, m$, whence follows (10). $\square$

IV. DELAY-INDEPENDENT RESULTS

Consider the following Lyapunov candidate functional:

$$V(\xi_t) = \xi(t)'P\xi(t) + \int_{t-\tau}^{t} \xi(\theta)'S\xi(\theta)d\theta \tag{11}$$

with $P = P' > 0, \;\; S = S' > 0$ and where ξ_t, $\forall t \geq t_0$, denotes the restriction of ξ to the interval $[t-\tau \, , \, t]$ translated to $[-\tau \, , \, 0]$, that is, $\xi_t = \xi(t+\theta) \; , \; \forall \theta \in [-\tau \, , \, 0]$.

Proposition 1: If there exist symmetric positive definite matrices $W \in \Re^{(n+n_c)\times(n+n_c)}$ and $R \in \Re^{(n+n_c)\times(n+n_c)}$, a diagonal positive definite matrix $G \in \Re^{m \times m}$, matrices $Y \in \Re^{m \times (n+n_c)}$ and $Z \in \Re^{n_c \times m}$ satisfying:

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W + R & \mathbb{A}_d W & \mathbb{B}G + \mathbb{R}Z - Y' \\ W\mathbb{A}_d' & -R & 0 \\ G\mathbb{B}' + Z'\mathbb{R}' - Y & 0 & -2G \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} W & W\mathbb{K}_{(i)}' - Y_{(i)}' \\ \mathbb{K}_{(i)}W - Y_{(i)} & u_{0(i)}^2 \end{bmatrix} \geq 0, \; i = 1, ..., m \tag{13}$$

then, for $E_c = ZG^{-1}$, it follows that for all initial conditions $\phi_\xi(\theta) \in B(\delta) = \{\phi_\xi \in \mathcal{C}_\tau ; \| \phi_\xi \|_c^2 \leq \delta\}$, $\forall \theta \in [-\tau, 0]$ with

$$\delta = (\lambda_{max}(W^{-1}) + \tau \lambda_{max}(W^{-1}RW^{-1}))^{-1} \tag{14}$$

the corresponding trajectories of system (8) converge asymptotically to the origin.

Proof. The satisfaction of relations (13) with implies that the set $\mathcal{E}(W^{-1}, 1) \stackrel{\triangle}{=} \{\xi \in \Re^{n+n_c} ; \; \xi' W^{-1} \xi \leq 1\}$ is included in the polyhedral set $\mathcal{S}$ with $F = YW^{-1}$ [19]. Hence, $\forall \xi(t) \in \mathcal{E}(W^{-1}, 1)$ it follows, from Lemma 1, that $\psi(\mathbb{K}\xi(t)) = \mathbb{K}\xi(t) - sat(\mathbb{K}\xi(t))$ satisfies the sector condition (10).

By considering a Lyapunov candidate function as defined in (11), and by computing its time-derivative along the trajectories of system (8) one gets: $\dot{V}(\xi_t) = \xi(t)'(\mathbb{A}'P + P\mathbb{A})\xi(t) - 2\xi(t)'P(\mathbb{B} + \mathbb{R}E_c)\psi(\mathbb{K}\xi(t)) + 2\xi(t)'P\mathbb{A}_d\xi(t-\tau) + \xi(t)'S\xi(t) - \xi(t-\tau)'S\xi(t-\tau)$. Thus, by using the sector condition (10), it follows, $\forall T$ diagonal and positive definite, that

$$\dot{V}(\xi_t) \leq \dot{V}(\xi_t) - 2\psi(\mathbb{K}\xi(t))'T[\psi(\mathbb{K}\xi(t)) - F\xi(t)] \tag{15}$$

Since $S > 0$ and $T > 0$ it follows that $-\xi(t-\tau)'S\xi(t-\tau) + 2\xi(t)'P\mathbb{A}_d\xi(t-\tau) \leq \xi(t)'P\mathbb{A}_d S^{-1}\mathbb{A}_d' P\xi(t)$ and $-2\psi(\mathbb{K}\xi(t))'T\psi(\mathbb{K}\xi(t)) + 2\xi'(t)(F'T - P(\mathbb{B} + \mathbb{R}E_c))\psi(\mathbb{K}\xi(t)) \leq 0.5\xi(t)'(F'T - P(\mathbb{B}+\mathbb{R}E_c))T^{-1}(F'T - P(\mathbb{B} + \mathbb{R}E_c))'\xi(t)$. Hence, from (15) one has

