Gain Tuned Internal Model Control for Handling Saturation in Actuators

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Abstract—**Many compensation techniques for handling saturation non-linearity have been devised, ranging from anti-windup, polynomial-based conditioning techniques, to non-linear design techniques using state-space models. In this paper, a new approach to handling actuator saturation based on Internal Model Control (IMC) is proposed. Within the proposed framework, a gain term and first-order exponential filter are tuned to 'desaturate' the actuator. A stability analysis shows the resulting system is always stable. Examples show the simplicity of design and the validity of proposed framework.**

I. Introduction

The existence of control signal saturation within a practical control system may result in instability (e.g. 'reset-windup' of integral action phenomenon in a classical PID control framework) [1-4]. For instance, even a stable system without control signal saturation would become destabilized when the constraints of input become active. In some other cases, the instability caused by saturation leads to the occurrence of limit cycles, if the saturation non-linearity is not being compensated, as shown in [5].

The influence of saturation non-linearity is apparent when the required control signal cannot be achieved because of actuator saturation, leading to incorrect updates of the internal controller states. The inconsistencies between the internal controller states and the control signal is minimized using the generalized anti-windup compensator (GAWC) in [4]. Although the research in control signal saturation has reached the level of maturity where ad-hoc compensation schemes are widely available, the compensator's design algorithms (such as pole placement, H_∞ control) still remain mathematically intense.

Internal Model Control (IMC), illustrated in Fig. 1, is a control technique that guarantees system stability while offering good tracking performance. Extensive use of IMC in non-linear system control has generated great interest among the researchers in various aspects. Over the last two decades, the increased popularity of IMC in process and system control has been mainly due to the fact that the IMC structure offers high robustness for disturbance and uncertainty rejection, as well as offering global stability for both linear and non-linear systems. The only restrictions on the IMC scheme are that the plant needs to be open loop stable and minimum phase. The use of IMC for control input saturation compensation, on the other hand is relatively rare compared to robust process control. The IMC structure was never intended to be an anti-windup scheme as pointed out in [2] and [6]. However, it has been suggested that IMC can be used to solve anti-windup problems, for example in [7] and [8]. An example was given in [8] to show the inherent anti-windup property of IMC structure. Error-offset in IMC was investigated in [9], and a design method that possesses robust servo characteristics, employing the idea of Internal Perturbed-Model Control, was proposed. Similar work was done in [10], where an anti-windup control method using modified form of Internal Model Control for an unstable plant was proposed. In [11], non-linear control laws within the setting of IMC for a single-input single output chemical reactor were developed.

In this paper, a novel approach to provide good tracking performance even when the system is susceptible to input saturation non-linearity is introduced. The controller design consists of computing the inverse of the internal model, then selecting a first order exponential filter parameter. Finally, a gain is tuned to reduce the control action needed until the optimum response for the plant under saturation is obtained. The proposed method not only reduces the complexity of currently available saturation compensation schemes, but also guarantees global stability for bounded input bounded output (BIBO) systems. Robustness and stability issues are also addressed. Global asymptotic stability for a control system with control input saturation non-linearity is shown for the proposed IMC framework

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using the off-axis circle criterion.

The paper is organized as follows. Section II outlines the fundamental properties of the IMC structure, which forms the basis of the proposed IMC structure. The proposed restructured IMC framework, together with the controller design procedure, is described in the same section. The stability of the compensated system is investigated in section III. Section IV contains the simulation results of an example system using the proposed method, showing the effectiveness of gain changing for saturation non-linearity compensation. In section V, conclusions are drawn and the simulation results examined.

