Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

FrA15.5

PARTIAL PCA-BASED OPTIMAL STRUCTURED RESIDUAL DESIGN FOR FAULT ISOLATION

Jin Cao and Janos Gertler

Electrical and Computer Engineering, George Mason University, Fairfax, VA 22030, USA

jgertler@gmu.edu

Keywords. Fault isolation, Structured residuals, Principal components, Max-min optimization.

Abstract. We are dealing with structured residuals, generated for fault detection and isolation in static linear systems by partial PC models. Structured design is combined with sensitivity optimization, using a max-min criterion applied to the fault-to-noise response ratio in the residual. A graphic representation of the optimization problem is given, together with illustrative examples.

1. INTRODUCTION

In model-based fault detection and diagnosis, residuals are generated, using the plant observations and the model, that are nominally zero in the absence of faults (Willsky, 1976). The model may be explicit, obtained from first principles or systems identification (Gertler, 1998), or implicit, obtained by principal component transformation (Kourti and MacGregor, 1995). For fault isolation, these residuals are enhanced, usually by a transformation. One of the main enhancement techniques, structured residuals, involves residuals that selectively respond to subsets of faults (Gertler and Singer, 1990). Such residuals may be generated by algebraic transformation, or by a direct technique. In the principal component framework, the direct approach involves structured partial PC models (Gertler and McAvoy, 1997). Alternative fault isolation techniques, in the PCA framework, involve contribution charts (Kourti and MacGregor, 1995) and statistical distance and angle measures (Raich and Cinar, 1997).

It has been proposed recently (Qin and Li, 1999) that structured design be combined with optimization for fault sensitivity. In this paper, this idea is extended in two respects:

(i) instead of an average sensitivity measure, proposed in (Qin and Li, 1999), we apply a max-min criterion (see also Xu and Kwan, 2003) to the fault-response to noise-response ratio;

(ii) we apply the combined approach to partial PC models where the computation of the optimum is relatively straightforward.

2. BACKGROUND

2.1. System Description

Consider a k-input-m-output linear static system:

$$\mathbf{y}^0(t) = \mathbf{A}\,\mathbf{u}^0(t) \tag{1}$$

where $\mathbf{u}^0(t) = [u_1^0(t)\;\; u_2^0(t)\;\dots\; u_k^0(t)]^{\mathrm{T}}$ represents the true inputs, $\mathbf{y}^0(t) = [y_1^0(t)\;\; y_2^0(t)\;\dots\; y_m^0(t)]^{\mathrm{T}}$ represents the true outputs, and $\mathbf{A}$ is the $m \times k$ system parameter matrix. A full model can be written as:

$$\mathbf{B}\,\mathbf{x}^0(t) = \mathbf{0} \tag{2}$$

where $\mathbf{B} = [-\mathbf{A}\;\;\mathbf{I}]$, which is $m \times (k + m)$, and $\mathbf{x}^0(t)$ is the combination of $\mathbf{u}^0(t)$ and $\mathbf{y}^0(t)$:

$$\mathbf{x}^0(t) = \begin{bmatrix} \mathbf{u}^0(t) \\ \mathbf{y}^0(t) \end{bmatrix} \tag{3}$$

With faults $\Delta\mathbf{x}(t) = [\Delta\mathbf{u}^{\mathrm{T}}(t)\;\; \Delta\mathbf{y}^{\mathrm{T}}(t)]^{\mathrm{T}} = [\Delta u_1(t)\;\; \Delta u_2(t)\;\dots\; \Delta u_k(t)\;\; \Delta y_1(t)\;\; \Delta y_2(t)\;\dots\; \Delta y_m(t)]^{\mathrm{T}}$ present in the system, the observed variables can be expressed as:

$$\mathbf{x}(t) = \mathbf{x}^0(t) + \Delta\mathbf{x}(t) \tag{4}$$

Apply the model (2) to the observed variables:

$$\mathbf{B}[\mathbf{x}(t) - \Delta\mathbf{x}(t)] = \mathbf{0} \;\Rightarrow\; \mathbf{B}\,\mathbf{x}(t) = \mathbf{B}\,\Delta\mathbf{x}(t) = \mathbf{o}(t) \tag{5}$$

where $\mathbf{o}(t)$ is the set of *primary residuals* which are computed from the observations but depend only on the faults. Enhanced residuals are then obtained by some transformation:

$$\boldsymbol{r}(t) = \mathbf{W}\mathbf{o}(t) = \mathbf{W}\,\mathbf{B}\,\Delta\mathbf{x}(t) \tag{6}$$

2.2. Structured Residual Design

Structured residuals are so designed that each residual responds to a distinctive subset of faults and is insensitive to the others. When a particular fault occurs, some of the residuals do respond and others do not. Then the pattern of the response set, the Boolean fault code, is characteristic of the specific fault.

