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# Recursive Identification of Systems with Hard Input Nonlinearities of Known Structure

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***Abstract*— We are considering nonlinear system identification using Hammerstein model, where the linear and nonlinear elements are both of known structure. The static gain may be discontinuous and is not linear in the (unknown) parameters. The focus is made on the case of a two-segment piecewise-linear gain with a preload and/or a dead zone. A recursive identification scheme is designed to determine the models of both the plant dynamics and the static gain. It involves a gradient algorithm, a singular value decomposition and an ad-hoc input sequence that ensures persistent excitation.**

## I. Introduction

We are considering the problem of nonlinear system identification based on Hammerstein model as shown in (fig. 1), where v(t), y(t) and z(t) are respectively the system input, output and disturbance. Most of the studies devoted to such a problem have supposed the nonlinear element characteristic u=F(v, θ) to be analytic, continuous in v and linear in the unknown parameters θ. Generally, F(v, θ) is a (truncated) polynomial or Fourier series in the variable v (e.g. [2]-[6]). The case of hard input nonlinearities have been considered in [7]-[8]. Then, F(v, θ) may be discontinuous in v and nonlinear in θ. Furthermore, F(v, θ) may not be written as an analytic function of v. On the other hand, control design involving these hard nonlinearities is becoming an active research area (e.g. see [9] and reference list therein). Surprisingly, there is a comparatively less interest paid to identification of systems with hard nonlinearities. In [7] the case of a two-segment piece-wise linear input nonlinearity with preload and dead-zone is considered. System identification is carried out based on a pseudolinear regression, which necessitates an alternative estimation of the relevant parameters and some auxiliary variables. The effectiveness of the proposed solution is illustrated by simulations but no formal analysis is made to prove the algorithm consistency. In [8] a separable least squares approach is proposed to deal with a specific set of nonlinearities involving one unknown parameter. It is shown that the identification problem is equivalent to a one-dimensional minimization problem. For a general input nonlinearity the author proposes a correlation analysis approach involving repeated applications of inputs. The first approach is interesting since the nonlinear element may be nonstatic; however, it is limited to one unknown parameter and the determination of the global minimum may be problematic in practice. The second approach is limited to noise-free situations and involves complex computations.

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In this paper, an identification scheme is built-up to deal with the case where the nonlinear block is a two-segment piecewise-linear function with a preload and/or a dead-zone (fig 2). Such a scheme allows to identify perfectly all the plant unknown parameters i.e. those of the linear dynamics as well as those of the nonlinear gain F i.e. (D1, D2, K1, K2, L1, L2). It is worth noting that a major difficulty related to hard input nonlinearities lies in the fact that the latter involve for different input intervals different mathematical formulations.

Furthermore, the mentioned input intervals involve uncertain parameters. For instance, in the present case, these intervals are (−∞, −D2], (−D2, D1) and [D1, +∞). Therefore, a crucial step in any solution to the identification issue is to develop for the nonlinear element a unique mathematical formulation i.e. that holds for all input intervals. In the present paper, a polynomial formulation is developed, making full use of the input sequence nature. The involved polynomial P perfectly coincides with the nonlinearity for the considered input sequence i.e. there is no approximation error. More precisely, the proposed identification scheme has the following features:

(i) to identify the two segments of F(v) (In the latter the argument θ is removed in F(.) ) one just needs to determine two points in each segment; in effect, these four points, say (Vi, F(Vi)) (i=1, …, 4), uniquely determine the four unknown parameters of the nonlinear element i.e. the two segment slopes and the two dead-zone sizes;

(ii) for any choice of (V1, V2, V3, V4), such that Vi≠ Vj for i≠j, corresponds a unique 4th-degree polynomial P(v) such that P(0)=F(0)=0 and F(Vi)=P(Vi) for i=1, …, 4;

 

letting the plant input be chosen in the set {0, V1, V2, V3, V4}, allows substitution of P(v) to F(v) in the plant model; so doing, the initial identification problem is converted to one where the nonlinear element is polynomial; if the polynomial coefficients could be perfectly identified then the parameters of the initial nonlinear elements (i.e. dead-zones and slopes) would be precisely determined;