II. RESTRUCTURED IMC

A. *Single-degree-of-freedom IMC with Saturation*

A typical single-degree-of-freedom IMC is shown in Fig. 1. In this figure, Gp(z-1) is the plant, Gm(z-1) the internal model, and Gc(z-1) is the inverse model controller. The main characteristic of an IMC configuration is that the plant model (strictly speaking, the inverse plant model) forms part of the controller. The saturation block can be either part of the controller, or of the plant. From Fig. 1, dropping the arguments for brevity, the following equation relating the inputs and outputs can be derived [6]:

$$y_t = \frac{G_c G_p}{1 + G_c (G_p - G_m)} r_t + \frac{1 - G_m G_c}{1 + G_c (G_p - G_m)} d_t \qquad (1)$$

The sensitivity function, $\varepsilon(z^{-1})$, that relating the disturbance d_t to output y_t is given by:

$$\varepsilon(z^{-1}) = \frac{y_t}{d_t} = \frac{1 - G_m G_c}{1 + G_c (G_p - G_m)} \qquad (2)$$

The complementary sensitivity function $\eta(z^{-1})$ (or output sensitivity function) by subtracting $\varepsilon(z^{-1})$ from 1 to give:

$$\eta(z^{-1}) = \frac{y_t}{r_t} = \frac{G_c G_p}{1 + G_c (G_p - G_m)} \qquad (3)$$

Equation (2) can be also be derived from equation (1) by omitting the reference signal term while deriving the relationship between y_t and d_t. Equation (3) is derived from (1) in a similar way but omitting the disturbance related term.

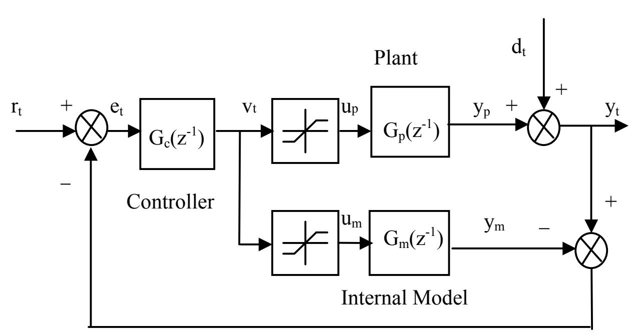

Fig. 1 Single-degree-of-freedom IMC with control signal saturation

One can thus conclude from (2) that the feedback control is used for disturbance rejection and from (3) that the feedforward control is used for set-point tracking. Both set-point tracking performance and disturbance rejection robustness are governed closely by perfect modeling (i.e. G_p=G_m), as shown in (2) and (3). If perfect modeling is achievable, with G_p=G_m, (2) and (3) can be simplified to:

$$\varepsilon(z^{-1}) = 1 - G_m G_c \qquad (4)$$

$$\eta(z^{-1}) = G_p G_c = G_m G_c \qquad (5)$$

Equation (4) determines the disturbance rejection performance, while (5) governs the set-point tracking robustness of the IMC structure shown in Fig. 1. The set-point tracking robustness filter $q_r(z^{-1})$ and disturbance rejection robustness filter $q_d(z^{-1})$ are incorporated to achieve good disturbance and tracking control.

Referring to Fig. 1, within a discrete-time IMC structure, an augmented feedback signal which consists of the effect of a disturbance signal and the plant/model mismatch is generated at each sample time. From (2), $\varepsilon(z^{-1})$ is a direct indicator of the system performance under the effect of disturbance. To make this sensitivity function relatively small so as to achieve perfect control (ideally $\varepsilon(z^{-1})$=0), the model inverse (i.e. G_c=1/G_m) can be chosen as the controller.

B. *Two-degrees-of-freedom IMC*

A typical discrete-time two-degrees-of-freedom IMC structure is shown in Fig. 2. In this figure, the extra 'freedom' is introduced by the inclusion of $q_d(z^{-1})$, which is the disturbance rejection robustness filter and $q_r(z^{-1})$, which is the tracking performance robustness filter. In practice, the best control performance depends on how well the robustness controllers and the arrangement of other *ad-hoc* controllers are being assigned. This arrangement is superior to that of Fig. 1 if the two inputs r_t and d_t are dynamically different [6].