With structured residuals, threshold testing is implemented separately on each component of the residual set. The outcome of the test applied to residual $r_i(t)$ is a binary variable $\varepsilon_i(t)$ so that:

$$\varepsilon_i(t) = \begin{cases} 0 & if\,|r_i(t)| < k_i \\ 1 & if\,|r_i(t)| \geq k_i \end{cases} \qquad i = 1 \ldots n \tag{7}$$

The vector $\boldsymbol{\varepsilon} = [\varepsilon_1 \quad \varepsilon_2 \quad \ldots \quad \varepsilon_n]^T$ is the fault signature or fault code. Now fault isolation is the comparison of the actually obtained code to a pre-defined set of codes.

A structure is "isolating" if each column is different and "strongly isolating" if the columns stay different from valid columns when 1s are turned into 0s. Strong isolation is best achieved by *column canonical structures*, in which each fault code has the same number of 0s, each in a different pattern. Table 1 is a column canonical structure for four faults Δx_1, Δx_2, Δx_3, Δx_4 and four residuals r_1, r_2, r_3, r_4. Note that such structures are normally designed for single faults.

	Δx_1	Δx_2	Δx_3	Δx_4
r_1	0	0	1	1
r_2	1	0	0	1
r_3	1	1	0	0
r_4	0	1	1	0

Table 1: A structured residual set

When structured residuals are generated by Eq. (6), then the ith row of $\mathbf{W}$, $\mathbf{w}_i^T$ is so designed that the zeros assigned for the ith row of the structure matrix be implemented, that is, the ith residual be decoupled from all faults which appear with 0s in that row. This requires that:

$$\mathbf{w}_i^T \mathbf{B}^{i\#} = \mathbf{0} \tag{8}$$

where $\mathbf{B}^{i\#}$ contains those columns of the $\mathbf{B}$ matrix which belong to the faults assigned for zero response in the ith residual structure. On the other hand, the rest of matrix $\mathbf{B}$, $\mathbf{B}^i$, should satisfy:

$$\mathbf{w}_i^T \mathbf{B}^i = \mathbf{v}_i^T \tag{9}$$

where each element of the vector $\mathbf{v}_i$ must be nonzero (Gertler and Singer, 1990; Gertler, 1998). Note that the elements of $\mathbf{v}_i$ are the gains for the fault responses.

3. MAXIMIZED SENSITIVITY FOR STRUCTURED RESIDUALS

When certain rank conditions on B are satisfied, the structured residual design based on Eqs. (8) and (9) can be obtained. However, some gains in the vector v_i may be very small, resulting in poor fault sensitivity. This calls for the inclusion of sensitivity considerations in the design.

3.1. Maximizing the Number of Zero Responses

The fundamental approach to structured residual design (Gertler, 1998) implies maximizing the number τ of zero elements in each column of the structure matrix. In a system containing m linear relations, we have:

$$\text{Max}\ (\tau) = m - 1 \tag{10}$$

Based on this consideration, canonical structures can be pre-defined in terms of the number of outputs, m, and the number ρ of faults (Gertler, 1998).

This approach results in simple isolation schemes. Besides, a fault gain intentionally set to zero does not pose sensitivity problems. However, this design does not actively address the possible small nonzero gains.

Recall Eq.(8), the vector $\mathbf{w}_i^T$ is $1 \times m$ and the matrix $\mathbf{B}^{i\#}$ is $m \times \tau$. By (10), we have:

$$1 \leq \tau \leq m - 1 \tag{11}$$

If the rank of the matrix $\mathbf{B}^{i\#}$ is also τ (full column rank), then the degree of freedom of the vector $\mathbf{w}_i^T$ is $m - \tau$. Obviously, with $\tau = m-1$, the degree of freedom of vector $\mathbf{w}_i^T$ is 1. That implies that $\mathbf{w}_i^T$ is unique, apart from normalization. Thus the vector $\mathbf{v}_i$ is fixed.

