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Stabilization of input delayed systems via memoryless state feedback

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Abstract— **This paper considers the problem of stabilization for input delayed systems. Using redundant descriptor representation, we derive a sufficient condition for stability of input delayed systems via memoryless (static) state feedback. Furthermore, we give a robust stabilization condition for input delayed system whose coefficient matrices depend rationally on real uncertainty parameters.**

I. INTRODUCTION

Recently, increasing attention has been paid to the problems of stability and stabilization of linear systems with time-delay. For state-delayed systems, a number of results have been reported. Some of the results are delay-independent, and some of the results are delay-dependent. However for the systems with input delay, the results are almost delay-independent [1], [2]. A few of the results are delay-dependent, however the results are obtained by designing dynamic state-feedback controller [3], [4]. In this paper, first, we derive a delay-dependent sufficient condition for stabilization of input and state delayed systems via static state-feedback. This result is obtained by using redundant descriptor form. Applying a new descriptor variables which is substituted for derivative of the state[6], [5], we can design a static state-feedback controller. Second, we give a delay-dependent robust stabilization condition for input and state delayed systems whose coefficient matrices depend rationally on uncertain parameters. These conditions are derived by computing a finite set of linear matrix inequalities.

II. STABILIZATION VIA STATIC STATE FEEDBACK

Consider linear time delay systems described by

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + B_u u(t-g) \quad (1)$$

where $x(t) = 0$, $t < 0$, $h > 0$ and $g > 0$ are constant time delays. For this system, we design the static state-feedback controller. When we apply the controller $u = Kx$ to the system (1), the closed loop system is given by

$$\dot{x}(t) = (A + A_h + B_u K)x(t) - (A_h + B_u K)\int_{t-h}^{t} \dot{x}(\tau)d\tau. \quad (2)$$

Now we apply $y(t) := \dot{x}(t)$, and represent an extended closed loop via descriptor form:

$$E\dot{\xi}(t) = (\bar{A} + \bar{B}_u\bar{K})\xi(t) - \begin{bmatrix} 0 \\ A_h \end{bmatrix}\int_{t-h}^{t}\begin{bmatrix} 0 & I \end{bmatrix}\xi(\tau)d\tau - \begin{bmatrix} 0 \\ B_u K \end{bmatrix}\int_{t-g}^{t}\begin{bmatrix} 0 & I \end{bmatrix}\xi(\tau)d\tau, \quad (3)$$

$$\bar{A} = \begin{bmatrix} 0 & I \\ A + A_h & -I \end{bmatrix},\ \bar{B}_u = \begin{bmatrix} 0 \\ B_u \end{bmatrix}, \xi(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix},$$
$$\bar{K} = \begin{bmatrix} K & 0 \end{bmatrix},\ E = \mathrm{diag}\{I, 0\}. \quad (4)$$

Using the representation, we obtain the following theorem.

Theorem 1: The system (1) is stabilizable by $u = Kx$ if there exist matrices $X_{11} > 0$, $Q > 0$, $R > 0$, Y and positive scalars n_1 and n_2 satisfying the following matrix inequality:

$$\begin{bmatrix} \Psi_1 & \Psi_2^T & \Psi_3^T \\ \Psi_2 & -hQ & 0 \\ \Psi_3 & 0 & -gR \end{bmatrix} < 0, \quad (5)$$

$$\Psi_1 := \mathrm{He}\{\bar{A}X + \bar{B}_u\bar{Y}\} + hQ + gR, \quad (6)$$

$$\Psi_2 := h\begin{bmatrix} 0 & 0 \\ -n_1 A_h X_{11} & n_2 A_h X_{11} \end{bmatrix}, \quad (7)$$

$$\Psi_3 := g\begin{bmatrix} 0 & 0 \\ -n_1 B_u Y & n_2 B_u Y \end{bmatrix}, \quad (8)$$

$$X := \begin{bmatrix} X_{11} & 0 \\ -n_1 X_{11} & n_2 X_{11} \end{bmatrix},\ \bar{Y} := \begin{bmatrix} Y & 0 \end{bmatrix}. \quad (9)$$

The state feedback gain is then given by $K = YX_{11}^{-1}$.