(iii) the plant representation obtained when replacing F(.) by P(.) in the plant model is bilinear in the relevant unknown parameters (the polynomial coefficients and the linear dynamic parameters); then a least-squares type algorithm only allows estimation of composite parameters; to recover the relevant parameters from (estimates) of the composites, a procedure is built up based on matrix algebraic tools such us singular values decomposition;

(iv) a persistently exciting input sequence, of the impulse type, is designed and shown to guarantee the convergence of the composite parameter estimates, and so those of the relevant parameters, to their true values.

Finally, it is worth noting that the polynomial formulation proposed in this paper is much more simpler than those used in [7]-[8]. Furthermore, the input sequence used in this paper to ensure persistent excitation is also simpler than those used in [7]-[8].

## II. Identification Problem Statement

### A. *Class of identified plants*

We are considering plants that can be described by the following Hammerstein model (fig. 1):

$$y(t)=\frac{B(q-1)}{A(q-1)}u(t)+z(t) \tag{2.1a}$$

$$u(t) = F(v(t)) \tag{2.1b}$$

with $A(q^{-1}) = 1 + a_1q^{-1} + ... + a_{na}q^{-na}$

and $B(q^{-1}) = b_1q^{-1} + ... + b_{nb}q^{-nb}$ (2.2)

The function F represents a static gain and is described by fig. 2. The plant dynamics are submitted to the following assumptions:

**A1.** There is a known integer n such that: $n \geq \max\{degree\ A(q^{-1}), degree\ B(q^{-1})\}$.

**A2.** $A(q^{-1})$ and $B(q^{-1})$ are coprime.

**A3.** All zeroes of $q^{na}A(q^{-1})$ are strictly inside the unit circle.

**A4.** $D_M > \max(D_1, D_2)$ for some known real $D_M$.

**Remarks 2.1**.

--Assumption A2 ensures the controllability of the transfer function B(q-1)/A(q-1).

--Except for assumptions A1-A3, the plant is arbitrary: the dynamic parameters (ai, bi) are unknown and the leading coefficients (b1, b2, ...) may be null i.e. the true plant delay is also unknown (but not greater than n).

--As mentioned in the introduction, the identification method proposed in [8] involves a minimization problem with respect to the nonlinearity unknown parameter. The only global minimum search method explicitly presented by the author is a graphical type. For this to work an upper bound of the unknown parameter should be known.

### B. *Identification Objective*

Our purpose is to design an on-line identification scheme that provide asymptotically the true plant dynamics model $B(q^{-1})/A(q^{-1})$ and the four parameters of the nonlinear element F(.), i.e. $D_1, D_2, K_1, K_2, L_1, L_2$. One major difficulty comes from the fact that the internal sequence u(t) is not measurable, i.e. only the input sequence {v(t)} and the output sequence {y(t)} should be used in the identification scheme (fig. 1).

## III. Basic Formulas for Plant Model Identification

### A. *Plant Model Parameterization*

First, notice that identifying the function F(.) amounts to identifying four points $(V_j, F(V_j))$ (j=1, ..., 4), where the $V_j$'s are arbitrarily chosen such that:

$$V_1 < V_2 < -D_M < 0 < D_M < V_3 < V_4 \tag{3.1}$$

Using a polynomial interpolation, one associates to the couples $(V_j, F(V_j))$ a unique $4^{th}$-order polynomial P such that:

$$P(0)=0 \quad \text{and} \quad P(V_j) = F(V_j) \quad \text{for } j=1, ..., 4 \tag{3.2}$$

Such a polynomial can be given in the following form:

$$P(v) = v\sum_{j=1}^{4} d_j P_j(v) \quad \text{with} \quad P_j(v) = \prod_{\substack{i=1 \\ i\neq j}}^{4} \frac{v - V_i}{V_j - V_i} \tag{3.3}$$

