

# Design of Observers for Descriptor Systems with delayed state and unknown inputs

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**Abstract**—This note deals with the problem of full-order observers design for a class of linear descriptor systems with delayed state and Unknown Inputs (UI). The objectives are twofolds. Firstly, the design of an full-order Unknown Input Observer for descriptor systems with delays is studied. Secondly, if disturbance rejection is not possible, an  $H_\infty$  observer for such systems is proposed. The existence conditions of these observers are given and proved. An illustrative example is included.

## I. INTRODUCTION

Many engineering processes can be modelled using time-delay in the dynamics equations [2]. In lots of them the delay effects have to be taken into account when one aims to ensure robustness and good performance for the closed-loop system. Let us cite for instance controlled network systems where communication delays may lead to system instability, or automotive industry where engine control is faced to delay in the control input application or in the measurements (for instance of the air-to-fuel ratio)[9].

In the last ten years lots of results have thus been obtained concerning stability, stabilizability and state feedback control, for instance in [6], [10]. Concerning observer design for usual system, few works have been devoted to time-delay systems. When the system is perturbed by an external unknown input, two kinds of methods may be used. The first one is the design of an unknown input observer (UIO), whose goal is to ensure the state estimation despite the presence of external disturbances [8], [3]. Let us recall that the use of UIO is of great interest in the field of Fault Detection and Isolation (FDI) in order to design a residual that is sensitive to a set of faults while being insensitive to the disturbances and other faults [1]. For time-delay systems the UIO design has been solved in [11] using a dynamic gain Luenberger-type observer. The second method is needed when the disturbance cannot be rejected. The purpose of the so called  $H_\infty$  observer design is to bound the  $H_\infty$ -norm of the transfer function from the disturbance to the estimation [7].

The aim of this work is to generalize the results of [5], [4] to the case of descriptor time-delay systems with disturbance. In [4] a design method of reduced-order observers without internal delays for nonsingular systems with delayed state is given. The design of observer for UI

descriptor systems has been studied in [5], but delay were not considered. To the best of our knowledge, no result have been presented concerning the observer design for descriptor systems with delays and disturbance.

The contribution of this paper is then to propose a full-order observer for descriptor systems with delays and with disturbance. Two types of observers will be considered : Unknown Input observers and  $H_\infty$  observer. Under a sufficient condition for disturbance decoupled estimation, the design of a full-order Unknown Input Observer for descriptor systems with UI and delayed state is proposed. If the decoupling condition is not satisfied, an  $H_\infty$  observer can be designed in order to ensure disturbance attenuation. The minimization of the transfer from the UI to the estimate is based on LMI optimization.

The outline of the paper is the following. In section II the problem formulation and main result are presented. Section III is devoted to the design of UIO and  $H_\infty$  observer for descriptor systems with delays and UI. In section IV an example is given to illustrate our results.

## II. PROBLEM FORMULATION AND MAIN RESULTS

### A. Preliminaries

Consider the linear time-invariant descriptor system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_h x(t-h) + Bu(t) + Ww(t) \\ x(t) &= \Phi(t), \quad t \in [-h, 0] \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

where  $h > 0$  denotes the state delay,  $\Phi(t)$  is a continuous vector-valued initial function and  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^k$ ,  $w \in \mathbb{R}^q$  and  $y \in \mathbb{R}^p$  are the plant state vector, the control input vector, the UI vector and the measured output vector, respectively.  $E \in \mathbb{R}^{m \times n}$  is known constant matrix with  $\text{rank}(E) = r$ . The other matrices are known constant with appropriate dimensions and satisfy the following assumption:

$$\text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n \quad (2)$$

We propose the full-order observer for system (1) :

$$\begin{aligned} \dot{z}(t) &= Fz(t) + F_h z(t-h) + TBu(t) + G_1 y(t) \dots \\ &\quad + G_{h_1} y(t-h) + G_2 y(t) + G_{h_2} y(t-h) \\ \hat{x}(t) &= z(t) + Ny(t) \end{aligned} \quad (3)$$

where  $z \in \mathbb{R}^n$  is the state observer and  $F, F_h, T, G_1, G_{h_1}, G_2, G_{h_2}$  and  $N$  are constant matrices of appropriate dimensions to be designed.