$$\dot{V}(\xi_t) \leq \xi(t)'\mathcal{L}\xi(t) \;, \quad \forall \xi(t) \in \mathcal{S} \tag{16}$$

with $\mathcal{L} \stackrel{\triangle}{=} \mathbb{A}'P + P\mathbb{A} + S + P\mathbb{A}_d S^{-1}\mathbb{A}_d' P + 0.5(F'T - P(\mathbb{B} + \mathbb{R}E_c))T^{-1}(F'T - P(\mathbb{B} + \mathbb{R}E_c))'$.

Consider now inequality (12). Pre and post-multiplying this inequality by $\begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & T \end{bmatrix}$, and considering $P^{-1} =$

W, $T^{-1} = G$, $Y = FW$, $E_c = ZG^{-1}$ and $S = PRP$, it follows that (12) is equivalent to

$$\begin{bmatrix} \mathbb{A}'P + P\mathbb{A} + S & P\mathbb{A}_d & P\mathbb{B} + P\mathbb{R}E_c - F'T \\ \mathbb{A}_d'P & -S & 0 \\ \mathbb{B}'P + E_c'\mathbb{R}'P - TF & 0 & -2T \end{bmatrix} < 0$$

and, from Schur's complement, we can conclude that (12) is equivalent to $\mathcal{L} < 0$. Hence, provided that $\xi(t) \in \mathcal{S}$, if (12) is verified one gets:

$$\begin{array}{c} (i)\ \dot{V}(\xi_t) < \pi_1 ||\xi(t)||^2 < 0 \\ (ii)\ \pi_2 ||\xi(t)||^2 \leq V(\xi_t) \leq \pi_3 ||\xi_t||_c^2 \end{array}$$

with $\pi_2 = \lambda_{min}(P)$ and $\pi_3 = \lambda_{max}(P) + \tau\lambda_{max}(S)$. From (14) and (ii), it follows that for $\phi_\xi(\theta) \in B(\delta)$, $\theta \in [-\tau, 0]$, one gets $\xi(t)'P\xi(t) \leq V(\xi_t) \leq V(\xi_{t_0}) \leq 1$, $\forall t \geq t_0$. Hence, for any initial condition in the ball $B(\delta)$, one has $\xi(t) \in \mathcal{E}(W^{-1}, 1) \subset \mathcal{S}$, $\forall t \geq t_0$, provided that (13) is satisfied. Thus, for any initial condition belonging to $B(\delta)$ conditions (i) and (ii) of the Krasovskii Theorem [25] are verified, ensuring the asymptotic stability of the closed-loop system (8). □

Proposition 1 provides a condition in a local context of stability. Under the assumption of open-loop stability, this result can be extended to ensure global stability as follows.

Corollary 1: If there exist symmetric positive definite matrices $W \in \Re^{(n+n_c)\times(n+n_c)}$ and $R \in \Re^{(n+n_c)\times(n+n_c)}$, a diagonal positive definite matrix $G \in \Re^{m\times m}$, and a matrix, $Z \in \Re^{n_c \times m}$ satisfying:

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W + R & \mathbb{A}_d W & \mathbb{B}G + \mathbb{R}Z - W\mathbb{K}' \\ W\mathbb{A}_d' & -R & 0 \\ G\mathbb{B}' + Z'\mathbb{R}' - \mathbb{K}W & 0 & -2G \end{bmatrix} < 0 \quad (17)$$

then, for $E_c = ZG^{-1}$, it follows that the origin of the closed-loop system (8) is globally asymptotically stable.

Proof: Consider $F = \mathbb{K}$. It follows that (10) is verified for all $\xi \in \Re^{n+n_c}$. In this case, (12) corresponds to (17) and the global asymptotic stability follows. □