It can be easily seen that:

$$P_j(V_i)=\delta_{ij}=\begin{cases}1 & \text{if } i=j \\ 0 & \text{if } i\neq j\end{cases} \tag{3.4}$$

and

$$d_j = P(V_j)\big/V_j \tag{3.5}$$

Then, identifying the four couples $(V_j, F(V_j))$ (j=1, …, 4) amounts to identifying the polynomial P(v) i.e. the coefficients $d_j$ (j=1, …, 4). Furthermore, if the plant input v(t) belongs to the set $\{0, V_1, …, V_4\}$, for all t, then P(v(t))= F(v(t)), due to (3.2). Therefore, the plant (2.1a-b) can also be represented by the following model:

$$A(q^{-1})y(t) = B(q^{-1})u(t)+ A(q^{-1})z(t) \tag{3.6a}$$

$$u(t) = P(v(t)) \tag{3.6b}$$

A regressive form can now be derived for (3.6a-b), as follows:

$$A(q^{-1})y(t) = B(q^{-1})P(v(t)) + A(q^{-1})z(t) = \sum_{i=1}^{n}\sum_{j=1}^{4}\mu_{ij}v(t-i)P_j(v(t-i)) + \eta(t) \quad (3.7a)$$

where $\eta(t) = A(q^{-1})z(t)$ and:

$$\mu_{ij} = b_i d_j \quad (i=1, ..., n; \; j=1, ..., 4) \quad (3.7b)$$

Equation (3.7a) can be given the following regressive form:

$$y(t)=\Phi(t)^T\Theta^*+\eta(t) \quad (3.8a)$$

$$\Phi(t)^T=[-y(t-1) \ldots -y(t-n) \;\; v(t-1)P_1(v(t-1)) \ldots v(t-n)P_1(v(t-n)) \;\ldots\; v(t-1)P_4(v(t-1)) \;\ldots\; v(t-n))\,P_4(v(t-n))] \quad (3.8b)$$

$$\Theta^* = [a_1 \ldots a_n \;\; \mu_{11} \ldots \mu_{n1} \;\; \ldots \ldots \;\; \mu_{14} \ldots \mu_{n4}]^T \quad (3.8c)$$

As $\Theta^*$ comes in linearly, equation (3.8a) turns out to be an adequate parameterization to get estimates of the parameters $a_i$ and $\mu_{ij}$, using a least squares type algorithm. It is worth noticing that the substitution of P(.) to F(.) in (3.6a-b) generates no new error. As long as v(t) belongs to $\{0, V_1, \ldots, V_4\}$, the models (2.1a-b) and (3.6a-b) are equivalent and, consequently, the initial identification problem (that consisted in identifying the 4 couples $(V_j, F(V_j))$ and the parameters $(a_i, b_i)$) amounts to identifying the parameters $a_i$, $b_i$ and $d_j$.

*B. Relations between Estimated and relevant Parameters*

From estimates $\hat{\theta}(t)$ of $\theta^*$, one has to get estimates $(\hat{b}_i(t), \hat{D}_j(t), \hat{K}_j(t), \hat{L}_j(t))$ of $(b_i, D_j, K_j, L_j)$ for i=1, …, n and j=1, 2. We will first construct a procedure to go back from the $\mu_{ij}$'s to the $b_i$'s and the $d_j$'s. Then, relations to get $(D_i, K_i, L_i$: i=1, 2) from ($d_j$: j=1, …, 4) will be established.