*Definition 1:* System (3) is said to be a full-order UIO without internal state delay  $z(t-h)$  if  $\hat{x}$  is an asymptotic estimate of  $x(t)$  for any  $\Phi(t), z(0), y(t), y(t-h), u(t), w(t), F_h = 0$  and  $TW = 0$ .

*Definition 2:* System (3) is said to be a full-order  $H_\infty$  observer if, under zero initial condition, the  $H_\infty$  norm of the transfer function between the disturbance and the estimated error is bounded, in other words  $\|H_{ew}\|_\infty < \gamma$  holds, for  $\gamma$  a real positive scalar.

### B. Main result

The existence condition of a solution to the state estimation problem defined in definition 1 and 2 are given respectively in the following theorems, which are proved in section III.

*Theorem 1:* Under assumption

$$\text{rank} \begin{bmatrix} E & W & A_h \\ C & 0 & 0 \\ 0 & 0 & C \end{bmatrix} = n + \text{rank} \begin{bmatrix} W & A_h \\ 0 & C \end{bmatrix} = 2n + q \quad (4)$$

there exists for system (1) an UIO (3) according to definition 1 if and only if

$$\text{rank} \begin{bmatrix} sI_n - TA \\ C \end{bmatrix} = n, \quad \forall \Re(s) \geq 0 \quad (5)$$

where (5) is the usual detectability condition of the pair  $(TA, C)$  needed to stabilize the estimation error.

*Lemma 1:* Condition (5) can be given in terms of matrices of the original system (1) as

$$\text{rank} \begin{bmatrix} sE - A & A_h & -W \\ C & 0 & 0 \\ 0 & C & 0 \end{bmatrix} = 2n + q \quad \forall \Re(s) \geq 0 \quad (6)$$

*Proof:* See the appendix. ■

*Theorem 2:* There exists an  $H_\infty$  observer (3) for system (1) according to definition 2 if (2) holds and if there exist positive definite matrices  $P$  and  $Q$ , matrices  $U$  and  $U_h$  and a positive scalar  $\gamma$  made as small as possible, verifying the following LMI

$$\begin{bmatrix} \Theta & PTA_h - U_h C & -PTW \\ (PTA_h - U_h C)^T & -Q & 0 \\ -(TW)^T P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (7)$$

where  $\Theta = (PTA - UC)^T + PTA - UC + I + Q$

*Proof:* Is derived along the section III. B ■

*Remark 1:* The regularity assumption (i.e.  $A, E$  are square) is not required and system (1) should not be necessarily impulse free. The UI and state decoupling condition (4) needed for designed an UIO can be relaxed like a  $H_\infty$  attenuation problem using the LMI resolution (7).

*Remark 2:* (4) represents the condition of disturbance and state delay decoupling of observer. For  $E = I_n, A_h = 0$  we obtain the usual UI decoupling condition  $\text{rank}[CW] = \text{rank}W = q$  [3].

*Proof:* For  $E = I_n, A_h = 0$  and the regular matrix  $U = \begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$

$$\begin{aligned} (4) &\Leftrightarrow \text{rank} \begin{bmatrix} I_n & W \\ C & 0 \end{bmatrix} = n + \text{rank}[W] = n + q \\ &\Leftrightarrow \text{rank}U \begin{bmatrix} I_n & W \\ C & 0 \end{bmatrix} = n + \text{rank}[W] = n + q \\ &\Leftrightarrow \text{rank}CW = \text{rank}[W] = q \end{aligned}$$

■

## III. OBSERVER DESIGN

### A. UIO design

When the observer (3) is applied to the system (1), the estimation error  $e(t) = \hat{x}(t) - x(t)$  becomes

$$\begin{aligned} e &= z + NCx - x \\ &= z - (I_n - NC)x \end{aligned} \quad (8)$$