In order to avoid very small gains in $\mathbf{v}_i^T$ by manipulating $\mathbf{w}_i^T$, one should make its freedom greater than 1. Hence, the number τ of zeros in the structure should be less than its possible maximum.

3.2. Maximizing the Sum of Squared Nonzero Gains

In the framework of optimally sensitive structured residuals, one would choose $\mathbf{w}_i^T$, the rows of the transformation matrix $\mathbf{W}$, such that the residuals are insensitive to a certain subset of faults but "most sensitive" to the rest. A performance index proposed by Qin and Li (1999) involved the sum of squared nonzero gains:

$$\max_{\mathbf{w}_i}\left(\sum_{j=1}^{k+m-\tau} v_{ij}^2\right) = \max_{\mathbf{w}_i}(\mathbf{v}_i^T \mathbf{v}_i) = \max_{\mathbf{w}_i}[\mathbf{w}_i^T \mathbf{B}^i (\mathbf{B}^i)^T \mathbf{w}_i] \tag{12}$$

The optimization is subject to:

$$\text{(a)}\ \mathbf{w}_i^T \mathbf{B}^{i\#} = \mathbf{0}; \quad \text{(b)}\ \|\mathbf{w}_i\| = 1; \quad \text{(c)}\ \|\mathbf{b}_l\| = 1.$$

where $i = 1, 2, \ldots, n$, with n being the number of the residuals, $\mathbf{b}_l^T$ is a $1 \times (k + m)$ row vector of the matrix $\mathbf{B}$, and $l = 1, 2, \ldots, m$.

This approach employs the largest possible freedom, which is $m-1$. Correspondingly, the number of the designed zeros is $\tau=1$.

Qin and Li (1999) also provided a closed-form solution for Eq.(12). However, as pointed out in (Xu and Kwan, 2003), this approach only optimizes the *sum* of squared nonzero gains in each row; the magnitude of some gains may be still very small while the others are large.

3.3. Max-Min Nonzero Gains

Clearly, nonzero gains with extremely small magnitude should be avoided. This calls for a max-min criterion for optimization. That is, $\mathbf{w}_i^T$ is to be so chosen that, while satisfying Eqs.(8) and (9), the smallest element in $\mathbf{v}_i^T$ is made as large as possible. This max-min criterion has been presented by Xu and Kwan (2003) as follows:

$$\max_{\mathbf{w}_i}[\min_j(|v_{ij}|)] \tag{13}$$

The optimization is subject to:

$$\text{(a) } \mathbf{w}_i^T\mathbf{B}^{i\#} = \mathbf{0}; \quad \text{(b) } \|\mathbf{w}_i\| = 1; \quad \text{(c) } \|\mathbf{b}_l\| = 1.$$

where v_{ij} are elements of the vector $\mathbf{v}_i^T$.

Note that the above performance index does not consider the effect of the noise. Since the test thresholds are determined by the magnitude of the noise-induced part of the residual, a more accurate optimization criterion should concern the fault-effect to noise-effect ratio in the residual. Thus, the max-min criterion can be revised as:

$$\max_{\mathbf{w}_i}[\min_j(\frac{|v_{ij}|}{\sigma_{\mathbf{w}_i}})] = \max_{\mathbf{w}_i}[\frac{1}{\sigma_{\mathbf{w}_i}}\min_j(|v_{ij}|)] \tag{14}$$

where $\sigma_{\mathbf{w}_i}$ is the standard deviation of the noise in the j-th transformed residual.

4. PARTIAL PCA-BASED STRUCTURED RESIDUAL DESIGN

4.1. LPC Modeling

The full model in Eq.(2) may be expanded as **BX**=**0**, where the matrix **X** contains N sets of process variables. **X** can be treated by *Singular Value Decomposition* (SVD):

$$\mathbf{X} = \mathbf{P}\,\Sigma\mathbf{Q}^{\mathrm{T}} \tag{15}$$

or by eigenstructure decomposition:

$$\mathbf{R} = \mathbf{Q}\Lambda\mathbf{Q}^{\mathrm{T}} \quad \text{where } \mathbf{R} = \frac{1}{N}\mathbf{X}^T\mathbf{X} \tag{16}$$

The eigenvector matrix $\mathbf{Q}=[\mathbf{Q}_1 \quad \mathbf{Q}_2]$ consists of the representation subspace $\mathbf{Q}_1$ and the residual subspace $\mathbf{Q}_2$. The number of the columns in the sub-matrix $\mathbf{Q}_2$, which is a column-normalized matrix, is equal to the number of the linear relations (that is, the outputs) in the system. By the subspace theory, the rows of $\mathbf{Q}_2$ span the same subspace as those of $\mathbf{B}^{\mathrm{T}}$. Thus, by a transformation, $\mathbf{Q}_2$ can be considered as an estimate of the real parameter matrix $\mathbf{B}^{\mathrm{T}}$. This PCA-based modeling approach is referred to as the *Last Principal Components* (LPC) modeling.