V. Delay-dependent results

Since $\xi(t)$ is continuously differentiable for $t \geq 0$, from the Leibnitz-Newton formula, it follows that

$$\xi(t-\tau) = \xi(t) - \int_{-\tau}^{0} \dot{\xi}(t+\beta)d\beta \quad (18)$$

Hence, from [25], to ensure the stability of the closed-loop system (8) it suffices to ensure the stability for the following system:

$$\begin{array}{l} \dot{\xi}(t) = (\mathbb{A} + \mathbb{A}_d)\xi(t) - (\mathbb{B} + \mathbb{R}E_c)\psi(\mathbb{K}\xi(t)) \\ \quad - \int_{-\tau}^{0} [\mathbb{A}_d\mathbb{A}\xi(t+\beta) - \mathbb{A}_d(\mathbb{B} + \mathbb{R}E_c)\psi(\mathbb{K}\xi(t+\beta))]d\beta \\ \quad - \int_{-\tau}^{0} \mathbb{A}_d\mathbb{A}_d\xi(t+\beta-\tau)d\beta \end{array} \quad (19)$$

with the initial data $\xi(t_0 + \theta) = \phi_\xi(\theta), \forall\theta \in [-2\tau, 0]$.

Consider the Lyapunov-Krasovskii functional

$$V(\xi_t) = \xi(t)'P\xi(t) + Q(\xi_t) \quad (20)$$

where $P = P' > 0$ and $Q(\xi_t)$ is a positive definite quadratic form that will be defined in the sequel.

Proposition 2: Given $\tau > 0$, if there exist symmetric positive definite matrices $W \in \Re^{(n+n_c)\times(n+n_c)}$, $X \in \Re^{(n+n_c)\times(n+n_c)}$, $R \in \Re^{(n+n_c)\times(n+n_c)}$ and $H \in \Re^{(n+n_c)\times(n+n_c)}$, a diagonal positive definite matrix $G \in \Re^{m\times m}$, matrices $Y \in \Re^{m\times(n+n_c)}$ and $Z \in \Re^{n_c\times m}$ satisfying[1]:

$$\begin{bmatrix} \Gamma & \tau W\mathbb{A}' & \tau W\mathbb{A}_d' & \Xi & 0 \\ \star & -\tau X & 0 & 0 & 0 \\ \star & \star & -\tau R & 0 & 0 \\ \star & \star & \star & -2G & \tau(G\mathbb{B}' + Z'\mathbb{R}') \\ \star & \star & \star & \star & -\tau H \end{bmatrix} < 0 \quad (21)$$

$$\begin{bmatrix} W & W\mathbb{K}_{(i)}' - Y_{(i)}' \\ \mathbb{K}_{(i)}W - Y_{(i)} & u_{0(i)}^2 \end{bmatrix} \geq 0,\ i = 1, ..., m \quad (22)$$

where $\Gamma = W(\mathbb{A} + \mathbb{A}_d)' + (\mathbb{A} + \mathbb{A}_d)W + \tau\mathbb{A}_d(X + R + H)\mathbb{A}_d'$ and $\Xi = -Y' + \mathbb{B}G + \mathbb{R}Z$, then, for $E_c = ZG^{-1}$, it follows that for all initial conditions $\phi_\xi(\theta) \in B(\delta) = \{\phi_\xi \in \mathcal{C}_{2\tau};\, \| \phi_\xi \|_c^2 \leq \delta\}$, $\forall\theta \in [-\tau, 0]$ with $\delta = (\lambda_{max}(W^{-1}) + \frac{3\tau^2}{2}\lambda_{max}(\mathbb{A}_d'R^{-1}\mathbb{A}_d) + \frac{\tau^2}{2}\lambda_{max}(\mathbb{A}'X^{-1}\mathbb{A}) + \frac{\tau^2}{2}\lambda_{max}((\mathbb{B} + \mathbb{R}E_c)'H^{-1}(\mathbb{B} + \mathbb{R}E_c)) \| YW^{-1} \|^2)^{-1}$, the corresponding trajectories of system (8) converge asymptotically to the origin.

Proof. The satisfaction of relations (22) with implies that the set $\mathcal{E}(W^{-1}, 1) \subset \mathcal{S}$ with $F = YW^{-1}$. Hence, $\forall\xi(t) \in \mathcal{E}(W^{-1}, 1)$ it follows, from Lemma 1 that $\psi(\mathbb{K}\xi(t)) = \mathbb{K}\xi(t) - sat(\mathbb{K}\xi(t))$ satisfies the sector condition (10).