From (2) there exists a full row rank matrix  $[T \ N]$  such that

$$[T \ N] \begin{bmatrix} E \\ C \end{bmatrix} = I_n \quad (9)$$

is satisfied and (8) is reduced to

$$e = z - TEx \quad (10)$$

then, the dynamic of this observer error is

$$\begin{aligned} \dot{e}(t) &= Fe(t) + F_h e(t-h) - TWw(t) \dots \\ &\quad + (F_h TE + (G_{h_1} + G_{h_2})C - TA_h)x(t-h) \dots \\ &\quad + (FTE + (G_1 + G_2)C - TA)x(t) \end{aligned} \quad (11)$$

Let us note

$$G_1 = FN \quad (12)$$

$$G_{h_1} = F_h N \quad (13)$$

substitute (12) and (13) in (11) and from (9) we obtain

$$\begin{aligned} \dot{e}(t) &= Fe(t) + F_h e(t-h) + (F + G_2 C - TA)x(t) \dots \\ &\quad + (F_h + G_{h_2} C - TA_h)x(t-h) - TWw(t) \end{aligned} \quad (14)$$

If

$$F = TA - G_2 C \quad (15)$$

$$F_h = TA_h - G_{h_2} C \quad (16)$$

$$TW = 0 \quad (17)$$

$$F_h = 0 \quad (18)$$

equation (14) is reduced to the homogeneous equation

$$\dot{e}(t) = Fe(t)$$

In this case the system dynamics  $F$  can be stabilized by selecting the gain  $G_2$  due to the detectability condition (5).

In the sequel we will give a solution of  $T$ ,  $N$  and  $G_{h_2}$  which satisfies constraints (9), (18) and (17). For that, rewrite (9), (18) and (17) like

$$\begin{bmatrix} T & N & -G_{h_2} \end{bmatrix} \begin{bmatrix} E & W & A_h \\ C & 0 & 0 \\ 0 & 0 & C \end{bmatrix} = \begin{bmatrix} I_n & 0 & 0 \end{bmatrix} \quad (19)$$

and assuming that (4) holds, then a particular solution of (19) is given by (see [3] for  $Y=0$ )

$$\begin{bmatrix} T & N & -G_{h_2} \end{bmatrix} = \begin{bmatrix} I_n & 0 & 0 \end{bmatrix} \psi^+ \quad (20)$$

where

$$\psi = \begin{bmatrix} E & W & A_h \\ C & 0 & 0 \\ 0 & 0 & C \end{bmatrix} \quad (21)$$

$\psi^+$  is the generalized inverse matrix of  $\psi$ , given by  $\psi^+ = (\psi^T \psi)^{-1} \psi^T$ , since  $\psi$  is of full column rank. That completes the design of UIO and it can be summarized by the following algorithm.

*Algorithm 1:* Under (4) compute  $\begin{bmatrix} T & N & -G_{h_2} \end{bmatrix}$  from (20). If the pair  $(TA, C)$  is detectable, find the gain  $G_2$  by any pole placement algorithm which stabilizes (15) and deduce  $F$ ,  $G_1 = FN$ ,  $F_h = 0$  and  $G_{h_1} = F_h N = 0$ .

#### B. $H_\infty$ observer design

In this section the main objective is to relax the assumption (4) by an  $H_\infty$  transfer attenuation.