4.2. Partial PCA-based Structured Residuals

Structured residuals may be generated by PCA without any algebraic transformation, using the idea of partial PCA models (Gertler and McAvoy, 1997). Partial PCA models, obtained directly from training data, describe the relationships among subsets of variables, according to a specific Boolean structure like Table 1.

Consider the system as Eq.(2). It contains m equations and $m + k$ variables. Apply a transformation $\mathbf{w}_i^T$ to it:

$$\mathbf{w}_i^T\mathbf{B}\mathbf{x}^0(t) = \mathbf{0} \tag{17}$$

With Eq.(8), Eq. (17) becomes:

$$\mathbf{w}_i^T\begin{bmatrix}\mathbf{B}^i & \mathbf{B}^{i\#}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i^0(t)\\ \mathbf{x}_{i\#}^0(t)\end{bmatrix} = \mathbf{w}_i^T\mathbf{B}^i\mathbf{x}_i^0(t) + \mathbf{w}_i^T\mathbf{B}^{i\#}\mathbf{x}_{i\#}^0(t) = \mathbf{w}_i^T\mathbf{B}^i\mathbf{x}_i^0(t) = \mathbf{0} \tag{18}$$

Define $\mathbf{v}_i^T = \mathbf{w}_i^T\mathbf{B}^i$, then Eq.(18) is re-written as:

$$\mathbf{v}_i^T\mathbf{x}_i^0(t) = \mathbf{0} \tag{19}$$

Here $\mathbf{x}_i^0(t)$ is a subset of $\mathbf{x}^0(t)$, containing at least $k + 1$ elements. This procedure can also be regarded as using τ (which is at most m−1 according to Eq.(11)) original equations to eliminate τ variables. The model represented by Eq.(19) can be obtained directly by the LPC approach. In order to distinguish this approach from the previous ones, we introduce a new vector symbol $\mathbf{q}_i^T$ to replace the vector $\mathbf{v}_i^T$:

$$\mathbf{q}_i^T\mathbf{x}_i^0(t) = \mathbf{0} \tag{20}$$

Note that the vector $\mathbf{q}_i^T$ is an eigenvector and, as such, it has been normalized.

Consider Eq.(20) with (4); a residual arises due to faults as:

$$r_i(t) = \mathbf{q}_i^T\,\mathbf{x}_i(t) = \mathbf{q}_i^T\,[\mathbf{x}_i^0(t) + \Delta\mathbf{x}_i(t)] = \mathbf{q}_i^T\,\Delta\mathbf{x}_i(t)$$

That is, the residual only responds to the faults $\Delta\mathbf{x}_i(t)$ but is completely decoupled from all the other faults. Thus, this is a structured residual. Several partial PCA models, each in a different structure following a structure matrix, can be created to lead to a structured fault isolation scheme.

If one considers the design of maximizing the number of zeros, the smallest number, k+1, of the variables should be selected. As per the previous discussion, it implies a single linear relation among those k+1 variables. If one needs more freedom for manipulating vector $\mathbf{q}_i^T$, then more than k+1 variables need to be chosen. If k+p ($1 < p \le m - 1$) variables are selected, there exist p linear relations among the k+p variables, and the vector $\mathbf{q}_i^T$ has p degrees of freedom, that is, there are p independent normalized vectors $\mathbf{q}_i^T$ that satisfy Eq.(20). Obviously, they are a p basis in the (k+p)-dimensional space.

5. PARTIAL PCA-BASED MAX-MIN DESIGN

The partial PCA-based structured residual design has some good features, such as freedom from algebraic transformation and normalization. We may enjoy these advantages also in the max-min optimization framework.

