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Optimal Guaranteed Cost Control of Singular Systems with Delayed State and Parameter Uncertainties

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Abstract—**The guaranteed cost control problem via memoryless state feedback controllers is studied in this paper for a class of linear singular systems with delayed state and norm-bounded time-varying parameter uncertainties. A sufficient condition for the existence of guaranteed cost controllers is derived, and it is shown that the condition is equivalent to the solvability of a certain linear matrix inequality (LMI). Furthermore, a convex optimization problem with LMI constraints is formulated to design the optimal guaranteed cost controller which minimizes the guaranteed cost of the closed-loop uncertain system.**

I. Introduction

CONTROL of delay systems has been a topic of recurring interest over the past decades since time delays are often the main causes for instability and poor performance of systems and encountered in various engineering systems such as chemical processes, long transmission lines in pneumatic systems, and so on (Malek-Zavarei and Jamshidi, 1987). Recently, the problems of robust analysis and robust synthesis for uncertain time-delay systems have been studied and many techniques have been proposed via Riccati equation and LMI-based approach (Silviu-Iulian Niculescu, 2001). Furthermore, the problem of guaranteed cost control (Chang and Peng, 1972) which considers both robust stability and robust performance of closed-loop uncertain systems has also been extended to uncertain time-delay systems (Moheimani and Petersen, 1997). An LMI approach to the design of guaranteed cost controllers was proposed for uncertain continuous time-delay systems (Yu and Chu, 1999) and uncertain discrete time-delay systems (Yu and Gao, 2001).

On the other hand, in recent years, there has been a growing interest in the system-theoretic problems of singular systems (or generalized state-space systems, descriptor systems) due to their extensive applications in power system (Stott, 1979), in large-scale systems, singular space perturbation theory (Wang, *et al*., 1988), circuits theory (Newcomb and Dziurla, 1989), and other areas. A great number of results based on the theory of regular systems have been extended to the area of singular systems (Dai, 1989). More recently, the robust control problems have been considered for uncertain singular time-delay systems, the notions of quadratic stability and quadratic stabilizability for uncertain regular time-delay systems have been extended, an LMI approach to robust stability analysis and the design of robust stabilizing controllers was proposed (Xu, 2002). To the best of our knowledge, however, few results have been reported on the guaranteed cost control problem for singular systems with delayed state and parameter uncertainties in the literature.

In this paper, we are concerned with the guaranteed cost control problem for a class of linear singular time-delay systems with time-varying norm-bounded parameter uncertainties via memoryless state feedback. A sufficient condition for the existence of guaranteed cost controllers is derived. Furthermore, it is shown that this condition is equivalent to the solvability of a certain LMI, and its feasible solutions provide a parametrized representation of guaranteed cost controllers. This important advantage is exploited to design the optimal guaranteed cost controller which minimizes the guaranteed cost of the closed-loop uncertain system. Finally, an example is given to illustrate the usefulness of the proposed results.

II. Problem Description And Preliminaries

Consider a linear singular system with state delay and parameter uncertainties described by

$$E\dot{x}(t) = (A+\Delta A)x(t) + (A_d + \Delta A_d)x(t-d) + (B+\Delta B)u(t) \quad (1)$$

$$x(t) = \varphi(t), \quad t \in [-d, 0] \quad (2)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input. A, A_d and B are known real constant matrices with appropriate dimensions. $d > 0$ is a constant time delay, $\varphi(t)$ is a compatible vector valued continuous function. $\Delta A, \Delta A_d$ and ΔB are unknown matrices representing norm-bounded parameter uncertainties, and are assumed to be