Using the previous developed results, when assumption (2) holds and (4) is not satisfied, the estimation error satisfies the following system:

$$\dot{e}(t) = Fe(t) + F_h e(t-h) - TWw(t) \quad (22)$$

From (2), (9) becomes

$$\begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^+ \quad (23)$$

and let  $G_1 = FN$ ,  $G_{h_1} = F_h N$  in order to satisfies (15) and (16). Let  $V(e, t)$  be the usual Lyapunov-Krasovskii functional of the form

$$V(e, t) = e^T(t)Pe(t) + \int_{t-h}^t e^T(\theta)Qe(\theta)d\theta \quad (24)$$

where  $P > 0$  and  $Q > 0$ . In order to establish sufficient conditions for the existence of (3) according to the definition 2, we should verify the following inequality

$$H(e, w) = \dot{V}(e, t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0 \quad (25)$$

By differentiating  $V(e, t)$  along the solution (22), we obtain

$$\begin{aligned} H(e, w) &= e^T(t)(F^T P + PF + I)e(t) - \gamma^2 w^T(t)w(t) \dots \\ &+ e^T(t-h)F_h^T P e(t) - w^T(t)(TW)^T P e(t) \dots \\ &+ e^T(t)PF_h e(t-h) - e^T(t)PTWw(t) \dots \\ &+ e^T(t)Qe(t) - e^T(t-h)Qe(t-h) < 0 \quad (26) \end{aligned}$$

and it can be rewritten like

$$v^T \begin{bmatrix} F^T P + PF + I + Q & PF_h & -PTW \\ F_h^T P & -Q & 0 \\ -(TW)^T P & 0 & -\gamma^2 I \end{bmatrix} v < 0 \quad (27)$$

where  $v^T = [e^T(t) \quad e^T(t-h) \quad w(t)]$ .

From (27),  $\dot{H}(e, w) < 0$  if

$$\begin{bmatrix} F^T P + PF + I + Q & PF_h & -PTW \\ F_h^T P & -Q & 0 \\ -(TW)^T P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (28)$$

Substituting (15) and (16) in (28) and let  $U = PG_2$ ,  $U_h = PG_{h_2}$  we obtain the LMI (7) on  $P$ ,  $U$ ,  $Q$  and  $U_h$  where

$$G_2 = P^{-1}U \quad (29)$$

$$G_{h_2} = P^{-1}U_h \quad (30)$$

are the filter gain which minimizes  $\|H_{ew}\|_\infty < \gamma$  and stabilizes (22).

*Algorithm 2:* Under (2) compute  $\begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^+$  and solve the LMI (7) on  $P$ ,  $U$ ,  $Q$  and  $U_h$  with  $0 < P = P^T$  and  $0 < Q = Q^T$ . The matrix gain  $G_2$  and  $G_{h_2}$  are deduced from  $G_2 = P^{-1}U$  and  $G_{h_2} = P^{-1}U_h$ . From (15), (16), (12) and (13) deduce  $F$ ,  $F_h$ ,  $G_1$  and  $G_{h_1}$  respectively.

## IV. EXAMPLE

In this section the two design algorithms are illustrated by simple numerical examples. Let consider the observable system (1) defined by

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 0 & 1 & 0 \\ -1 & -2 & 0 & 1 \\ 0 & -1 & -3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \\ A_h &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}, \quad W = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (31)$$

#### A. UIO illustration

Assumption (4) holds. From algorithm 1 we obtain

$$\begin{aligned} T &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -0.2 & -1 \\ 0 & 0 & -0.4 & 0 \\ 0 & 0 & -0.2 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.2 & -0.2 \\ 0 & 0.6 & -0.6 \\ 0 & 0.2 & 0.8 \end{bmatrix}, \\ G_{h_2} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & -0.4 & 0.4 \\ 0 & 0.2 & -0.2 \end{bmatrix} \end{aligned} \quad (32)$$

and since the pair  $(TA, C)$  is observable we choose to place the eigenvalues in  $[-50 \ -51 \ -52 \ -53]$ , we obtain

$$\begin{aligned} G_2 &= \begin{bmatrix} 52 & 0 & 0 \\ -3.8 & -4811 & 7531 \\ 0 & 89 & -111 \\ 0 & -19 & 81 \end{bmatrix}, \\ F &= \begin{bmatrix} -52 & 0 & 0 & 0 \\ 2.8 & -1.8 & 4811 & -2717 \\ 0 & -0.4 & -90.2 & 22.1 \\ 0 & 0.2 & 19.6 & -62 \end{bmatrix}, \\ G_1 &= \begin{bmatrix} -52 & 0 & 0 \\ 2.8 & 2343 & -5060 \\ 0 & -49.8 & 71.8 \\ 0 & 0.6 & -61.4 \end{bmatrix}, \\ F_h &= 0 \text{ and } TW = 0 \end{aligned} \quad (33)$$

