

A direct approach to identify closed loop Wiener systems, whose linear dynamics are open-loop unstable

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Abstract -- A Wiener system is a series connection of a linear dynamic system followed by a static non-linearity. The identification of Wiener systems has been an active research topic for years. In this paper, we extend the algorithm proposed by Zhao & Westwick, [Proceedings, ACC2003] [1] to identify Wiener systems that are unstable in open loop, but being operated stably in a closed-loop configuration. The variant of the MOESP (Multivariable Output-Error State Space) algorithm developed in [1] will be used to identify a state space model of the linear part of a Wiener system operating in closed loop. Since the linear dynamics of the Wiener system are unstable in open loop, the output of the linear subsystem cannot be obtained by direct simulation. Without an estimate of the linear output, the nonlinearity can't be estimated. The main contribution of this paper is the design of an extended Kalman filter which is used to estimate the states of the linear subsystem as well as the parameters of the nonlinearity.

1. INTRODUCTION

Due to their relative simplicity and varied applications, Wiener systems have been considered as useful structures in nonlinear system identification. For the identification of Wiener systems in open loop, there are already a number of methods, including a subspace method [2]. However, if the Wiener system is operated in closed-loop, the algorithm in [2] will produce biased estimates because it requires the input signal to the Wiener system to be Gaussian. The input to the Wiener system is generated by the closed loop system and it can't be guaranteed to be Gaussian. Chou and Verhaegen developed an indirect approach [3] to solve this closed-loop identification problem. In that algorithm, the open-loop subspace algorithm proposed by Westwick and Verhaegen [2] was used to get an initial, but likely biased, estimate of the Wiener system. Then a nonlinear minimization was performed to refine this estimate. We have developed a direct method to estimate the Wiener system in closed loop [1], provided that the Wiener system itself is stable both in open loop and in closed loop. The reference signal is used as an instrumental variable. Selecting a persistently exciting, Gaussian signal as the reference input, and hence as the instrumental variable, reduces the bias in the subspace method's estimate of the linear dynamics. However, the algorithms in [1] and [2]

will fail if the Wiener system is unstable in open loop since they both require open loop simulation of the linear system before the nonlinearity can be fitted. In this paper, we will extend the algorithm in [1] and provide an alternative identification method to work when the Wiener system is unstable in open loop, but exists with a stabilizing controller together in closed loop.

The outline of the paper is as follows. In section 2, we review the algorithm in [1] and explain why it fails when the Wiener system is unstable in open loop. Section 3 will introduce a Kalman type filter based technology that estimates the parameters of the nonlinearity, provided the system is stable in closed loop. Finally, two simulated examples will be presented.

2. IDENTIFICATION METHODS FOR OPEN-LOOP STABLE WIENER SYSTEMS OPERATING IN CLOSED LOOP

A Wiener system is a series connection of a linear dynamic system followed by a static non-linearity. In some cases, particularly when the linear dynamics are unstable, it may not be possible to perform an open-loop experiment, in which case, we must deal with a closed loop system, such as Figure 1.

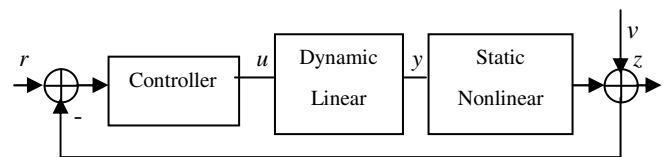


Figure 1. Wiener system operated in closed-loop system. r , is the reference input. u , is the input to the Wiener system. y , is the output from the linear part and the input to the nonlinear part as well. z , is the output of the whole system. v is the output noise. In this paper, we assume r , u and z can be measured. v , is a white noise and independent with the reference input, r . y is internal and can't be measured.

Mathematically the above closed loop Wiener system can be expressed by the following equations:

Linear part:

$$x_{k+1} = Ax_k + Bu_k \quad (1)$$

$$y_k = Cx_k + Du_k \quad (2)$$

∇ Nonlinear part:

$$z_k = F(y_k) + v_k \quad (3)$$

∇ Controller:

$$u_k = H_{(q)}(r_{k-j} - z_{k-j}) \quad (4)$$

where $x_k \in R^n$, $u_k \in R^m$, $y_k \in R^l$, $r_k \in R^l$, $z_k \in R^l$ and $F(\cdot)$ is a nonlinear function which maps $R^l \rightarrow R^l$. $H_{(q)}$ is an operator expression for a linear controller which stabilizes the system in closed loop. The measurement noise, $v_k \in R^l$, is assumed to be a white-noise sequence that is independent of the reference input r_k , and of the current value and the past value of u_k . The goal of the identification is to estimate the linear system matrices (A, B, C, D) and the parameters of the nonlinearity from measurements of the input-output data (r_k, u_k, z_k) .