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of the following form:

$$[\Delta A \quad \Delta A_d \quad \Delta B] = MF[N_1 \quad N_d \quad N_2] \tag{3}$$

where M, N_1, N_2 and N_d are known real constant matrices with appropriate dimensions. F is the uncertain matrix satisfying

$$FF^T \le I \tag{4}$$

in which I denotes the identity matrix of appropriate dimension. The uncertainties $\Delta A, \Delta A_d$ and ΔB are said to be admissible if both (3) and (4) hold. The matrix $E \in R^{n\times n}$ may be singular and assume that rank $E = r \le n$. Without loss of generality, we may assume that

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

where I_r denotes the $r \times r$ identity matrix. In fact, for any matrix E whose rank is r, there exist two invertible matrices G and $H \in R^{n\times n}$ such that

$$\overline{E} = GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

Let

$$\overline{A} = GAH, \quad \overline{A}_d = GA_dH, \quad \overline{B} = GB$$

$$[\Delta\overline{A} \quad \Delta\overline{A}_d \quad \Delta\overline{B}] = GMF[N_1H \quad N_dH \quad N_2], \quad \overline{x} = H^{-1}x$$

Then the system (1) is equivalent to

$$\overline{E}\dot{\overline{x}}(t) = (\overline{A} + \Delta\overline{A})\overline{x}(t) + (\overline{A}_d + \Delta\overline{A}_d)\overline{x}(t-d) + (\overline{B} + \Delta\overline{B})u(t)$$

$$\overline{x}(t) = H^{-1}\varphi(t), \quad t \in [-d, \quad 0]$$

In the sequel, we always assume that the matrix E is of the form (5).

Associated with the system (1) is the cost function

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \tag{6}$$

where Q and R are given symmetric positive definite matrices.

Definition 1: Consider the uncertain system (1), if there exist a control law $u^*(t)$ and a positive scalar J^* such that for all admissible uncertainties, the closed-loop system is asymptotically stable and the closed-loop value of the cost function (6) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law of the uncertain system (1).

The objective of this paper is to develop a procedure to designing a memoryless state feedback guaranteed cost control law $u(t) = Kx(t)$ for the uncertain singular time-delay system (1) and cost function (6).

Lemma 1: [Yu, 2001] Given matrices S, M, N of appropriate dimensions and with S symmetric, then

$$S + MFN + N^TF^TM^T < 0$$

for all F satisfying $FF^T \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$S + \varepsilon MM^T + \varepsilon^{-1}N^TN < 0 .$$

III. Main Results

Lemma 2: The nominal unforced singular system of (1)

$$E\dot{x}(t) = Ax(t) + A_dx(t-d) \tag{7}$$

is regular, impulse free and stable if there exist a symmetric positive definite matrix S and an invertible matrix P of the form

$$P = \begin{bmatrix} P_{11} & P_{12} \\ 0 & P_{22} \end{bmatrix} \tag{8}$$

where $P_{11} \in R^{r\times r}$ is a symmetric positive definite matrix, $P_{22} \in R^{(n-r)\times(n-r)}$ is an invertible matrix, $P_{12} \in R^{r\times(n-r)}$, such that

$$PA + A^TP^T + S + PA_dS^{-1}A_d^TP^T < 0 \tag{9}$$

Proof: From the structure forms of the matrix P and E, and Theorem 1 in [Xu, 2002], we can conclude the results of this lemma.

We first present a sufficient condition for the existence of memoryless state feedback guaranteed cost control laws for the uncertain system (1).

Theorem 1: The state feedback control law $u(t) = Kx(t)$ is a guaranteed cost controller of the system (1) and cost function (6) if there exist a symmetric positive definite matrix S and an invertible matrix P of the form (8) such that

$$\begin{aligned} &Q + K^TRK + P(A_c + \Delta A_c) + (A_c + \Delta A_c)^TP^T \\ &\quad + S + P(A_d + \Delta A_d)S^{-1}(A_d + \Delta A_d)^TP^T < 0 \end{aligned} \tag{10}$$

for all the admissible uncertainties $\Delta A, \Delta A_d$ and ΔB ,where

$$A_c = A + BK, \quad \Delta A_c = \Delta A + \Delta BK = MF(N_1 + N_2K) \tag{11}$$