The results obtained with the designed UI observer are displayed on figure 1. The estimated state variable are initialized by  $\hat{x}(0) = 1$ . One can see the perfect disturbance decoupling and the quality of the estimation.

### B. $H_\infty$ Observer illustration

Let consider an observable system (1), defined by the matrices given in the previous example, except the disturbance distribution matrix  $W$  which is replaced by

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Since assumption (4) is not satisfied an UIO can not be implemented. Since (2) holds, an  $H_\infty$  observer, according to definition 2, can be designed. Applying algorithm 2 we obtain

$$T = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2/3 & 0 \\ 0 & 0 & -1/3 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/3 & -1/3 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

Solve the LMI (7) on  $P, U, Q$  and  $U_h$  with  $0 < P = P^T$  and  $0 < Q = Q^T$  we obtain

$$\begin{aligned} F &= 10^4 \begin{bmatrix} -8.3714 & 0 & 0.0299 & -0.0626 \\ -0.0000 & -0.0002 & -0.0000 & 0.0001 \\ -0.0000 & -0.0001 & -0.0003 & -0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.0002 \end{bmatrix}, \\ F_h &= 10^3 \begin{bmatrix} 0.1806 & 0 & 1.0467 & -0.5514 \\ -0.0001 & -0.0010 & -0.0000 & 0.0002 \\ 0.0000 & 0 & -0.0005 & 0.0000 \\ -0.0000 & 0 & 0.0002 & -0.0000 \end{bmatrix}, \\ G_2 &= 10^4 \begin{bmatrix} 8.3713 & -0.0299 & 0.0925 \\ -0.0001 & 0.0000 & 0.0000 \\ 0.0000 & 0.0001 & -0.0000 \\ -0.0000 & 0.0001 & 0.0001 \end{bmatrix}, \\ G_{h_2} &= 10^3 \begin{bmatrix} -0.1811 & -1.0467 & 1.5981 \\ 0.0001 & 0.0000 & -0.0002 \\ -0.0000 & -0.0001 & 0.0001 \\ 0.0000 & 0.0001 & -0.0001 \end{bmatrix}, \\ G_1 &= 10^4 \begin{bmatrix} -4.1857 & -0.0109 & -0.0517 \\ -0.0000 & 0.0000 & 0.0000 \\ -0.0000 & -0.0001 & 0.0001 \\ 0.0000 & -0.0000 & -0.0001 \end{bmatrix}, \\ G_{h_1} &= \begin{bmatrix} 90.3015 & 165.0975 & -716.4995 \\ -0.0368 & 0.0435 & 0.1230 \\ 0.0054 & -0.1683 & 0.2124 \\ -0.0045 & 0.0715 & -0.0799 \end{bmatrix} \end{aligned}$$

with  $\gamma = 0.0005$ . Furthermore we obtain the following static transfer function from  $w$  to  $e$

$$\begin{aligned} e_\infty &= -(F + F_h)^{-1} TW w_\infty \\ &= 10^{-5} \begin{bmatrix} 0.5992 & 0 \\ -0.1119 & 0 \\ 0.0263 & 0 \\ -0.0144 & 0 \end{bmatrix} w_\infty \end{aligned}$$

and the result of the estimation are displayed on figure 2. The  $H_\infty$  observer is initialized by  $\hat{x}(0) = 1$ .