Mathematically, we can express a Wiener system as a series connection of the linear subsystem and nonlinear subsystem. However, in practice, the linear part and the nonlinear part are integrated together and there is no way to measure the linear output experimentally. This makes the identification of a Wiener system difficult. To estimate a Wiener system operating in closed-loop, Zhao and Westwick [1] developed a subspace method based on an extension to Bussgang's theorem [4]. Specifically, if the reference input is a Gaussian signal, due to central limit theory, the input to the nonlinear part will be an approximately Gaussian signal. Thus the cross-correlation between the reference input and nonlinear output will be approximately proportional to the cross-correlation between the reference input and the linear output. Based on these observations, the PR-MOESP [5] (using the past reference signal as an instrumental variable) can be then extended to Wiener systems.

In practice, this can be accomplished by using an LQ factorization as follows:

$$\begin{bmatrix} U_f \\ R_p \\ Z_f \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \quad (5)$$

where U_f , Z_f and R_p are Hankel matrices constructed from the future input, the future output and the past reference input respectively [6]. And the subspace L_{32} will be proportional to the subspace that only contains the zero-input response of the linear system.

$$L_{32} \approx k \Gamma_i X_{i,N} Q_2 \quad (6)$$

The column space of L_{32} is spanned by the extend Observability matrix Γ_i . We can then use SVD (Singular Value Decomposition) to estimate Γ_i . Moreover, following the algorithm in [2], we can easily obtain the linear system

matrices $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$. To get the input to the nonlinearity, we have to simulate this linear model in an open-loop experimental environment using the input signal, u . We can then fit the nonlinearity between this simulated output and each of the true observed outputs using Chebyshev polynomials as basis functions.

Now it is clear why this algorithm is not suitable for systems whose linear dynamics are unstable in open loop. Fitting the nonlinearity requires an open-loop simulation of the estimated linear system. However, if the system is unstable, this simulation can't be performed. Without the intermediate signal, we can't estimate the nonlinearity.

3. DUAL EXTENDED KALMAN FILTER

In this section, we will mainly focus on how to estimate the parameters of the nonlinearity after we estimate the linear part of the Wiener system using the algorithm in [2].

The instability of the Wiener system in open loop makes it impossible to simulate the system in open loop to get the estimated linear output. However, its linear states remain bounded when the system is operating in closed-loop with a stabilizing controller. With the estimated linear system, if we can estimate the bounded linear states in closed loop, we can calculate the estimated linear output. Kalman type filters have been viewed as standard tools to estimate the states of linear or nonlinear systems. In our case, we need to estimate both linear states and the nonlinear parameters, since we don't know the nonlinear parameters so far. As proposed in [7] [8] [9], the extended Kalman filter can also be used to estimate the parameters of nonlinear models. This type of Kalman filter is also known as the dual extended Kalman filter, proposed by Wan and Nelson [10]. The dual extended Kalman filter basically combines two extended Kalman filters. The first one estimates the states of the systems, while the second one estimates the parameters of the nonlinearity. Those two extended Kalman filters run concurrently. The first Kalman filter requires the nonlinear parameters for the time-update, while the second Kalman filter needs the linear states for the measurement update.

The Wiener system can be represented by a set of state-space equations:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ z_k &= W(Cx_k + Du_k) + v_k \end{aligned}$$

where (A, B, C, D) are the linear system matrices, $W(\bullet)$ is the nonlinear function and v_k is the measurement noise.

To estimate the nonlinearity, the first step is to represent it as a weighted sum of basis functions:

$$z(k) = W(\hat{y}_{D_F}) \bar{\theta} + v_k \quad (7)$$

where $\bar{\theta}_n = [\theta_1 \ \theta_2 \ \dots \ \theta_n]^T$ contains the parameters of the nonlinear function, and $W(\cdot)$ contains basis functions.