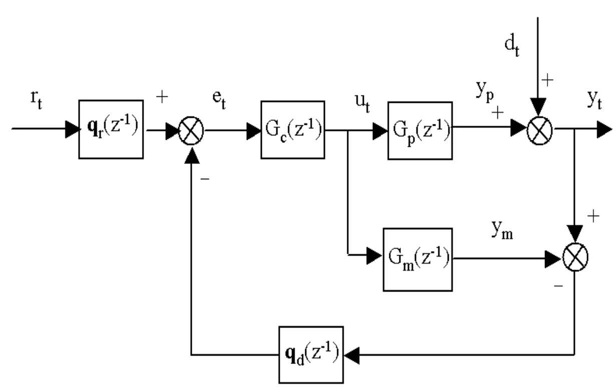

Fig. 2 Two-degrees-of-freedom IMC structure

C. *Proposed IMC Framework*

In an IMC setting, control signal saturation would not cause any 'reset-windup' problem, so long as the saturated input u_t is fed into the model G_m rather than the control signal, v_t computed by the controller [6]. The proposed IMC structure with control input saturation is thus constructed in accordance with this proposition with the saturated input fed into the plant and model.

The saturation function is given by:

$$u(t) = \begin{cases} +u_{sat} & v(t) > +u_{sat} \\ v(t) & -u_{sat} \le v(t) \le +u_{sat} \\ -u_{sat} & v(t) < -u_{sat} \end{cases} \quad (6)$$

As discussed earlier, to achieve near perfect control, one would ideally like to use the model inverse as the controller. Since the saturation nonlinearity forms part of the plant, it is impossible to invert the saturation function, as doing so will yield 'infinite' values when control signal exceeds $-u_{sat}$ or $+u_{sat}$. To achieve perfect control, a fully realizable IMC controller incorporating with the inverse of the saturation function has to be made available.

If no limitation was placed on the demand signal r_t or its derivatives, v_t would be unbounded. A single static saturation non-linearity function defined by (6) can be characterized by:

$$0 < \frac{u_t}{v_t} \le 1 \quad (7)$$

The saturation non-linearity represented by (7) is said to be bounded by the conic sector (0,1].

The standard IMC model of Fig. 1 can be restructured to give the proposed IMC structure shown in Fig. 3. In Fig. 3, if assumption on the plant and internal model are exact (or equal), then

$$G_p(z^{-1}) = G_m(z^{-1}) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{nb} z^{-nb}}{a_0 + a_1 z^{-1} + \ldots + a_{na} z^{-na}} \quad (8)$$

where a_0=1, nb=na or na-1.

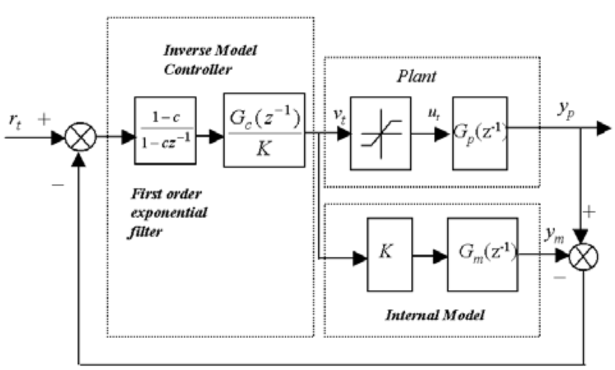

Fig. 3 Proposed IMC Structure

Within this new structure, a gain term K>0 is added to the internal model to replace the saturation function. This also results in the output signal from the internal model being scaled by a gain. When the feedback signal (the output difference between the plant and internal model) is fed back into the forward loop, the set-point entering the controller is modified. This approach is similar to the conditioning technique in [3], where tuning the gain in the internal model has the effect of de-saturating the actuator by modifying the set-point. For good set-point tracking and disturbance rejection, a first-order filter, $F_{IMC}(z^{-1})$ is introduced into the forward loop. This filter is given by:

$$F_{IMC}(z^{-1}) = \frac{1-c}{1-cz^{-1}} \quad (9)$$

with $0 \le c < 1$, the value of c to be chosen.