By considering the Lyapunov candidate function as defined in (20), and by computing its time-derivative along the trajectories of system (19) one gets[2]: $\dot{V}(\xi_t) = \dot{\xi}(t)'P\xi(t) + \xi(t)'P\dot{\xi}(t) + \dot{Q}(\xi_t) = \xi(t)'[P(\mathbb{A} + \mathbb{A}_d) + (\mathbb{A} + \mathbb{A}_d)'P]\xi(t) - 2\xi(t)'P(\mathbb{B} + \mathbb{R}E_c)\psi(t) + \mu(\xi_t) + \nu(\xi_t) + \zeta(\xi_t) + \dot{Q}(\xi_t)$ where $\mu(\xi_t) = -2\int_{-\tau}^{0} \xi(t)'P\mathbb{A}_d\mathbb{A}\xi(t+\beta)d\beta$, $\nu(\xi_t) = -2\int_{-\tau}^{0} \xi(t)'P\mathbb{A}_d\mathbb{A}_d\xi(t-\tau+\beta)d\beta$ and $\zeta(\xi_t) = 2\int_{-\tau}^{0} \xi(t)'P\mathbb{A}_d(\mathbb{B} + \mathbb{R}E_c)\psi(t+\beta)d\beta$.

Using the fact that $2u'v \leq u'Mu + v'M^{-1}v$ where M is any symmetric positive definite matrix and u and v are vectors of appropriate dimensions, it follows that $\mu(\xi_t) \leq \tau\xi(t)'P\mathbb{A}_d X\mathbb{A}_d'P\xi(t) + \int_{-\tau}^{0} \xi(t+\beta)'\mathbb{A}'X^{-1}\mathbb{A}\xi(t+\beta)d\beta$, $\nu(\xi_t) \leq \tau\xi(t)'P\mathbb{A}_d R\mathbb{A}_d'P\xi(t) + \int_{-\tau}^{0} \xi(t-\tau+\beta)'\mathbb{A}_d'R^{-1}\mathbb{A}_d\xi(t-\tau+\beta)d\beta$ and $\zeta(\xi_t) \leq \tau\xi(t)'P\mathbb{A}_d H\mathbb{A}_d'P\xi(t) + \int_{-\tau}^{0} \psi(t+\beta)'(\mathbb{B} + \mathbb{R}E_c)'H^{-1}(\mathbb{B} + \mathbb{R}E_c)\psi(t+\beta)d\beta$.

Defining now $Q(\xi_t) = \int_{-\tau}^{0}\int_{t+\beta}^{t} \xi(\theta)'\mathbb{A}'X^{-1}\mathbb{A}\xi(\theta)d\theta d\beta + \int_{-\tau}^{0}\int_{t-\tau+\beta}^{t} \xi(\theta)'\mathbb{A}_d'R^{-1}\mathbb{A}_d\xi(\theta)d\theta d\beta + \int_{-\tau}^{0}\int_{t+\beta}^{t} \psi(\theta)'(\mathbb{B} + \mathbb{R}E_c)'H^{-1}(\mathbb{B} + \mathbb{R}E_c)\psi(\theta)d\theta d\beta$ one obtains $\dot{Q}(\xi_t) = -\int_{-\tau}^{0} \xi(t+\beta)'\mathbb{A}'X^{-1}\mathbb{A}\xi(t+\beta)d\beta + \tau\xi(t)'\mathbb{A}'X^{-1}\mathbb{A}\xi(t) - \int_{-\tau}^{0} \xi(t-\tau+\beta)'\mathbb{A}_d'R^{-1}\mathbb{A}_d\xi(t-\tau+\beta)d\beta + \tau\xi(t)'\mathbb{A}_d'R^{-1}\mathbb{A}_d\xi(t) - \int_{-\tau}^{0} \psi(t+\beta)'(\mathbb{B} + \mathbb{R}E_c)'H^{-1}(\mathbb{B} + \mathbb{R}E_c)\psi(t+\beta)d\beta + \tau\psi(t)'(\mathbb{B} + \mathbb{R}E_c)'H^{-1}(\mathbb{B} + \mathbb{R}E_c)\psi(t)$.

Hence, considering the sector condition (10), it follows that $\dot{V}(\xi_t) \leq \xi(t)'[P(\mathbb{A} + \mathbb{A}_d) + (\mathbb{A} + \mathbb{A}_d)'P]\xi(t) - 2\xi(t)'P(\mathbb{B} + \mathbb{R}E_c)\psi(t) + \tau\xi(t)'P\mathbb{A}_d X\mathbb{A}_d'P\xi(t) + \tau\xi(t)'\mathbb{A}'X^{-1}\mathbb{A}\xi(t) +$

[1] $\star$ stands for symmetric blocks.