Proof: Consider the following Lyapunov function candidate for the system (3)

$$V := V_1 + V_2 + V_3 \quad (10)$$

$$V_1 := \xi^T(t)EP\xi(t) \quad (11)$$

$$V_2 := \int_{-h}^{0}\int_{t+s}^{t}\xi^T(\theta)\begin{bmatrix} 0 \\ I \end{bmatrix}\begin{bmatrix} 0 & A_h^T \end{bmatrix}Q^{-1} \times \begin{bmatrix} 0 \\ A_h \end{bmatrix}\begin{bmatrix} 0 & I \end{bmatrix}\xi(\theta)d\theta ds \quad (12)$$

$$V_3 := \int_{-g}^{0}\int_{t+s}^{t}\xi^T(\theta)\begin{bmatrix} 0 \\ I \end{bmatrix}\begin{bmatrix} 0 & K^T B_u^T \end{bmatrix}R^{-1} \times \begin{bmatrix} 0 \\ B_u K \end{bmatrix}\begin{bmatrix} 0 & I \end{bmatrix}\xi(\theta)d\theta ds, \quad (13)$$

$$P := \begin{bmatrix} P_{11} & 0 \\ \frac{n_1}{n_2}P_{11} & \frac{1}{n_2}P_{11} \end{bmatrix},\ P_{11} = P_{11}^T > 0. \quad (14)$$

The time derivatives of V_1, V_2 and V_3 along the solution of (3) are given by the followings

$$\begin{aligned}\frac{dV_1}{dt} &= \xi^T(t)\{(\bar{A} + \bar{B}_u\bar{K})^T P + P^T(\bar{A} + \bar{B}_u\bar{K})\}\xi(t) \\ &\quad - \int_{t-h}^{t} 2\xi^T(t)P^T\begin{bmatrix} 0 & 0 \\ 0 & A_h \end{bmatrix}\xi(\tau)d\tau \\ &\quad - \int_{t-g}^{t} 2\xi^T(t)P^T\begin{bmatrix} 0 & 0 \\ 0 & B_u K \end{bmatrix}\xi(\tau)d\tau \\ &\le \xi^T(t)\{(\bar{A} + \bar{B}_u\bar{K})^T P + P^T(\bar{A} + \bar{B}_u\bar{K})\}\xi(t) \\ &\quad + \int_{t-h}^{t}\xi^T(t)P^T QP\xi(t)d\tau + \int_{t-g}^{t}\xi^T(t)P^T RP\xi(t)d\tau \\ &\quad + \int_{t-h}^{t}\xi^T(\tau)\begin{bmatrix} 0 & 0 \\ 0 & A_h \end{bmatrix}^T Q^{-1}\begin{bmatrix} 0 & 0 \\ 0 & A_h \end{bmatrix}\xi(\tau)d\tau\end{aligned}$$

$$+\int_{t-g}^{t}\xi^T(\tau)\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}^T R^{-1}\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}\xi(\tau)d\tau$$

$$\frac{dV_2}{dt}=h\xi^T(t)\begin{bmatrix}0&0\\0&A_h\end{bmatrix}^T Q^{-1}\begin{bmatrix}0&0\\0&A_h\end{bmatrix}\xi(t)$$
$$-\int_{t-h}^{t}\xi^T(\tau)\begin{bmatrix}0&0\\0&A_h\end{bmatrix}^T Q^{-1}\begin{bmatrix}0&0\\0&A_h\end{bmatrix}\xi(\tau)d\tau$$

$$\frac{dV_3}{dt}=g\xi^T(t)\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}^T R^{-1}\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}\xi(t)$$
$$-\int_{t-g}^{t}\xi^T(\tau)\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}^T R^{-1}\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}\xi(\tau)d\tau$$

Hence, the upper bound of $\dot{V}$ is given by $\frac{dV}{dt}\le\xi^T(t)M\xi(t)$,

$$M:=\mathrm{He}\{(\bar{A}+\bar{B}_u\bar{K})^TP\}+h\begin{bmatrix}0&0\\0&A_h\end{bmatrix}^T Q^{-1}\begin{bmatrix}0&0\\0&A_h\end{bmatrix}$$
$$+hP^TQP+gP^TRP+g\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}^T R^{-1}\begin{bmatrix}0&0\\0&B_uK\end{bmatrix}.$$

If $M<0$ has solutions, system (3) is asymptotically stable. Pre- and post-multiplying $X^T=P^{-T}$ and $X=P^{-1}$ to M, and applying Schur complement, we obtain (5). ■

III. Robust Stabilization

Now, we derive a delay-dependent robust stabilization condition for input and state delayed systems with real rational parametric time-varying uncertainty described by

$$\begin{cases}\dot{x}(t)=Ax(t)+A_hx(t-h)+Bw(t)+B_uu(t-g)\\ z(t)=Cx(t)+C_hx(t-h)+Dw(t)+D_uu(t-g)\\ w(t)=\Delta(t)z(t),\quad \Delta(t)\in U\end{cases}\qquad(15)$$
$$U:=\{\mathrm{diag}\{r_1I,r_2I,\cdots\}\mid r_i\in[-1,1]\}$$

where $h>0$ and $g>0$ are constant time delays. Note that the system has D matrix, i.e., real rational uncertainties. Synthesis for the system is difficult even without time delay.