The first-order exponential filter, $F_{IMC}(z^{-1})$ is added to make the inverse of the internal model viable and causal. This filter thus ensures that the implementation of the inverse model is practically feasible. The pole position of the first-order filter, c, is selected to provide the desired closed-loop bandwidth [12]. In the event that nb=na-1 in (8), taking G_c as the inverse of the internal model (i.e. $G_c = G_m^{-1}$) results in a non-causal inverse model controller. This situation is resolved by having the first-order exponential filter, $F_{IMC}(z^{-1})$ in the feedforward controller. This makes the implementation of the inverse model feasible. The configuration of the proposed IMC structure is shown in Fig. 3.

Remarks: The performance of the compensated system under the influence of the saturating non-linearity is closely related to the chosen gain in the internal model. A rule of thumb for tuning the gain is first to select a large enough gain to maximize the full potential of the actuator. The gain can then be reduced to improve the speed of the response until overshoot starts to occur. Alternatively, the gain can be fine-tuned so as to minimize a defined function, e.g. the sum of squared errors.

III. Stability Analysis

In the proposed IMC structure, the closed-loop system remains globally stable as long as the plant model is equal to the nominal model ($G_p = G_m$). This global stability property will be shown in this section.

Within the system shown in Fig. 3, an equivalent feedback connection represented by two subsystems, one with a dynamic linear transfer function, and another with the static saturation nonlinearity, can be established. This is as shown in Fig. 4. The dynamic linear transfer function relating the unsaturated signal v_t and the saturated signal u_t is given by:

$$M = -\frac{v_t}{u_t} \quad (10)$$

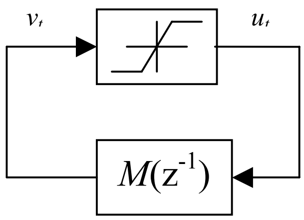

Fig. 4 Modeling loop as one linear and one static nonlinear subsystem

Also, the following equations can be derived from Fig. 3:

$$y_p = G_p u_t \quad (11)$$

$$y_m = K G_m v_t \quad (12)$$

Now, consider the feedback loop of Fig. 3 ignoring the reference signal r_t as this does not contribute to the loop gain. The calculated unsaturated control signal v_t is given by:

$$v_t = F_{IMC}(G_m^{-1}K^{-1})(-y_p + y_m) \quad (13)$$

Substituting (11) and (12) into (13), and rearranging gives:

$$v_t = -F_{IMC}G_m^{-1}G_pK^{-1}u_t + F_{IMC}G_mG_m^{-1}K^{-1}Kv_t \quad (14)$$

Assuming perfect modeling, with G_p=G_m, then $G_m^{-1}G_p$=1, so, (13) can be rewritten as:

$$v_t = -F_{IMC}K^{-1}u_t + F_{IMC}v_t \quad (15)$$

Rearranging (15) to give

$$(1 - F_{IMC})v_t = -F_{IMC}K^{-1}u_t \quad (16)$$

Then use (10) and (16) gives

$$M = -\frac{v_t}{u_t} = \frac{F_{IMC}}{K(1 - F_{IMC})} \quad (17)$$

Consider a single-input-single-output nonlinear system (in this case saturation) represented by the feedback connection as shown in Fig. 5, where $G(z^{-1})$ is strictly proper (or Hurwitz, i.e. has its eigenvalues strictly in the left half-plane), and ψ is a single time-invariant, memoryless nonlinearity.