Choosing the nonlinear basis function is not the main topic of this paper. In this paper, we used Chebyshev polynomials [11] of orders 1 through n to represent the nonlinearity. Thus,

$$W(x) = [T_1(x) \ T_2(x) \ \cdots \ T_n(x)]$$

where $T_i(x)$ are Chebyshev polynomials defined by the recursive relationship

$$\begin{aligned} T_1(x) &= 1 \\ T_2(x) &= x \\ T_i(x) &= 2 \cdot x \cdot T_{i-1}(x) - T_{i-2}(x) \end{aligned} \quad (8)$$

After choosing the nonlinear basis functions, the next step is to estimate the parameters of the nonlinearity. To achieve this using the Kalman filter, we must somehow add those parameters to the state equation so that they can be updated. To achieve this, we change the system matrices and now we consider the parameters of the nonlinear model as part of the states. So we can have a new state space model:

$$\begin{aligned} \begin{bmatrix} \bar{x}_{k+1} \\ \bar{\theta}_{k+1} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & I_k \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ \bar{\theta}_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \\ z_k &= W(Cx_k + Du_k)\bar{\theta}_k + v_k \end{aligned} \quad (9)$$

where $\bar{\theta}_k = [\theta_{k,1} \ \theta_{k,2} \ \cdots \ \theta_{k,n}]^T$ contains the time-varying parameters of the nonlinear function, I_k is an identity matrix with order equal to the order of the nonlinearity and W is the nonlinear basis function (8).

As to the newest states space model (9), we have known (A, B, C, D) matrices and we have chosen a nonlinear basis function. Though we don't know the parameters of this basis function, those parameters are included in the state vectors and we can estimate them after we run the filter. So for the above model, with the measured input and output data, we can design an extended Kalman filter by the following steps:

Initialize the filter with

$$\begin{aligned} \hat{x}_0 &= E[x_0] \\ P_{x_0} &= E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \end{aligned}$$

For $k \in \{1, \dots, \infty\}$, the time update equations of the extended Kalman filter are:

$$\begin{aligned} \hat{x}_k^- &= \hat{A}\hat{x}_{k-1}^- + \hat{B}u_k \\ P_{x_k}^- &= \hat{A}_{k-1}P_{x_{k-1}}\hat{A}_{k-1}^T + R^w \end{aligned}$$

and the measurement update equations:

$$\begin{aligned} K_k^x &= P_{x_k}^- C_k^T (C_k P_{x_k}^- C_k^T + R^v)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k^x (y_k - W(C\hat{x}_k^- + Du_k)\bar{\theta}_k) \\ P_{x_k} &= (I - K_k^x C_k) P_{x_k}^- \end{aligned}$$

where $C_k \equiv \left. \frac{\partial \Phi(x)}{\partial x} \right|_{\hat{x}_k}$ and R^w, R^v are the covariances of w_k and v_k respectively.

Remark: Ideally we want to use $E(x_0)$ to initialize the Kalman filter. However this is usually unavailable. In practice, we set the initial linear states equal to zeros. As to the nonlinear parameters, we use a small random number. As to P_{x_0} , we choose a relatively small value for the state estimates and a relatively large value for the parameter estimates. R^w reflects the process noise. We use a diagonal matrix with all zeros for the nonlinear parameters and a small value for the linear states. Then R^v is decided from the output SNR.

Now we briefly list the algorithm:

ALGORITHM

1. Using the algorithm in [1] to estimate the linear part of the Wiener system based on the experimental data (r_k, u_k, z_k) .
2. Design an extended Kalman filter.
3. Run the extended Kalman filter. The first n (n is the order of the linear system) states are the estimation of the linear states and the rest are the parameters of the nonlinearity.

4. EXAMPLE:

In this section, we provide a simulation example to show how our algorithm works. The identification procedure contains two steps. In the first step, we use the algorithm in [1] to estimate the linear part of the Wiener system. With the estimated linear dynamics, we design an extended Kalman filter to estimate the parameters of the nonlinearity.

The transfer function of the linear part of the Wiener system is

$$P(z) = \frac{z(z+11.7)(z+1.39)(z+0.216)}{(z-1.056)(z^2-1.487z+0.5807)(z^2-1.465z+0.9042)}$$

The plant has a pole at 1.056 which makes the plant unstable in open loop. So a discrete PD controller is designed and its transfer function is as the following:

$$c(Z) = 0.181 \times \frac{1.22z - 0.22}{z}$$

The gain of Proportional controller is 0.181 and the gain for Derivative controller is 1.99×10^{-4} .