Proof: Applying the state feedback control law $u(t) = Kx(t)$ in the system (1) results in the closed-loop system

$$E\dot{x}(t) = (A_c + \Delta A_c)x(t) + (A_d + \Delta A_d)x(t-d) \tag{12}$$

Suppose that there exist a symmetric positive definite matrix S and an invertible matrix P of the form (8) such that the matrix inequality (10) holds for all the admissible uncertainties, then the matrix inequality (10) implies

$$\begin{aligned} &P(A_c + \Delta A_c) + (A_c + \Delta A_c)^TP^T + S \\ &\quad + P(A_d + \Delta A_d)S^{-1}(A_d + \Delta A_d)^TP^T < 0 \end{aligned}$$

It follows from Lemma 2 that the system (12) is regular, impulse free and stable for any admissible uncertainties $\Delta A, \Delta A_d$ and ΔB.

In order to derive the upper bound over the closed-loop value of the cost function, consider the following Lyapunov functional:

$$V(x) = x^T(t)PEx(t) + \int_{t-d}^{t} x^T(\sigma)Sx(\sigma)d\sigma$$

Then the time derivative of $V(\cdot)$ along any trajectory of the closed-loop system (12) is given by

$$\begin{aligned}\frac{dV(x)}{dt} &= L(x) \\ &= x^T(t)P[E\dot{x}(t)] + [E\dot{x}(t)]^T Px(t) \\ &\quad + x^T(t)Sx(t) - x^T(t-d)Sx(t-d) \\ &= x^T(t)[P(A_c+\Delta A_c) + (A_c+\Delta A_c)^T P^T + S]x(t) \\ &\quad + 2x^T(t)P(A_d+\Delta A_d)x(t-d) - x^T(t-d)Sx(t-d)\end{aligned}$$

Since

$$\begin{aligned}&2x^T(t)P(A_d+\Delta A_d)x(t-d) \\ &\le x^T(t)P(A_d+\Delta A_d)S^{-1}(A_d+\Delta A_d)^T P^T x(t) + x^T(t-d)Sx(t-d)\end{aligned}$$

holds for any symmetric positive definite matrix S. By using the above inequality and the inequality (10), it follows that

$$\begin{aligned}\frac{dV(x)}{dt} &= x^T(t)[P(A_c+\Delta A_c) + (A_c+\Delta A_c)^T P^T + S \\ &\quad + P(A_d+\Delta A_d)S^{-1}(A_d+\Delta A_d)^T P^T]x(t) \\ &< -x^T(t)(Q + K^T RK)x(t)\end{aligned}$$

Furthermore, by integrating both sides of the above inequality from 0 to T and using the initial condition, we obtain

$$\begin{aligned}&-\int_0^T x^T(t)(Q+K^T RK)x(t)dt \\ &> x^T(T)PEx(T) - x^T(0)PEx(0) + \int_{T-d}^{T} x^T(\sigma)Sx(\sigma)d\sigma \\ &\quad - \int_{-d}^{0} x^T(\sigma)Sx(\sigma)d\sigma\end{aligned}$$

As the closed-loop system (12) is asymptotically stable, when $T \to \infty$

$$x^T(T)Px(T) \to 0,$$
$$\int_{T-d}^{T} x^T(\sigma)Sx(\sigma)d\sigma \to 0.$$

Hence, we get

$$\int_0^\infty x^T(t)(Q+K^T RK)x(t)dt \le \varphi^T(0)PE\varphi(0) + \int_{-d}^{0}\varphi^T(\sigma)S\varphi(\sigma)d\sigma$$

It follows from Definition that the result of the theorem is true. This completes the proof.

In the following, we show that the above sufficient condition for the existence of guaranteed cost controllers is equivalent to the solvability of a certain LMI.