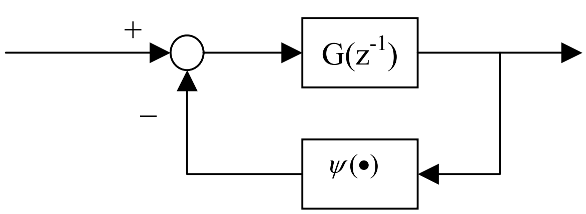

Fig.5 Nonlinearity Feedback Connection

Popov's criterion gives a sufficient condition for asymptotic stability. Given that the saturation nonlinearity belongs to the conic sector (0,1], Popov's criterion requires the following inequality to be fulfilled [13]:

$$\mathrm{Re}[(1 + j\omega)G(j\omega)] > -1, \quad \forall \omega \geq 0 \quad (18)$$

Equation (18) can be rewritten as:

$$\mathrm{Re}(M(j\omega)) + \omega\,\mathrm{Im}(M(j\omega)) > -1, \ \forall \omega \geq 0 \quad (19)$$

Define N as:

$$N = M + 1 = \frac{F_{IMC}}{K(1 - F_{IMC})} + 1 \quad (20)$$

From the definition given above, the following stability theorem can be established:

Theorem: *For any choice of K and c chosen for the proposed IMC synthesis, global asymptotic stability can be achieved in the presence of control signal saturation provided that that Re(N($e^{-j\omega T}$) >0 whenever Im(N($e^{-j\omega T}$) = 0 for any frequency* $\omega \geq 0$.

Substitute (9) into (20) gives:

$$N = \frac{1 - c}{Kc(1 - z^{-1})} + 1 \quad (21)$$

Let

$$a = \frac{1 - c}{Kc} > 0 \quad (22)$$

Thus (21) can be rewritten as,

$$N = \frac{a}{1 - z^{-1}} + 1 \quad (23)$$

It can be shown that,

$$\mathrm{Re}(\mathrm{N}(e^{-j\omega T})) = \frac{a}{2} + 1 \quad \text{for } \forall \omega \geq 0 \quad (24)$$

Inspection of (24) implies that the proposed IMC structure is robustly stable for all frequencies range, thus global asymptotic stability is achieved for all positive *K* and 0<*c*<1.

IV. Examples and Simulation Results

The plant model used in this example is taken from [5], itself adapted from [12]. This plant is a fourth-order lead-lag Butterworth, with a pair of lightly damped poles. The original continuous-time plant given in [5] is:

$$G_p(s) = 0.2\left(\frac{s^2 + 2\zeta_1\omega_1 s + \omega_1^2}{s^2 + 2\zeta_1\omega_2 s + \omega_2^2}\right)\left(\frac{s^2 + 2\zeta_2\omega_1 s + \omega_1^2}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2}\right)$$

with ζ_1=0.3827, ζ_2=0.9239, ω_1=0.2115 , ω_1=0.0473.

For digital controller implementation, this plant is discretized using a bilinear transformation with a sampling time of 1s to give,

$$G_p(z^{-1}) = \frac{0.2478 - 0.8547\, z^{-1} + 1.114\, z^{-2} - 0.6492\, z^{-3} + 0.1427 z^{-4}}{1 - 3.876 z^{-1} + 5.637 z^{-2} - 3.644 z^{-3} + 0.8837 z^{-4}}$$

Next, controller design using the proposed IMC framework is demonstrated in the following 3 examples. The assumption of exact modeling is made for all these examples, or mathematically, the internal model, $G_m(z^{-1})$ is the same as the transfer function for the plant $G_p(z^{-1})$.

A. *Example 1(Varying K)*

The controller design begins with choosing a value of *c* that provides the desire closed-loop bandwidth. Choose *c* = 0.2 and this gives the following IMC filter $F_{IMC}(z^{-1})$:

$$F_{IMC}(z^{-1}) = \frac{0.8}{1 - 0.2 z^{-1}}$$

With this fixed filter structure, the remaining task is to selecting an appropriate gain term, *K.* With a saturation limits set at ±1, the effect of changing gain *K* is shown in the simulation response plots in Fig. 6. For *K*= 20, the resulting controller no longer causes actuator saturation. It is clear that once the gain is optimally tuned, a good tracking performance can be achieved. This inherent property of a classical IMC scheme is being exhibited using the proposed IMC structure. For larger *K* values, the responses become sluggish, as the control signals are reduced.