***Going back from ($\mu_{ij}$: i=1, …, n; j=1, …, 4) to ($b_i$: i=1, …, n) and ($d_j$: j=1, …, 4)***

Observe that (3.7b) can be rewritten as follows:

$$M \stackrel{def}{=} \begin{bmatrix} \mu_{11} & \cdots & \mu_{14} \\ \vdots & & \vdots \\ \mu_{n1} & \cdots & \mu_{n4} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} [d_1 \;\; \cdots \;\; d_4] \quad (3.9)$$

Notice that since $M$ is a rank-1 matrix, the $b_i$'s and the $d_j$'s cannot be determined uniquely from the $\mu_{ij}$'s, unless extra conditions are imposed on the $b_i$'s or $d_j$'s. Uniqueness of the solution of (3.9) can be achieved by imposing, for instance, the following couple of conditions:

$$\sum_{i=1}^{n} b_i^2 = 1 \quad \text{and} \quad \rho([b_1 \;\; \cdots \;\; b_n]) > 0 \quad (3.10)$$

where $\rho([b_1 \;\; \cdots \;\; b_n])$ denotes the first component of the vector $[b_1 \;\; \cdots \;\; b_n]^T$ that satisfies:

$$|\rho([b_1 \;\; \cdots \;\; b_n])| = \sup_{1\le j\le n}|b_j|$$

i.e the first component with a greatest absolute value.

Based on the above observations a procedure to solve (3.9) is designed using singular value decomposition (SVD). This is described in the following proposition.

**Proposition 3.1** Let $M \in R^{nx4}$ be any rank-1 real matrix. Then its SVD decomposition has the following form:

$$M = \Gamma \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \Sigma \quad (3.11)$$

where $\Gamma \in R^{nxn}$, $\Sigma \in R^{4x4}$ and $\sigma_1$ is the unique nonzero singular value of $M$. Furthermore, $M$ can be uniquely decomposed as follows:

$$M = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} [d_1 \;\; \cdots \;\; d_4] \quad (3.12a)$$

with:

$$[b_1 \;\; \cdots \;\; b_n]^T = \mathrm{sign}(\gamma)\, \Gamma[\sigma_1 \; 0 \ldots 0]^T \Big/ \left\| \Gamma[\sigma_1 \; 0 \ldots 0]^T \right\|$$

$$[d_1 \;\; \cdots \;\; d_4]^T = \mathrm{sign}(\gamma) \left\| \Gamma[\sigma_1 \;\; 0 \;\; \cdots \;\; 0]^T \right\| . [1 \;\; 0 \;\; \cdots \;\; 0] \Sigma \quad (3.12b)$$

Where $\gamma = \rho(\Gamma[\sigma_1 \;\; 0 \;\; \cdots \;\; 0 \;\; 0])$.

The vector $[b_1 \;\; \cdots \;\; b_n]^T$ thus obtained is the only solution of (3.12a) that satisfies:

$$\sum_{i=1}^{n} b_i^2 = 1; \quad \rho([b_1 \;\; \cdots \;\; b_n]) > 0 \quad (3.13)$$

**Proof**. See [9].

***Going back, from ($d_j$: j=1, …, 4), to ($D_i$, $K_i$,$L_j$: i=1, 2)***

From (3.5) it follows that the four couples $(V_j, F(V_j))$ can be computed from the $d_j$'s as follows:

$$F(V_j) = P(V_j) = d_j . V_j \quad \text{for } j=1, ..., 4 \quad (3.14)$$

Furthermore, it follows from fig. 2 that:

$$F(v) = \begin{cases} K_1 v + H_1 & \text{if} \quad v > D_M \\ K_2 v + H_2 & \text{if} \quad v < -D_M \end{cases} \quad (3.15)$$

with:

$$K_1 = \frac{(F(V_3) - F(V_4))}{(V_3 - V_4)} \text{ and } K_2 = \frac{(F(V_1) - F(V_2))}{(V_1 - V_2)} \quad (3.16)$$

$$H_1 = \frac{(V_3F(V_4) - V_4F(V_3))}{V_3 - V_4}$$
$$\text{and } H_2 = \frac{(V_1F(V_2) - V_2F(V_1))}{V_1 - V_2} \quad (3.17)$$

Relations that give ($D_1$, $D_2$) are available in some cases. For dead-zone nonlinearities (L1=L2=0), one has:

$$D_1 = -H_1/K_1 \quad \text{and} \quad D_2 = -H_2/K_2 \quad (3.18a)$$

For preload nonlinearities ($D_1$=$D_2$=0), one has:

$L_i$=$H_i$ (i=1, 2) (3.18b)

For symmetric preload dead-zone nonlinearities (i.e. $D_1$=–$D_2$=D, $L_1$=–$L_2$=L and $K_1 \neq K_2$):

$$D = (H_1 + H_2)/(K_2 - K_1)$$
$$\text{and} \quad L = K_1D + H_1 \quad (3.18c)$$

## IV. Model Parameter Identification

### A. Parameter Estimation

***Estimation algorithm :*** Estimates $\hat{\theta}(t)$ of $\theta^*$ can be recursively obtained using, for instance, the following algorithm :

$$\theta'(t) = \hat{\theta}(t-1) + \frac{\Phi(t)e(t)}{1 + \Phi(t)^T\Phi(t)}$$
$$\hat{\theta}(t) = \min(\rho^*, \|\theta'(t)\|).\frac{\theta'(t)}{\|\theta'(t)\|} \quad (4.1a)$$

$$e(t) = y(t) - \Phi(t)^T\hat{\theta}(t-1) \quad (4.1b)$$

where the initial vector $\hat{\theta}(0)$ is arbitrary and the real constant $\rho^*$ is any upper bound on $\|\theta^*\|$. In fact, equations (4.1a-b) define a gradient algorithm with parameter projection on the sphere centered on the origin with radius $\rho^*$. Such a projection prevents $\|\hat{\theta}(t)\|$ from diverging, despite the modeling error $\{\eta(t)\}$. The quality of the estimates $\hat{\theta}(t)$ depends, at least partly, on the mean size of $\{\eta(t)\}$.

The mean size of real sequences can be evaluated using the smallness-in-the-mean concept [6]. Accordingly, a real sequence $\{s(t)\}$ is said to be α-small in the mean if:

$$\limsup_{k\to\infty} \frac{1}{k}\sum_{t=h+1}^{h+k}|s(t)| \le \alpha$$

For a given α, the set of all α-small in the mean sequences is noted S(α). Let μ be the smallest real such that:

$$\{\eta(t)\} \in S(\mu) \quad (4.1c)$$

Note that μ exists because $\{|\eta(t)|\}$ is bounded. The properties of the estimation algorithm (4.1a-b) can now be stated in terms of μ and the posterior prediction error defined by:

$$e_p(t) = y(t) - \Phi(t)^T\hat{\theta}(t) \quad (4.1d)$$

**Proposition 4.1**. (General properties of $\hat{\theta}(t)$ )

1) There exists a real constant $K_1$, independent of μ, such that:
$\{\| \hat{\theta}(t) - \hat{\theta}(t-1) \|\} \in \{K_1\mu\}$ and $\{e_p(t)\} \in S(K_1\mu)$
2) In the ideal case ({z(t)}={0}) one has:
$\{\| \hat{\theta}(t) - \hat{\theta}(t-1) \|\} \in l_2$, $\{e_p(t)\} \in l_2$.

The proof of this proposition can be found any many places; see for example [10]. It shows that the quality of the (asymptotic) input-output behaviour of $\{\hat{\theta}(t)\}$ depends on the (mean) size μ of the modelling error $\{\eta(t)\}$. The smaller μ the better the (asymptotic) model. Interestingly, the above properties hold whatever the input sequence {v(t)}, i.e. even if the latter is not sufficiently rich. However, $\hat{\theta}(t)$ will not converge to $\theta^*$ unless persistent excitation is ensured. An example of such sequence is proposed in subsection 5.