[2] For notational simplicity, we denote $\psi(\mathbb{K}\xi(t))$ as $\psi(t)$ throughout the proof.

$\tau\xi(t)'P\mathbb{A}_dR\mathbb{A}_d'P\xi(t) + \tau\xi(t)'\mathbb{A}_d'R^{-1}\mathbb{A}_d\xi(t) + \tau\xi(t)'P\mathbb{A}_dH\mathbb{A}_d'P\xi(t) + \tau\psi(t)'(\mathbb{B}+\mathbb{R}E_c)'H^{-1}(\mathbb{B}+\mathbb{R}E_c)\psi(t) - 2\psi(t)'T[\psi(t) - F\xi(t)]$, $\forall\xi(t) \in \mathcal{S}$, $\forall T > 0$ diagonal.

Following now a similar reasoning to the one done in the proof of Proposition 1, it is easy to show that

$$\dot{V}(\xi_t) \le \xi(t)'\mathcal{L}\xi(t) < 0, \quad \forall\xi(t) \in \mathcal{S} \tag{23}$$

with $\mathcal{L} \triangleq P(\mathbb{A} + \mathbb{A}_d) + (\mathbb{A} + \mathbb{A}_d)'P + \tau P\mathbb{A}_d(X + R + H)\mathbb{A}_d'P + \tau\mathbb{A}'X^{-1}\mathbb{A} + \tau\mathbb{A}_d'R^{-1}\mathbb{A}_d + (-F'T + P(\mathbb{B} + \mathbb{R}E_c))(2T - \tau(\mathbb{B}+\mathbb{R}E_c)H^{-1}(\mathbb{B}+\mathbb{R}E_c))^{-1}(-F'T + P(\mathbb{B} + \mathbb{R}E_c))'$, and that inequality (21) is equivalent to $\mathcal{L} < 0$.

Hence, provided that $\xi(t) \in \mathcal{S}$, if (21) is verified one gets: $\dot{V}(\xi_t) < \pi_1||\xi(t)||^2 < 0$ and $\pi_2||\xi(t)||^2 \le V(\xi_t) \le \pi_3||\xi_t||_c^2$, with $\pi_2 = \lambda_{min}(P)$. The computation of π_3 needs to study the overbounding of $V(\xi_t)$ and therefore that one of $Q(\xi_t)$. Thus, we have to express the upper bound on the norm of $\psi(t)$. For $\xi(t) \in \mathcal{S}$, one can verify that $\psi(\mathbb{K}\xi(t))$ satisfies $||\psi(\mathbb{K}\xi(t))|| \le ||F\xi(t)|| \le ||F||||\xi(t)||$

Hence, from (20) one gets $\pi_3 = \delta^{-1}$, and, for $\phi_\xi(\theta) \in B(\delta)$, $\forall\theta \in [-2\tau, 0]$, one gets $\xi(t)'P\xi(t) \le V(\xi_t) \le V(\xi_{t_0}) \le 1$, $\forall t \ge t_0$. Thus, following the same reasoning used in the proof of Proposition 1 we can conclude that $\forall\phi_\xi(\theta) \in B(\delta)$, $\theta \in [-2\tau, 0]$, the asymptotic stability of the system (19) is ensured and, as consequence, the asymptotic stability of system (8) is ensured $\forall\phi_\xi(\theta) \in B(\delta)$, $\theta \in [-\tau, 0]$. □

Concerning global stability, the following corollary can be stated.

Corollary 2: If there exist symmetric positive definite matrices $W \in \Re^{(n+n_c)\times(n+n_c)}$, $X \in \Re^{(n+n_c)\times(n+n_c)}$, $R \in \Re^{(n+n_c)\times(n+n_c)}$ and $H \in \Re^{(n+n_c)\times(n+n_c)}$, a diagonal positive definite matrix $G \in \Re^{m\times m}$, and a matrix $Z \in \Re^{n_c\times m}$ satisfying:

$$\begin{bmatrix} \Gamma & \tau W\mathbb{A}' & \tau W\mathbb{A}_d' & \tilde{\Xi} & 0 \\ \star & -\tau X & 0 & 0 & 0 \\ \star & \star & -\tau R & 0 & 0 \\ \star & \star & \star & -2G & \tau(G\mathbb{B}' + Z'\mathbb{R}') \\ \star & \star & \star & \star & -\tau H \end{bmatrix} < 0 \tag{24}$$

with $\tilde{\Xi} = -W\mathbb{K}' + \mathbb{B}G + \mathbb{R}Z$, then, for $E_c = ZG^{-1}$, it follows that the origin of the closed-loop system (8) is globally asymptotically stable.