Theorem 2: There exists a guaranteed cost controller of the system (1) and cost function (6) if there exist a scalar $\varepsilon > 0$, an invertible matrix $X \in R^{n\times n}$, a matrix $W \in R^{m\times n}$ and a positive definite matrix $Y \in R^{n\times n}$ such that

$$\begin{bmatrix} \tilde{A} & A_d Y & \Sigma_1^T & X^T & W^T & X^T \\ YA_d^T & -Y & YN_d^T & 0 & 0 & 0 \\ \Sigma_1 & N_d Y & -\varepsilon I & 0 & 0 & 0 \\ X & 0 & 0 & -Q^{-1} & 0 & 0 \\ W & 0 & 0 & 0 & -R^{-1} & 0 \\ X & 0 & 0 & 0 & 0 & -Y \end{bmatrix} < 0 \qquad (14)$$

where $\tilde{A} = AX + BW + (AX+BW)^T + \varepsilon MM^T$, $\Sigma_1 = N_1 X + N_2 W$, X is of the form

$$X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, \quad 0 < X_{11} = X_{11}^T \in R^{r\times r}. \qquad (15)$$

In this case, a guaranteed cost control law is given by

$$u(t) = WX^{-1}x(t) \qquad (16)$$

and a corresponding upper bound over closed-loop cost function is

$$J^* = \varphi_1^T(0)X_{11}^{-1}\varphi_1(0) + \int_{-d}^{0}\varphi^T(\sigma)Y^{-1}\varphi(\sigma)d\sigma \qquad (17)$$

where $\varphi(t) = [\varphi_1^T(t)\ \ \varphi_2^T(t)]^T$, $\varphi_1(t) \in R^r$

Proof: By the Schur complement, the matrix inequality (10) equals to

$$\begin{bmatrix} P(A_c+\Delta A_c) + (A_c+\Delta A_c)^T P + S + Q + K^T RK & P(A_d+\Delta A_d) \\ (A_d+\Delta A_d)^T P & -S \end{bmatrix} < 0$$

Using the expressions of the matrices ΔA_c and ΔA_d, the above matrix inequality can be equivalently written as

$$\begin{aligned}&\begin{bmatrix} \Pi & PA_d \\ A_d^T P^T & -S \end{bmatrix} + \begin{bmatrix} PM \\ 0 \end{bmatrix} F \begin{bmatrix} N_1 + N_2 K & N_d \end{bmatrix} \\ &\quad + \begin{bmatrix} N_1 + N_2 K & N_d \end{bmatrix}^T F^T \begin{bmatrix} PM \\ 0 \end{bmatrix}^T < 0\end{aligned} \qquad (18)$$

where $\Pi = S + Q + K^T RK + P(A+BK) + (A+BK)^T P^T$.

By lemma 1, it follows that there exists a scalar $\varepsilon > 0$ such that

$$\begin{aligned}&\begin{bmatrix} \Pi & PA_d \\ A_d^T P^T & -S \end{bmatrix} + \varepsilon \begin{bmatrix} PM \\ 0 \end{bmatrix}\begin{bmatrix} PM \\ 0 \end{bmatrix}^T \\ &\quad + \varepsilon^{-1}\begin{bmatrix} N_1 + N_2 K & N_d \end{bmatrix}^T \begin{bmatrix} N_1 + N_2 K & N_d \end{bmatrix} < 0\end{aligned} \qquad (19)$$