We use a polynomial, $F(x) = 0.2x^3 + 0.01x^2 + 0.5x + 0.1$, as the nonlinearity. As required in the algorithm [1], the reference input, $r(k)$, must be a Gaussian random sequence so that we use a Gaussian white noise of STD=1 (standard deviation) to excite the system. Considering real-world constraints, we filtered the input using a low-pass

filter with a normalized cut-off frequency of 0.6. The measurement noise is a zero-mean Gaussian signal and the SNR is 20 dB. The simulation runs for 50 seconds and provides 10,000 points with the sample time equal to 1/200 second. Figure 2 shows the first second of data set from this simulation.

To estimate (A,B,C,D) and the parameters of the nonlinearity numerically, we used the SMI Toolbox 2.0 [6] modified as modified as described in [1] and [5]. The (A,B,C,D) that we obtained from this identification is the open loop state space model of the linear subsystem, up to a similarity transformation and output scaling.

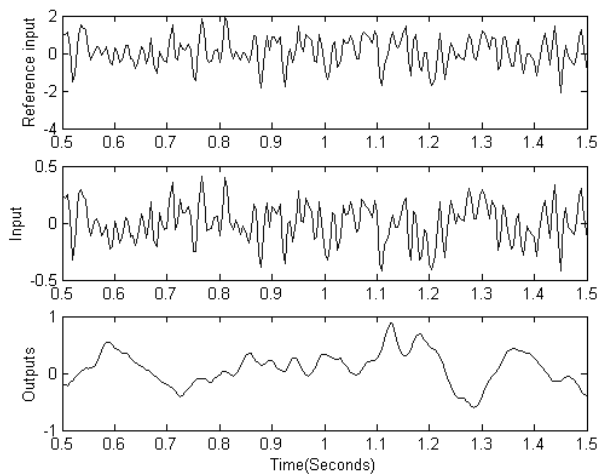


Figure 2. Simulation data used for identification

To test the estimation accuracy, we first compared the true and estimated poles, as shown in Figure 3. We can see that the algorithm estimates the poles pretty well, especially the most crucial pole which is outside the unit circle.

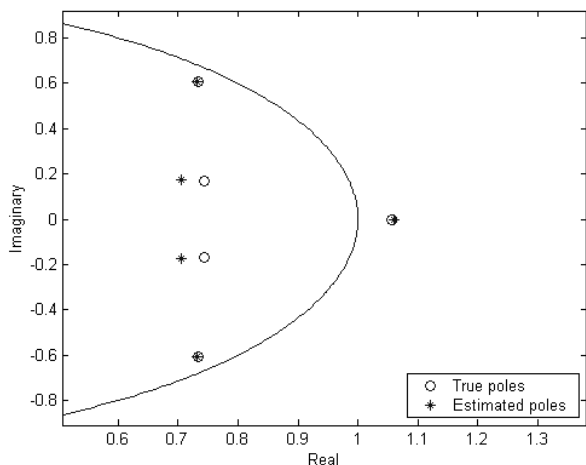


Figure 3. Comparison of the true poles with the estimated poles After estimating the linear part of the Wiener system, we set up a dual Kalman filter to estimate the parameters of the nonlinearity based on the input and output data with the estimated system matrices. To validate the nonlinear parameters, we compared the measured output and the estimated output from the Kalman filter. Figure 4 shows the

results. Moreover, we also simulated the true system and the estimated system in closed loop using a new reference input and then compared the output from the true system with the output from the estimated system. The result is shown in Figure 5. To compare the estimation accuracy, we use VAF (Variance Accounted For), which can be expressed by

$$VAF\% = \left(1 - \frac{\text{variance}(y - y_{est})}{\text{variance}(y)}\right) \times 100\%$$

Since all the simulations were performed in a computer-based environment, we could actually measure the output from the linear part, which would be unavailable in a real experiment. Using this signal, we measured the accuracy of the estimated nonlinearity. In Figure 6, we plot the true nonlinearity and the estimated nonlinearity. We can see the result is very good. Based on other simulation results, the algorithm still provides very good estimate when the cut-off frequency is as low as 0.2.