Remark 1: The results in [22] appear as particular cases of Propositions 1 and 2. In that paper, $\psi(\mathbb{K}\xi)$ satisfies the classical sector condition (see for instance [21] and references therein):

$$\psi(\mathbb{K}\xi)'T[\psi(\mathbb{K}\xi) - \Lambda\mathbb{K}\xi] \le 0, \quad \forall\xi \in S(\mathbb{K}, u_0^\lambda) \tag{25}$$

where Λ is a positive diagonal matrix and

$$S(\mathbb{K}, u_0^\lambda) \triangleq \{\xi \in \Re^{n+n_c}; |\mathbb{K}_{(i)}\xi| \le \tfrac{u_{0(i)}}{1-\Lambda_{(i,i)}},\ i = 1, ..., m\} \tag{26}$$

Considering this classical sector condition (25) and following a similar procedure to the one applied in the proof of Propositions 1 and 2, it is only possible to obtain conditions that are bilinear in variables W and Λ, i.e., the conditions are BMIs. On the other hand, by taking $G = \Lambda\mathbb{K}$, it is easy to see that the conditions obtained from (25) corresponds to particular cases of the ones stated in Proposition 1 and 2. Furthermore, the conditions of Propositions 1 and 2 appear directly in an LMI form.

VI. Computational issues

We discuss in this section, how to use these conditions in order to find, numerically, solutions for Problem 1.

A. Delay-independent case

Since the implicit objective is to obtain a set $B(\delta)$ with a significant size, we can consider an optimization problem with the following criterion: $\min\{\beta_1\lambda_{max}(W^{-1}) + \beta_2\lambda_{max}(R)\}$, where β_i, $i = 1, 2$ are tuning parameters. Note that by minimizing the function above we are, implicitly, maximizing δ. Thus, we propose the following convex optimization problem for providing a solution to Problem 1:

$$\begin{array}{l} \min\{\beta_1\lambda_W + \beta_2\lambda_R\} \\ \text{subject to} \\ \text{relations } (12), (13) \\ \begin{bmatrix} \lambda_W I_{n+n_c} & I_{n+n_c} \\ I_{n+n_c} & W \end{bmatrix} \ge 0, \quad \lambda_R I_{n+n_c} \ge R \end{array} \tag{27}$$

B. Delay-dependent case

In this case, we can consider the following optimization criterion, related, implicitly, to the maximization of the δ: $\min\{\beta_1\lambda_{max}(W^{-1}) + \beta_2\lambda_{max}(H^{-1}) + \beta_3\lambda_{max}(\mathbb{A}_d'R^{-1}\mathbb{A}_d) + \beta_4\lambda_{max}(\mathbb{A}'X^{-1}\mathbb{A})\}$, where β_i, $i = 1, ..., 4$ are weighting parameters. The following convex optimization problem can therefore be formulated in order to provide a solution to Problem 1:

$$\begin{array}{l} \min\{\beta_1\lambda_W + \beta_2\lambda_H + \beta_3\lambda_R + \beta_4\lambda_X\} \\ \text{subject to} \\ \text{relations } (21), (22) \\ \begin{bmatrix} \lambda_W I_{n+n_c} & I_{n+n_c} \\ I_{n+n_c} & W \end{bmatrix} \ge 0, \begin{bmatrix} \lambda_X I_{n+n_c} & \mathbb{A}' \\ \mathbb{A} & X \end{bmatrix} \ge 0 \\ \begin{bmatrix} \lambda_R I_{n+n_c} & \mathbb{A}_d' \\ \mathbb{A}_d & R \end{bmatrix} \ge 0, \begin{bmatrix} \lambda_H I_{n+n_c} & I_{n+n_c} \\ I_{n+n_c} & H \end{bmatrix} \ge 0 \end{array} \tag{28}$$

VII. Examples and Concluding Remarks

Example 1: Consider system (1) borrowed from [19], where: $A = \begin{bmatrix} 1 & 1.5 \\ 0.3 & -2 \end{bmatrix}$; $A_d = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$; $C = \begin{bmatrix} 5 & 1 \end{bmatrix}$; $u_0 = 15$ and $\tau = 0.5$. The open-loop matrix A is strictly unstable since its eigenvalues are $1.1432; -2.1432$.