## V. CONCLUSION

In this paper, we have presented a simple method for the design of a full-order observer without internal state delay and with  $H_\infty$  attenuation for descriptor systems with disturbance and delayed state. Existence conditions of the observer gains have been given and proved. When the UI decoupled condition needed in UI observer design is not satisfied it is relaxed by a  $H_\infty$  attenuation problem. This allows to provide a unified design procedure for both observer types. Further works may concern the robustness of such observers with respect to parameters variations (including the delay), using for instance some  $H_\infty$  criteria of robust stability as in [12].

## VI. APPENDIX

*Proof:* of lemma 1. From (19) we have  $\Psi^+ \Psi = I_{2n+q}$  with  $\psi^+ = \begin{bmatrix} T & N & -G_{h_2} \\ V_1 & V_2 & V_3 \\ V_4 & V_5 & V_6 \end{bmatrix}$ , hence define the full row

rank matrix

$$V = \begin{bmatrix} T & N & -G_{h2} & 0 \\ V_1 & V_2 & V_3 & 0 \\ V_4 & V_5 & V_6 & 0 \\ 0 & 0 & 0 & I_p \end{bmatrix}$$

Since

$$\begin{bmatrix} sE - A & A_h & -W \\ C & 0 & 0 \\ 0 & C & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} sE - A & W & A_h \\ sC & 0 & 0 \\ 0 & 0 & C \\ C & 0 & 0 \end{bmatrix}$$

the following equivalences hold

$$\begin{aligned} (6) & \iff \text{rank} V \begin{bmatrix} sE - A & W & A_h \\ sC & 0 & 0 \\ 0 & 0 & C \\ C & 0 & 0 \end{bmatrix} = 2n + q, \forall \Re(s) \geq c \\ & \iff \text{rank} \begin{bmatrix} sI_n - TA & 0 & 0 \\ * & I_q & 0 \\ * & 0 & I_n \\ C & 0 & 0 \end{bmatrix} = 2n + q \forall \Re(s) \geq 0 \\ & \iff (5) \end{aligned}$$

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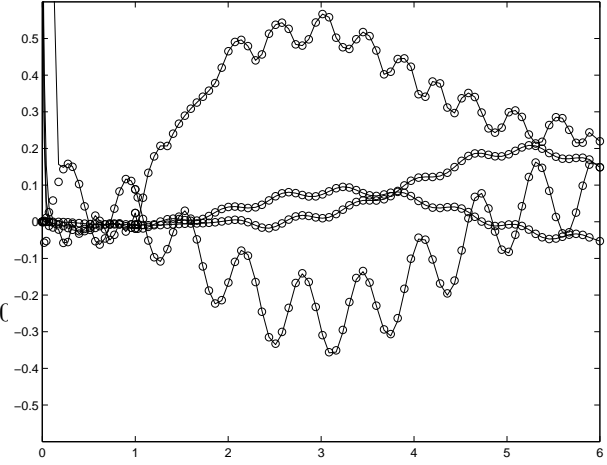


Fig. 1. Original (circles) and estimated (solid line) state variable obtained with an UI observer.

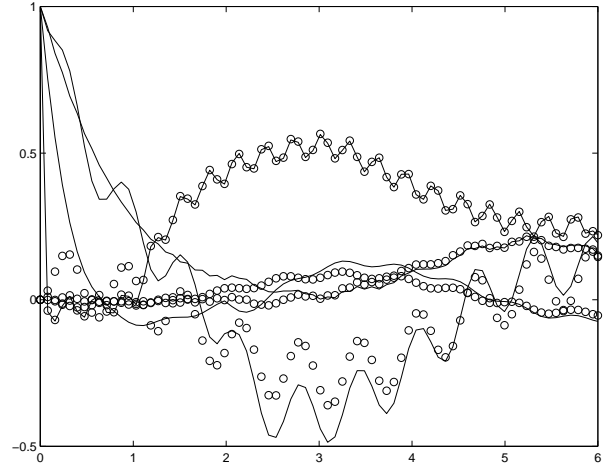


Fig. 2. Original (circles) and estimated (solid line) state variable obtained with an  $H_\infty$  observer.