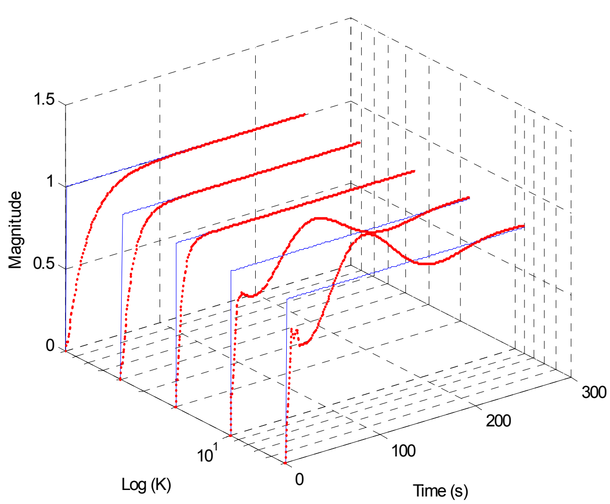

Fig. 6 Step responses with different *K* values.

B. *Example 2(Varying step size)*

In this example, the gain K is fixed at 20 while the reference step size take the values of 0.5,1, 2, 5 and 10. The simulation results are as shown in Fig. 7, with the reference and output being scaled down by their step sizes to aid comparison. It can be seen from these plots that the gain set at *K*=20 is only suitable for a step size no greater than 1. For step sizes above 1, the fixed gain is no longer able to 'desaturate' the actuator and hence the response deteriorates. When this happens, gain K must be increased to further reduce the control signal size. Fig. 8 shows the step response of the system with step size of 10, with gain *K* = 200.

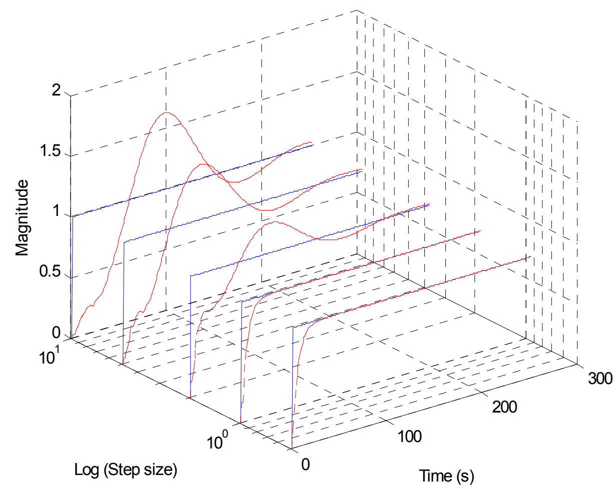

Fig 7. Step responses with varying saturation limit.