### B. Reconstruction of $A(q^{-1})$, $B(q^{-1})$ and F

Given the parameter vector estimate:

$$\hat{\theta}(t) = [\hat{a}_1(t)\cdots\hat{a}_n(t)\ \hat{\mu}_{11}(t)\cdots\hat{\mu}_{n1}(t)\cdots\hat{\mu}_{14}(t)\cdots\hat{\mu}_{n4}(t)]^T \quad (4.2a)$$

One immediately gets the following estimate of $A(q^{-1})$:

$$\hat{A}_t(q^{-1}) = 1 + \hat{a}_1(t)q^{-1} + \cdots + \hat{a}_n(t)q^{-n} \quad (4.2b)$$

Estimates $(\hat{b}_i, \hat{d}_j)$ of $(b_i, d_j)$ have now to be recovered the $\hat{\mu}_{ij}$'s. Following closely Proposition 3.1, one first considers the matrix:

$$\hat{M}(t) = \begin{bmatrix} \hat{\mu}_{11}(t) & \cdots & \hat{\mu}_{1N}(t) \\ & \cdots & \\ \vdots & & \vdots \\ \hat{\mu}_{n1}(t) & \cdots & \hat{\mu}_{1N}(t) \end{bmatrix} \quad (4.3)$$

This is a natural estimate of the matrix M defined by (3.9). Let the singular values decomposition of $\hat{M}(t)$ be written as follows:

$$\hat{M}(t) = \Gamma(t)\left[\begin{matrix} \sigma_1(t) & 0 & \cdots & 0 \\ 0 & \sigma_2(t) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \sigma_n(t) \end{matrix}\ \middle|\ 0\right]\Sigma(t) \quad (4.4)$$

Then, Proposition 3.1 suggests the following estimates for the parameters $(b_i, d_j)$:

$$\begin{bmatrix} \hat{b}_1(t) \\ \vdots \\ \hat{b}_n(t) \end{bmatrix} = \mathrm{sign}(\gamma(t)) \frac{\Gamma(t).[\sigma_1(t) \quad 0 \quad \cdots \quad 0]^T}{\left\| \Gamma(t).[\sigma_1(t) \quad 0 \quad \cdots \quad 0]^T \right\|} \qquad (4.5a)$$

$$\left[\hat{d}_1(t) \quad \cdots \quad \hat{d}_N(t)\right]^T =$$
$$\left\| \Gamma(t)\left[\sigma_1(t) \quad 0 \quad \cdots \quad 0\right]^T \right\| . [1 \quad 0 \quad \cdots \quad 0]\Sigma(t)\mathrm{sign}(\gamma(t)) \qquad (4.5b)$$

$$\gamma(t) = \rho\left(\Gamma\left[\sigma_1(t) \quad 0 \quad \cdots \quad 0\right]^T\right) \qquad (4.5c)$$

The estimates thus obtained are the only ones that satisfy the condition

$$\sum_{i=1}^{n} \hat{b}_i^2 = 1 \text{ and } \rho\left(\left[\hat{b}_1(t) \ldots \hat{b}_n(t)\right]\right) > 0 .$$

Notice that the singular values $\sigma_2(t)\ldots\ \sigma_n(t)$ have not been accounted for in the rules (4.5a-b). This has no effect when the $\hat{\mu}_{ij}$'s converge to there true values, because the rank of matrix $\hat{M}(t)$ then converges to 1. This is made precise in the next proposition.

**Proposition 4.2**. Let $\{\hat{M}(t)\}$ be the real matrix sequence defined by (4.3). Let $\left[\hat{b}_1(t) \quad \cdots \quad \hat{b}_n(t)\right]$ and $\left[\hat{d}_1(t) \quad \cdots \quad \hat{d}_N(t)\right]$ be the vectors obtained from $\hat{M}(t)$ according to the rules (4.5a-c). If the $\hat{\mu}_{ij}$'s converge to their true values $\mu_{ij}$, then the estimates $\left(\hat{b}_i(t), \hat{d}_j(t)\right)$ will converge to their true values $(b_i, d_j)$.