Invoking the Schur complement again, one obtains

$$\begin{bmatrix} \hat{A} & PA_d & \Sigma_2^T & I & K^T & I \\ A_d^T P^T & -S & N_d^T & 0 & 0 & 0 \\ \Sigma_2 & N_d & -\varepsilon I & 0 & 0 & 0 \\ I & 0 & 0 & -Q^{-1} & 0 & 0 \\ K & 0 & 0 & 0 & -R^{-1} & 0 \\ I & 0 & 0 & 0 & 0 & -S^{-1} \end{bmatrix} < 0 \qquad (20)$$

where $\hat{A} = P(A+BK)+(A+BK)^T P^T + \varepsilon PMM^T P^T$, $\Sigma_2 = N_1 + N_2 K$. Left- and right- multiplying both sides of the above inequality by matrix $T = diag(P^{-1}, S^{-1}, I, I, I, I)$ and its transpose and denoting $X = P^{-T}$, $W = KP^{-T}$, $Y = S^{-1}$ yield that the matrix inequality (20) is equivalent to (14). Furthermore, if there exist a scalar $\varepsilon > 0$, an invertible matrix $X \in R^{n\times n}$, a matrix $W \in R^{m\times n}$ and a positive definite matrix $Y \in R^{n\times n}$ such that the matrix inequality (14) holds, then $u(t) = WX^{-1}x(t)$ is a guaranteed cost control law, and J^* given in (17) is an upper bound over the corresponding closed-loop cost function.

Since (14) is a linear matrix inequality in ε, W, X, Y, (14) defines a convex solution set of (ε, W, X, Y), and therefore various efficient convex optimization algorithms can be used to test whether the LMI is solvable and to generate particular solutions. Moreover, its solutions parametrize the set of guaranteed cost controllers. This parametrized representation can be exploited to design the guaranteed cost controllers with some additional requirements. In particular, the optimal guaranteed cost control law which minimizes the value of the guaranteed cost for the closed-loop uncertain system can be determined by solving a certain optimization problem. This is the following theorem:

Theorem 3: Consider the system (1) and the cost function (6), if the following optimization problem

$$\min_{\varepsilon,\alpha,X,W,Y,Z} \alpha + Trace(Z) \tag{21}$$

s.t. (i) (14)

$$\text{(ii)} \begin{bmatrix} -\alpha & \varphi_1^T(0) \\ \varphi_1(0) & -X_{11} \end{bmatrix} < 0$$

$$\text{(iii)} \begin{bmatrix} -Z & J^T \\ J & -Y \end{bmatrix} < 0$$

has a solution $\hat{\varepsilon}, \hat{\alpha}, \hat{X}, \hat{Y}, \hat{W}, \hat{Z}$, where $Trace(\cdot)$ denotes the trace of the matrix. Then $u(t) = \hat{W}\hat{X}^{-1}x(t)$ is the optimal guaranteed cost control law which ensures the minimization of the guaranteed cost (17), where

$$\int_{-d}^{0} \varphi(\sigma)\varphi^T(\sigma)d\sigma = JJ^T .$$

Proof: By Theorem 2, the control law $u(t) = \hat{W}\hat{X}^{-1}x(t)$ is a guaranteed cost controller of system (1). It follows from the Schur complement that the constraint condition (ii) in (21) is equivalent to $\varphi_1^T(0)X_{11}^{-1}\varphi_1(0) < \alpha$, (iii) in (21) is equivalent to $J^T Y^{-1} J < Z$. On the other hand,

$$\begin{aligned}\int_{-d}^{0} \varphi^T(\sigma)Y^{-1}\varphi(\sigma)d\sigma &= \int_{-d}^{0} Trace(\varphi^T(\sigma)Y^{-1}\varphi(\sigma))d\sigma \\ &= Trace(JJ^T Y^{-1}) \\ &= Trace(J^T Y^{-1} J) < Trace(Z)\end{aligned}$$

So it follows from (17) that

$$J^* < \alpha + Trace(Z)$$

Thus, the minimization of $\alpha + Trace(Z)$ implies the minimization of the guaranteed cost for the uncertain system (1). The global optimality of the solution to the problem (21) follows from the convexity of the objective function and of the constraints. This completes the proof.

The convex optimization problem (21) can be easily solved by using the solver mincx in LMI Toolbox.