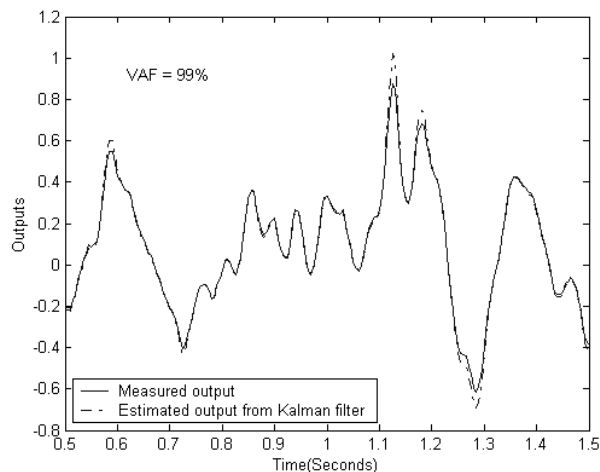


Figure 4. Comparison of the true output with the estimated output from the Kalman filter

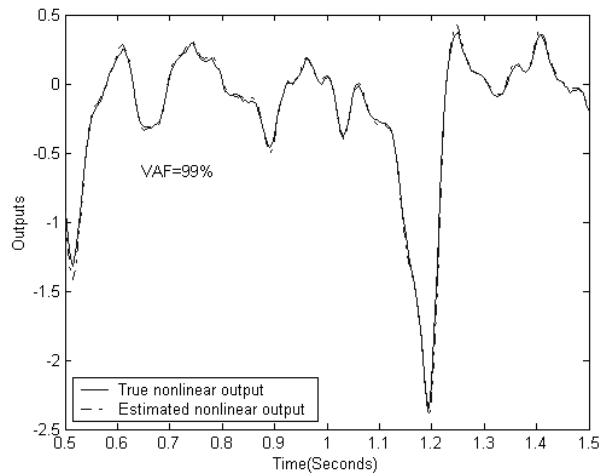


Figure 5 Comparison of the true and estimated system outputs obtained from a closed-loop simulation using a novel reference input.

Though our algorithm is based on the fact that the reference input must be Gaussian, some experiments have been done to test non-Gaussian inputs. In the following experiment, we will use a Bernoulli sequence, with zero mean and unit variance. Similar with the first simulation, we also filter the Bernoulli sequence using a low-pass filter. The cut-off frequency is 0.6. Figure 7 records the reference input signal, the input to the system and the output from the system.

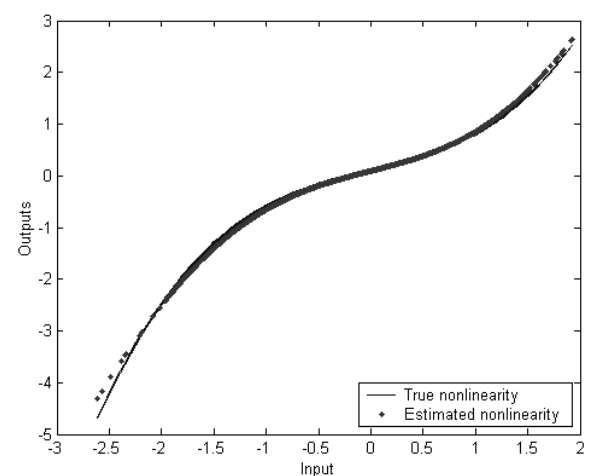


Figure 6. Comparison of the input-output characteristics of the true and estimated nonlinearities.

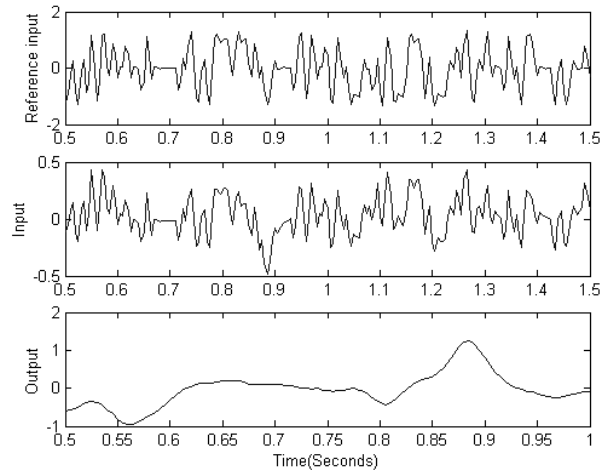


Figure 7. Reference input, input and output from the second simulation.

With the simulated data, we can now do the identification for both the linear and nonlinear parts. Figure 8 shows the estimated poles with the true poles from the linear part of the Wiener system.

To compare the nonlinearity, we first compare the measured output and the estimated output from the Kalman filter. Figure 9 shows the comparison. Then we used another set of data. Figure 10 compares the true output from the system and the simulated output from the estimated system. Figure 11 compares the true nonlinearity with the estimated nonlinearity.