Theorem 2: The system (15) is stabilizable by $u=Kx$, if there exist matrices $\tilde{Q}>0$, $\tilde{R}>0$, $\tilde{X}$, $\tilde{Y}$ and positive scalars n_1 and n_2 satisfying the following matrix inequality:

$$\begin{bmatrix}\tilde{\Psi}_1&\tilde{\Psi}_2^T&\tilde{\Psi}_3^T\\ \tilde{\Psi}_2&-h\tilde{Q}&0\\ \tilde{\Psi}_3&0&-g\tilde{R}\end{bmatrix}<0 \qquad (16)$$

$$\tilde{\Psi}_1:=\mathrm{He}\{\tilde{A}\tilde{X}+\tilde{B}_u\tilde{Y}\}+h\tilde{Q}+g\tilde{R}, \qquad (17)$$

$$\tilde{\Psi}_2:=h\begin{bmatrix}0&0&0\\-n_1A_hX_{11}&n_2A_hX_{11}&0\\-n_1C_hX_{11}&n_2C_hX_{11}&0\end{bmatrix}, \qquad (18)$$

$$\tilde{\Psi}_3:=g\begin{bmatrix}0&0&0\\-n_1B_uY&n_2B_uY&0\\-n_1D_uY&n_2D_uY&0\end{bmatrix}, \qquad (19)$$

$$\tilde{X}:=\begin{bmatrix}X_{11}&0&0\\-n_1X_{11}&n_2X_{11}&0\\X_{31}&X_{32}&X_{33}\end{bmatrix},\ X_{11}>0, \qquad (20)$$

$$\tilde{A}=\begin{bmatrix}0&I&0\\A+A_h&-I&B\Delta(t)\\C+C_h&0&-I+D\Delta(t)\end{bmatrix}, \qquad (21)$$

$$\tilde{B}_u=\begin{bmatrix}0&B_u^T&D_u^T\end{bmatrix}^T,\ \tilde{Y}=\begin{bmatrix}Y&0&0\end{bmatrix}. \qquad (22)$$

The state feedback gain is then given by $K=YX_{11}^{-1}$.

Proof: Similar to (2), we apply $y(t):=\dot{x}(t)$, and we represent an extended closed loop via descriptor form:

$$E\dot{\xi}(t)=(\tilde{A}+\tilde{B}_u\tilde{K})\xi(t)-\begin{bmatrix}0\\A_h\\C_h\end{bmatrix}\int_{t-h}^{t}\begin{bmatrix}0&I&0\end{bmatrix}\xi(\tau)d\tau$$
$$-\begin{bmatrix}0\\B_uK\\D_uK\end{bmatrix}\int_{t-g}^{t}\begin{bmatrix}0&I&0\end{bmatrix}\xi(\tau)d\tau \qquad (23)$$

where

$$E=\begin{bmatrix}I&0&0\\0&0&0\\0&0&0\end{bmatrix},\ \xi(t)=\begin{bmatrix}x(t)\\y(t)\\z(t)\end{bmatrix},\tilde{K}=\begin{bmatrix}K^T\\0\\0\end{bmatrix}^T \qquad (24)$$

Consider the following Lyapunov function candidate,

$$V:=\xi^T(t)E\tilde{P}\xi(t)+V_2+V_3, \qquad (25)$$

$$V_2:=\int_{-h}^{0}\int_{t+s}^{t}\xi^T(\theta)\begin{bmatrix}0\\I\\0\end{bmatrix}\begin{bmatrix}0&A_h^T&C_h^T\end{bmatrix}$$
$$\times Q^{-1}\begin{bmatrix}0\\A_h\\C_h\end{bmatrix}\begin{bmatrix}0&I&0\end{bmatrix}\xi(\theta)d\theta ds, \qquad (26)$$

$$V_3:=\int_{-g}^{0}\int_{t+s}^{t}\xi^T(\theta)\begin{bmatrix}0\\I\\0\end{bmatrix}\begin{bmatrix}0&K^TB_u^T&K^TD_u^T\end{bmatrix}$$
$$\times R^{-1}\begin{bmatrix}0\\B_uK\\D_uK\end{bmatrix}\begin{bmatrix}0&I&0\end{bmatrix}\xi(\theta)d\theta ds. \qquad (27)$$

As with the proof of Theorem 1, we obtain (16). ■

IV. Conclusion

In this paper, using descriptor form, we proposed a delay-dependent stabilization condition for systems with input and state delay via static state-feedback controller. Furthermore, we derived a delay-dependent robust stabilization condition for input and state delayed systems whose coefficient matrix depend rationally on uncertain parameters.

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