IV. Illustrative Example

Consider the linear uncertain singular delay system (1), where

$$E = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1.5 & 0.5 & 1 \\ -1 & 0 & 1 \\ 0.5 & 0 & 1 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 0.5 \\ 0.3 & 0.5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.1 \end{bmatrix},$$

$$N_1 = [0.2\ \ 0.1\ \ 0.3], \quad N_2 = [0.1\ \ 0.1], \quad N_d = [0.1\ \ 0.2\ \ 0.5].$$

the initial state $\varphi(t) = [e^{t+1.5}\ \ 0\ \ 0]^T$ ($t \in [-d, 0]$) . The weighting matrices in cost function (6) are chosen as $Q = I_3$ and $R = I_2$.

The matrix E is not of the form (8), by using matrices

$$G = \begin{bmatrix} 0.3384 & 0.2535 & -0.7327 \\ 0.7386 & -0.1704 & -0.1516 \\ -0.5774 & -0.5774 & 0.5774 \end{bmatrix},$$

$$H = \begin{bmatrix} -1.0483 & 0.3179 & 0.6928 \\ 0.2596 & 0.8559 & -0.6928 \\ -0.1298 & -0.4280 & -1.3855 \end{bmatrix}$$

we have

$$GEH = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Furthermore, using the matrices G and H, we get

$$\overline{A} = GAH = \begin{bmatrix} 0.1799 & 0.1694 & 0 \\ -1.2187 & 0.5201 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\overline{A}_d = GA_d H = \begin{bmatrix} 0.0909 & -0.8505 & -0.5037 \\ 1.1123 & 0.0651 & 0.2046 \\ 0.0807 & 0.9199 & -0.0800 \end{bmatrix},$$

$$\overline{B} = GB = \begin{bmatrix} 0.5919 & -0.3943 \\ 0.5682 & 0.5869 \\ -1.1547 & 0 \end{bmatrix},$$

$$\overline{M} = GM = \begin{bmatrix} 0.1466 \\ 0.3200 \\ -0.3464 \end{bmatrix},$$

$$\overline{N}_1 = N_1 H = [-0.2226 \ \ 0.0208 \ \ -0.3464],$$

$$\overline{N}_d = N_d H = [-0.1178 \ \ -0.0110 \ \ -0.7620],$$

$$\overline{N}_2 = N_2 = [0.1 \ \ 0.1],$$

$$\overline{Q} = \begin{bmatrix} 1.1832 & -0.0555 & -0.7263 \\ -0.0555 & 1.0168 & 0.2203 \\ -0.7263 & 0.2203 & 2.8796 \end{bmatrix}.$$

Using LMI Toolbox in MATLAB, it follows that the optimization problem (21) with $\overline{A}, \overline{A}_d, \overline{B}, \overline{M}, \overline{N}_1, \overline{N}_2, \overline{N}_d, \overline{Q}, R$ is feasible, and the optimal solution is as follows:

$$\hat{X} = \begin{bmatrix} 0.0671 & -0.0409 & 0 \\ -0.0409 & 0.0572 & 0 \\ 0.0293 & -0.0646 & -0.0746 \end{bmatrix},$$

$$\hat{Y} = \begin{bmatrix} 0.1358 & -0.1618 & 0.1553 \\ -0.1618 & 0.8728 & -0.4737 \\ 0.1553 & -0.4737 & 0.4873 \end{bmatrix},$$

$$\hat{W} = \begin{bmatrix} -0.5521 & -0.5048 & 1.0430 \\ 0.4342 & -0.5235 & -0.1117 \end{bmatrix},$$

$$\hat{\varepsilon} = 0.7339, \quad \hat{\alpha} = 531.2488, \quad \hat{Z} = 110.9080$$

The optimal guaranteed cost control law is given by

$$u(t) = \begin{bmatrix} 14.2738 & -40.7095 & 37.5896 \\ -3.5136 & -5.5701 & -0.5310 \end{bmatrix} x(t) \qquad (22)$$