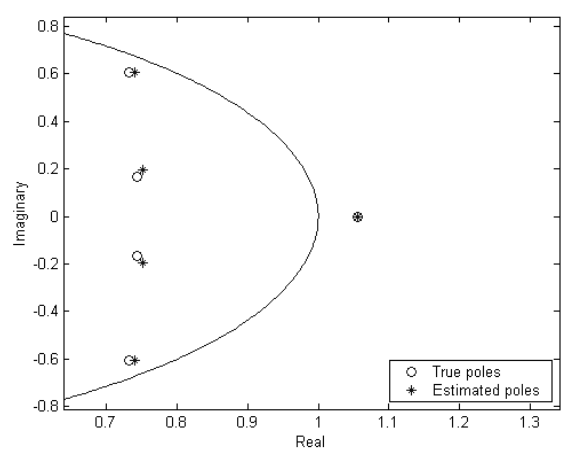


Figure 8. Comparison of the true poles with the estimated poles

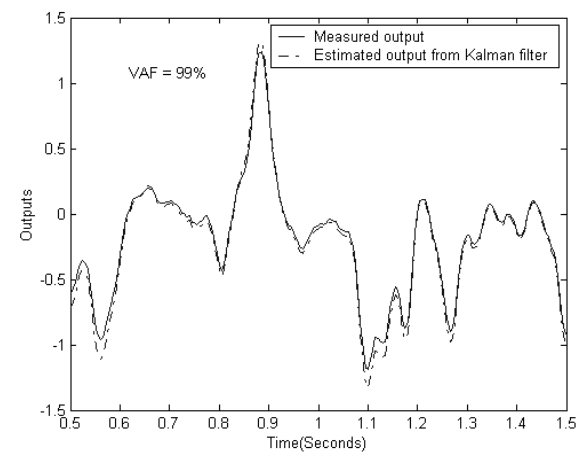


Figure 9 Comparison of the true output with the estimated output from the Kalman filter

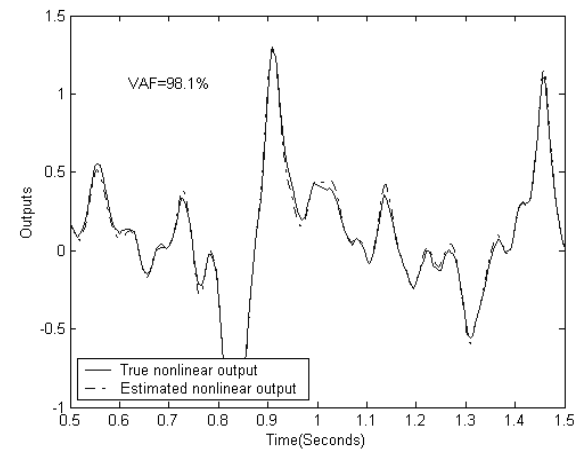


Figure 10. Comparison of the true output with the estimated output

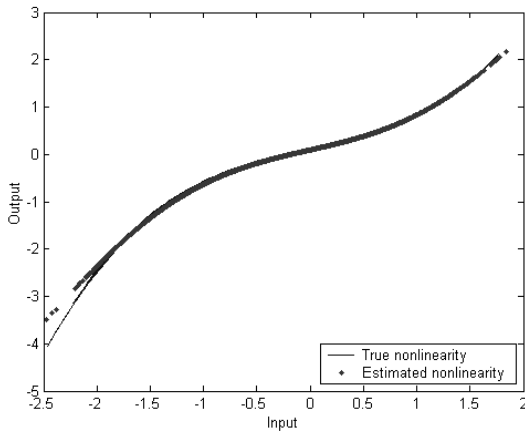


Figure 11. Comparison of the true nonlinearity with the estimated nonlinearity

5. CONCLUSION:

In this paper, we showed how to identify the Wiener operating in closed loop directly from the input-output data when the system is unstable in open loop. The first step is to use the algorithm in [1] to estimate the linear dynamics of the Wiener system. The second step is to setup a dual extended Kalman filter to estimate the linear states and the parameters of the nonlinearity. We presented two illustrative simulation results. Both simulation results show comparisons between the measured true nonlinear output and the one step ahead predicted output from the extended Kalman filter, and the comparison between the measured true nonlinear output and the simulated output from the estimated system operating in closed loop. The behaviour of the estimated system in closed loop is a much more valuable indication of the model accuracy than is a one step ahead prediction of the output. If the nonlinear basis functions fit the nonlinearity well, the simulated output from estimated system operating in closed loop tracks the true output pretty well: for example, we obtained VAFs of 99% and 98% in the two simulations.

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