Consider the following dynamic controller: $A_c = \begin{bmatrix} -20.2042 & 2.5216 \\ 2.1415 & -4.4821 \end{bmatrix}$; $B_c = \begin{bmatrix} 1.9516 \\ -0.0649 \end{bmatrix}$; $C_c = \begin{bmatrix} -0.9165 & 0.1091 \end{bmatrix}$. Note that matrix $\mathbb{A}$ is asymptotically stable. Hence, the problem can be addressed in the independent or in the dependent delay contexts.

Solving the optimization problem (27), with $\beta_1 = 1$ and $\beta_2 = 0$, one obtains $\delta = 4.520 \times 10^3$ and $E_c = \begin{bmatrix} -59.640 \\ -161.718 \end{bmatrix}$. On the other hand, if we consider problem (28), with all tuning parameters $\beta_i = 1$, one gets: $\delta =$

4.889×10^3 and $E_c = \begin{bmatrix} -76.201 \\ 70.413 \end{bmatrix}$. It is important to note that the obtained values of δ above are clearly less conservative than the ones obtained in [22] with the classical sector condition: $\delta = 637.824$ in the independent delay case and $\delta = 285.151$ in the dependent delay case.

Tables I show the values of δ resulting from the application of respectively the optimization problems (27) and (28) for different values of τ. Two cases are illustrated: with and without anti-windup gain (i.e with $E_c = 0$).

TABLE I

VALUES OF δ

	problem (27)		Problem (28)	
τ	$E_c \neq 0$	$E_c = 0$	$E_c \neq 0$	$E_c = 0$
0.1	$7.679e3$	$7.317e3$	$10.129e3$	$9.648e3$
0.5	$4.520e3$	$4.367e3$	$4.889e3$	$4.476e3$
1	$2.986e3$	$2.903e3$	$1.468e3$	$1.402e3$

We can notice, as expected, that greater is the time-delay, smaller is the region of stability obtained. Furthermore, it is possible to obtain an improvement of the size of the stability region by using the anti-windup gain. On the other hand, we can see that the delay-dependent condition can provide less conservative regions of stability for small delays.

Example 2: Consider system (1) with: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; A_d = \begin{bmatrix} 0 & 1.5 \\ 0.3 & -2 \end{bmatrix}; B = \begin{bmatrix} 10 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 5 & 1 \end{bmatrix}$ and $u_0 = 15$.

Consider the same dynamic controller as that one in Example 1. Note that matrix $\mathbb{A}$ is now unstable. Hence, we have to solve our problem in the delay-dependent context.

Table II show the obtained values of δ and E_c considering the optimization problem (28) with $\beta_i = 1$. The last column of the table shows the values for δ obtained in [22] with the classical sector condition

TABLE II

VALUES OF δ AND E_c

τ	E_c	δ	δ of [22]
0.1	$\begin{bmatrix} -81.950 \\ -235.487 \end{bmatrix}$	$7.682e3$	$1.512e3$
0.2	$\begin{bmatrix} -124.673 \\ -349.249 \end{bmatrix}$	$5.552e3$	977.840
0.4	$\begin{bmatrix} -310.232 \\ -558.027 \end{bmatrix}$	756.19	288.829

As in the previous example, for larger τ one obtains the guarantee of stability for a small region of stability. Moreover we can notice the improvement of the results when compared with the ones obtained using a classical sector condition.

REFERENCES

[1] J. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667–1604, 2003.

[2] H. H. Choi and M. J. Chung, "An LMI approach to $\mathcal{H}_\infty$ controller design for linear time-delay systems," *Automatica*, vol. 33, no. 4, pp. 737–739, 1997.

[3] B. Lehman and K. Shujaee, "Delay independent stability conditions and decay-estimates for time-varying functional-differential equations," *IEEE Transactions on Automatic Control*, vol. 39, no. 8, pp. 1673–1676, 1994.

[4] U. Shaked, I. Yaesh, and C. E. de Souza, "Bounded real criteria for linear time-delay systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 7, pp. 1016–1022, 1998.