**Proof:** this is a direct consequence of equation (3.9) and Proposition 3.1.

The sequence $\left(\hat{b}_i(t)\right)$ yields directly the following estimate for $B(q^{-1})$:

$$\hat{B}(q^{-1}) = \hat{b}_1(t)q^{-1} + \ldots + \hat{b}_n(t)q^{-n} \qquad (4.6a)$$

Equation (3.6c) suggests for $F(V_j)$ ( j= 1,..,4) the following estimate:

$$\hat{F}_t(V_j) = \hat{d}_j(t).V_j \qquad (4.7)$$

Then, according to (3.15), an estimate $\hat{F}_t(v)$ of $F(v)$ is obtained for $|v| \geq D_M$ :

$$\hat{F}_t(v) = \begin{cases} \hat{K}_1(t)v + \hat{H}_1(t) & \text{if} \quad v > D_M \\ \hat{K}_2(t)v + \hat{H}_2(t) & \text{if} \quad v < -D_M \end{cases} \qquad (4.8)$$

with: $\hat{K}_1(t) = (\hat{F}_t(V_4) - \hat{F}_t(V_3))/(V_4 - V_3)$,

and $\hat{K}_2(t) = (\hat{F}_t(V_2) - \hat{F}_t(V_1))/(V_2 - V_1)$ (4.9)

$$\hat{H}_1(t) = \frac{V_3\hat{F}_t(V_4) - V_4\hat{F}_t(V_3)}{V_3 - V_4}$$

$$\text{and} \quad \hat{H}_2(t) = \frac{V_1\hat{F}_t(V_2) - V_2\hat{F}_t(V_1)}{V_1 - V_2} \qquad (4.10)$$

Finally, estimates $\left(\hat{D}_1(t), \hat{D}_2(t), \hat{L}_1(t), \hat{L}_2(t)\right)$ of $(D_1, D_2, L_1, L_2)$ can then be obtained operating the rules (3.18) on $\left(\hat{K}_1(t), \hat{K}_2(t), \hat{H}_1, \hat{H}_2\right)$.

## V. Exciting Input Sequence and Convergence Analysis

### A. *Input Sequence*

The input sequence $\{v(t)\}$ should be generated so that its values belong to the set $\{0, V_1, \ldots, V_4\}$ and the resulting regression vectors $\Phi(t)$ be *persistently exciting* (PE). Bearing this in mind, $\{v(t)\}$ is chosen to be a periodic sequence, with period T=n(4+3), defined as follows:

for all integer k and all t in the interval $t_k \leq t < t_{k+1}$ with $t_k = kT$:

$$v(t) = \begin{cases} V_1 & \text{for} \quad t = t_k + 2n \\ V_j & \text{for} \quad t = t_k + (2+j)n \,;\; j = 2, \ldots, 4 \\ 0 & \text{otherwise} \end{cases} \qquad (5.1)$$

### B. *Convergence analysis of parameter estimates*

Due to paper length limitation the proofs of forthcoming propositions have been removed.

**Proposition 5.1**. The sequence $\{v(t)\}$ generated by (5.1) provides the regression vector $\{\Phi(t)\}$ with the PE property.

**Proposition 5.2**. (Convergence of $\hat{\Theta}(t)$ ).

1) In the general case, i.e. $\{z(t)\}$ arbitrary but bounded, there exists a real constant $K_2$, independent of $\mu$, such that: $\left\{ \left\| \hat{\Theta}(t) - \Theta^* \right\| \right\} \in S(K_2 \ \mu)$
2) In the ideal case ($\{z(t)\}=\{0\}$), $\left\{ \left\| \hat{\Theta}(t) - \Theta^* \right\| \right\}$ converge to zero.

### C. *Simulation*

The above results have been checked using many simulations. The system to be identified is characterized by the following parameters:

$A(q^{-1})=1-1{,}5q^{-1}+0{,}56q^{-1}$; $B(q^{-1})=q^{-1}-2q^{-1}$; $K_1=1$; $K_2=2$; $D_1=1$; $D_2=-2$; $L_1=1$; $L_2=-2$ (see fig. 2) ; $z(t)=0$ to prove the convergence of estimates to the values of parameters (Fig. 3a & Fig. 3b) .