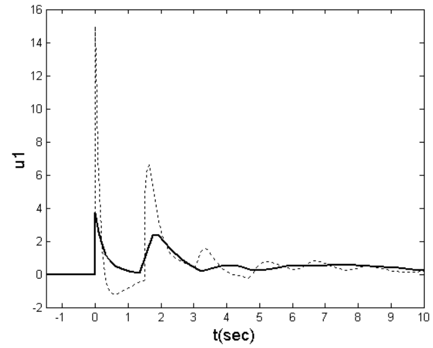
and the optimal guaranteed cost of the corresponding closed-loop system is $J^* = 642.1568$. The controller (22) ensures that, for all admissible parameter uncertainties, the resulting closed-loop system is asymptotically stable and the closed-loop cost function is no more than the guaranteed cost $J^* = 642.1568$, so the closed-loop system has not only robust stability, but also the given robust performance.

Xu *et al* (2002) gave a robust stabilizing control law

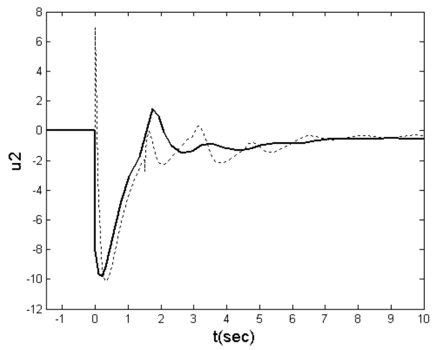
$$u(t) = \begin{bmatrix} -13.5354 & 19.4496 & -19.6474 \\ 6.7469 & -15.0227 & 11.8584 \end{bmatrix} x(t) \qquad (23)$$

To compare the effect of the controller (22) and (23) by simulation, we assume that the delay $d = 1.5$ sec , the uncertain matrix $F = \sin t$. The control law (solid line) proposed in this paper, Xu's control law (dot line) and the corresponding closed-loop system state are shown in Fig.1 and Fig 2, respectively.

It is clear from Fig.1 that the closed-loop system designed with the proposed approach in this paper has better transient properties.

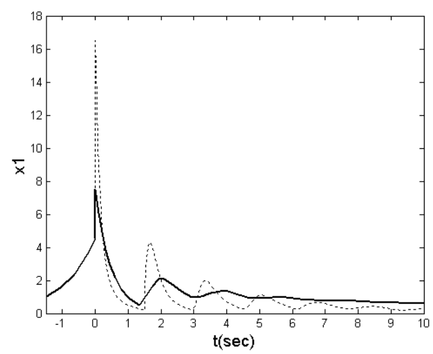


(a) control input $u_1(t)$

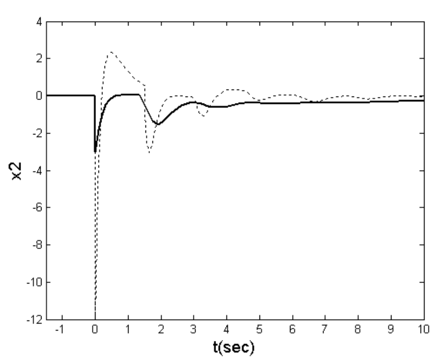


(b) control input $u_2(t)$

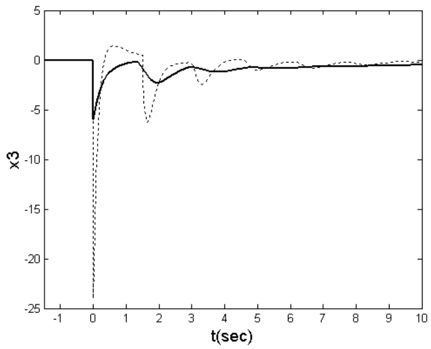
Fig.1 control law



(a) state $x_1(t)$



(b) state $x_2(t)$



(c) state $x_3(t)$

Fig. 2 The state of the closed-loop system

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