[5] Y. Y. Cao, Y. X. Sun, and C. Cheng, "Delay-dependent robust stabilization of uncertain systems with multiple state delays," *IEEE Transactions on Automatic Control*, vol. 43, no. 11, pp. 1608–1612, 1998.

[6] C. E. de Souza and X. Li, "Delay-dependent robust $\mathcal{H}_\infty$ control of uncertain linear state-delayed systems," *Automatica*, vol. 35, no. 7, pp. 1313–1321, 1999.

[7] H. Sussmann, E. Sontag, and Y. Yang, "A general result on the stabilization of linear systems using bounded controls," *IEEE Transactions on Automatic Control*, vol. 39, no. 12, pp. 2411–2425, 1994.

[8] Z. Lin and A. Saberi, "Semi-global exponential stabilization of linear systems subject to input saturation via linear feedback," *Systems & Control Letters*, vol. 21, pp. 225–239, 1993.

[9] J. M. Gomes da Silva Jr. and S. Tarbouriech, "Local stabilization of discrete-time linear systems with saturating controls: an LMI-based approach," *IEEE Transactions on Automatic Control*, vol. 46, no. 1, pp. 119–125, 2001.

[10] C. Burgat and S. Tarbouriech, "Intelligent anti-windup for systems with input magnitude saturation," *Int. J. Robust Nonlinear Control*, vol. 8, pp. 1085–1100, 1998.

[11] N. Kapoor, A. R. Teel, and P. Daoutidis, "An anti-windup design for linear systems with input saturation," *Automatica*, vol. 34, no. 5, pp. 559–574, 1998.

[12] M. V. Kothare and M. Morari, "Multiplier theory for stability analisys of anti-windup control systems," *Automatica*, vol. 35, pp. 917–928, 1999.

[13] A. R. Teel, "Anti-windup for exponentially unstable linear systems," *Int. J. Robust Nonlinear Control*, vol. 9, no. 10, pp. 701–716, 1999.

[14] E. F. Mulder, M. V. Kothare, and M. Morari, "Multivariable anti-windup controller synthesis using linear matrix inequalities," *Automatica*, vol. 37, pp. 1407–1416, 2001.

[15] S. Oucheriah, "Global stabilization of a class of linear continuous time delay systems with saturating controls," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 43, no. 12, pp. 1012–1015, 1996.

[16] B. S. Chen, S. S. Wang, and H. C. Lu, "Stabilization of time-delay systems containing saturating actuators," *International Journal of Control*, vol. 47, pp. 867–881, 1988.

[17] S.-I. Niculescu, J. M. Dion, and L. Dugard, "Robust stabilization for uncertain time-delay systems containing saturating actuators," *IEEE Transactions on Automatic Control*, vol. 41, no. 5, pp. 742–747, 1996.

[18] E. Tissir and A. Hmamed, "Further results on the stabilization of time delay systems containing saturating actuators," *Int. J. Systems Science*, vol. 23, pp. 615–622, 1992.

[19] S. Tarbouriech and J. M. Gomes da Silva Jr., "Synthesis of controllers for continuous-time delay systems with saturating controls via LMIs," *IEEE Transactions on Automatic Control*, vol. 45, no. 1, pp. 105–111, 2000.

[20] J.-K. Park, C.-H. Choi, and H. Choo, "Dynamic anti-windup method for a class of time-delay control systems with input saturation," *International Journal of Robust and Nonlinear Control*, vol. 10, pp. 457–488, 2000.

[21] T. Kiyama and T. Iwasaki, "On the use of multi-loop circle for saturating control synthesis," *Systems & Control Letters*, vol. 41, pp. 105–114, 2000.

[22] J. Gomes da Silva Jr., S. Tarbouriech, and G. Garcia, "Synthesis of anti-windup loops for enlarging the stability region of time-delay systems with saturating inputs," in *European Control Conference 2003 (ECC' 03)*, Cambridge, UK, 2003.

[23] M. Klaï, S. Tarbouriech, and C. Burgat, "Some independent-time-delay stabilization of linear systems with saturating controls," in *Proc. of IEE Control'94*, Coventry (U.K), March 1994, pp. 1358–1363.

[24] S.-I. Niculescu, *Delay Effects on Stability. A Robust Control Approach*. Berlin, Germany: Springer-Verlag, 2001.

[25] J. K. Hale, *Theory of Functional Differential Equations*. New York: Springer-Verlag, 1977.