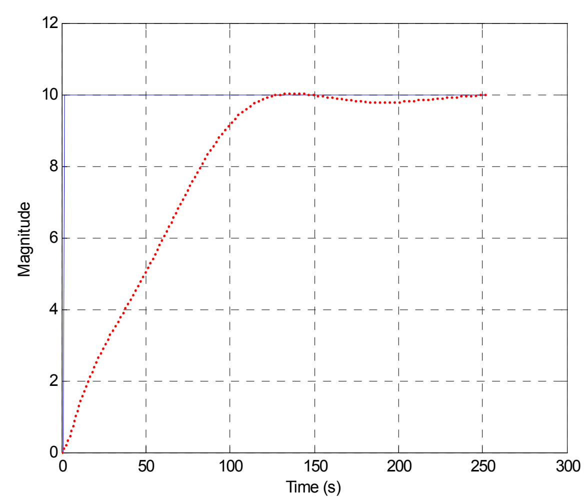

Fig. 8 Step response for step size of 10 and *K*=200

C. *Example 3(Disturbance rejection)*

In this example, a disturbance input with magnitude of 5 at t=0, switching to –5 at t=4 is introduced. Using the same saturation limits as in example 1, the controller design task is to re-design the first-order filter by choosing c and tune the gain to give the optimum response. Choosing *c*=0.4 and *K*=72, the response of the proposed IMC controller compared to the conventional IMC is shown in Fig. 9. Using the proposed IMC structure, significant improvement in the response over conventional IMC is obtained.

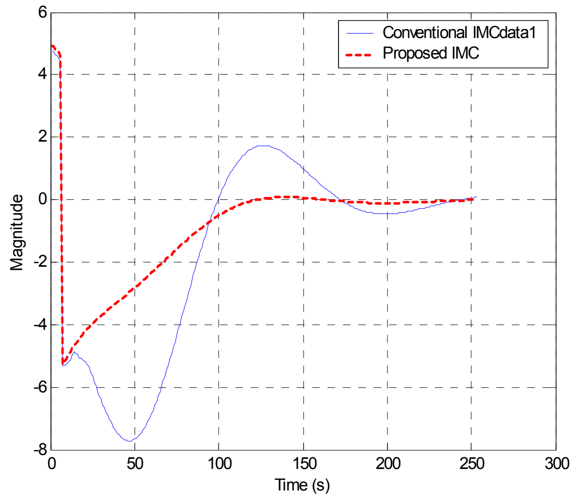

Fig. 9 Step disturbance rejection response.

In general, increasing *K* results in a more sluggish response, as the forward loop gain is decreased but a smaller control signal is required for the actuator. The proposed scheme has the advantage for the plant with limitation on the actuator's physical size or the output compliance. Under such circumstances, the gain can be tuned to tolerate the control limit while giving perfect tracking performance.

V. CONCLUSION

It has been shown through the simulation results the validity and superiority of the proposed method utilizing a IMC structure for control input constraints non-linearity. In attempts to compensate for the saturation non-linearity, a gain has been proposed to be included in the internal model. In effect, this arrangement generates a feedback signal that modifies the set-point that is fed into the feedforward controller, and reduces the control signal required under conventional IMC. From the simulation results, it is apparent that the gain tuning process in the proposed IMC structure fulfills the compensation objective for the control signal saturation non-linearity. Overshoot can be eliminated by increasing gain, *K*. The increase of gain is done at the expense of compromising the speed of the response. Larger gain results in more sluggish response as shown in example 1, but allows tighter actuator limits to be accommodated.

It is also apparent that under the proposed IMC method the entire process of designing a controller for saturation non-linearity is reduced to choosing an appropriate IMC filter $F_{IMC}(z^{-1})$ and gain, *K*. In the event that the step size changes, due to the change of the plant configuration or output requirement, only a single parameter (gain K) needs to be altered. A tuning process to find the optimum gain that meets the tracking performance criteria is required. By comparison, most of the conventional compensation techniques require the re-design of the entire set of controllers.

Also, the stability analysis in section III showed that the proposed IMC structure is globally asymptotic stable. This is indeed a special case of the off-axis circle criterion, where graphical interpretation (i.e. polar plot) is used to examine the global asymptotic stability of the proposed controller. Intrinsic robustness of proposed IMC scheme can also be shown, where global stability can be achieved for any choice of IMC filter term, c and gain, *K*. The effect of limit cycles or windup problem often encountered in a constraint system can be effectively eliminated with the proposed IMC structure.

Although the proposed method does not yield the fastest possible response, this approach has proven to be robust to step size variation, and is thus a general solution to the saturating actuator problem. The modified IMC synthesis can be used for the handling of other sector-bounded non-linearities, for example, a dead zone.

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