Bearing in mind condition (3.1), the input sequence used is characterized by: $V1=-6$; $V2=-3$; $V_3=3$; $V_4=6$. The resulting parameter vector $\theta^*$ is:

$\theta^*=[0{,}56\ \ -1{,}5\ \ 1{,}66\ \ -3{,}33\ \ 1{,}33\ \ -2{,}66\ \ 1\ \ -2\ \ 1\ \ -2]^T$

The estimated parameters are shown by fig. 3. It is seen that the estimates do converge to their true values.

## VI. Conclusion

We have considered system identification based on Hammerstein model (fig. 1) where the nonlinear element is defined by a two segments characteristic F (fig. 2). We have designed an identification scheme that determines precisely the linear dynamic parameters ($a_i$, $b_i$) (i=1, …, n) and those of the nonlinear characteristic. The class of nonlinear elements dealt with includes all dead-zone, all preload and some dead-zone/preload elements.

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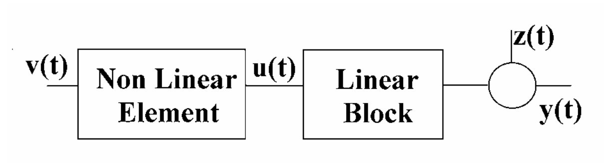


Fig. 1. Hammerstein Model.

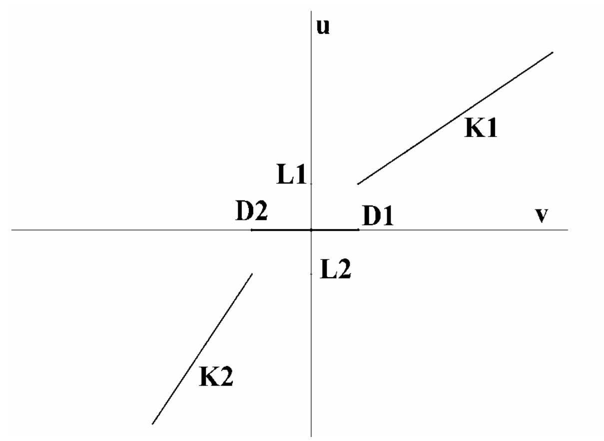


Fig. 2. Characteristic of the non linear element.

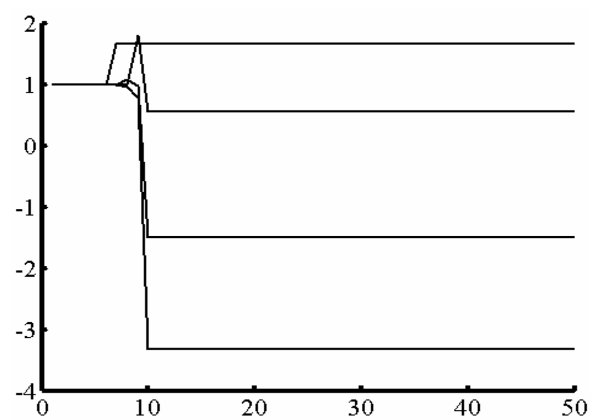

Fig. 3a. The estimates of the linear block parameters.

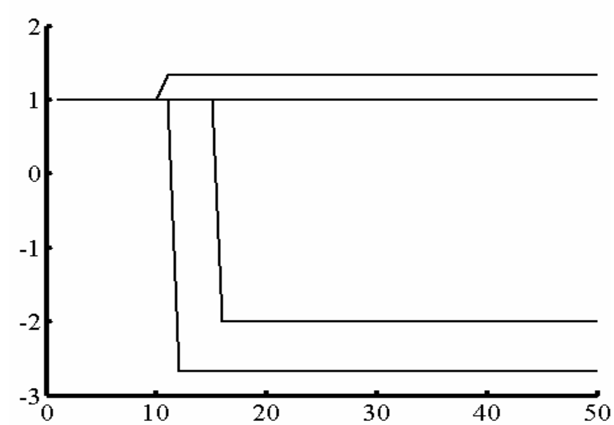

Fig. 3b. The estimates of the non linear elements parameters.