We still consider the k-input-m-output linear static system shown in Eq.(2). As we discussed before, if any $k+p$ ($1< p \le m-1$) selected variables are treated by LPC modeling, there will be p (close to) zero eigenvalues, which correspond to p linear relations. Accordingly, there is an eigenvector sub-matrix $\mathbf{Q}_2^{(p)}$, which is $(k+m)\times p$, associated with those eigenvalues. Any $\mathbf{v}_i^T$ can be obtained as a linear combination of all the column vectors in $\mathbf{Q}_2^{(p)}$, namely, $\mathbf{Q}_2^{(p)}\boldsymbol{\alpha}_i$. Thus, the max-min optimization criterion can be described as:

$$\max_{\boldsymbol{\alpha}_i}[\min_j(|v_{ij}|)] = \max_{\boldsymbol{\alpha}_i}[\min_j|\mathbf{Q}_2^{(p)}\alpha_{ji}|] \tag{21}$$

subject to:

$$\left\|\mathbf{Q}_2^{(p)}\boldsymbol{\alpha}_i\right\| = 1 \tag{22}$$

Here $\boldsymbol{\alpha}_i = [\alpha_{1i}, \alpha_{2i}, \ldots \alpha_{pi}]^T$ and $j = 1, 2, \ldots, p$.

Since $\mathbf{Q}_2^{(p)}$ is an eigenvector sub-matrix (its columns are eigenvectors), it has:

$$(\mathbf{Q}_2^{(p)})^T\mathbf{Q}_2^{(p)} = \mathbf{I} \tag{23}$$

To satisfy Eq.(22), we have:

$$\boldsymbol{\alpha}_i^T(\mathbf{Q}_2^{(p)})^T\mathbf{Q}_2^{(p)}\boldsymbol{\alpha}_i = 1 \tag{24}$$

With Eq.(23), we obtain:

$$\boldsymbol{\alpha}_i^T\boldsymbol{\alpha}_i = 1 \text{ or } \|\boldsymbol{\alpha}_i\| = 1 \tag{25}$$

Thus, the constraint in Eq.(22) becomes Eq. (25), which is simpler than the former.

Compared to the optimization criteria in Eqs.(12) and (13), this optimum criterion is much simpler since the constraints (a) and (c) of the previous cases have been implied in the partial PCA structured design.

If the effect of the noise is taken account, the max-min criterion should follow Eq.(14). The standard deviation, σ_i, of the transformed noise in $\mathbf{Q}_2^{(p)}\boldsymbol{\alpha}_i$ is quite straightforward in this framework, namely:

$$\sigma_i = \sqrt{\sum_{j=1}^{p}\alpha_{ji}^2\lambda_{k+j}} \tag{26}$$

where $\{\lambda_{k+j}\}$ are the last p (close to) zero eigenvalues. Thus, a more accurate max-min criterion is:

$$\max_{\boldsymbol{\alpha}_i}\left[\frac{1}{\sqrt{\sum_{j=1}^{p}\alpha_{ji}^2\lambda_{k+j}}}\min_j(|\mathbf{Q}_2^{(p)}\alpha_{ji}|)\right] \tag{27}$$

subject to Eq.(25).

If the noise power is small, the max-min criterion (21) is a reasonable approximation for (27).

The degrees of freedom may be chosen in the range of $1\le p\le m-1$, offering an additional level of optimization. When increasing the degrees of freedom, the search is moved to a higher dimensional subspace. However, it is not guaranteed that better optima may be found in higher dimensions, because of the constraints posed by the forced zero fault responses. Also, with various values of p, the number of possible residual structures, and the size of their subset needed for isolation, may also change; this makes this extension difficult to tract.

Special case: 3 outputs. Next, we will consider a special situation with a k-input 3-output linear system. Note that 3 is the minimal number of outputs which can offer two options, maximized zero design (p=1) and one-freedom max-min optimization design (p=2).

When p=2, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T$ and $\mathbf{Q}_2 = [\mathbf{q}_1 \quad \mathbf{q}_2]$, which is $(k+2)\times 2$. Define:

$$\mathbf{v} = \mathbf{Q}_2\boldsymbol{\alpha} = [\mathbf{q}_1 \quad \mathbf{q}_2][\alpha_1 \quad \alpha_2]^T = \alpha_1\mathbf{q}_1 + \alpha_2\mathbf{q}_2 \tag{28}$$

For each element of the vector $\mathbf{v}$, we have:

$$v_j = \alpha_1 q_{1j} + \alpha_2 q_{2j}, \quad (j = 1, 2, \ldots, k+2) \tag{29}$$

Note that the constraint in Eq.(25) becomes:

$$\alpha_1^2 + \alpha_2^2 = 1 \tag{30}$$

$$\text{Define: } \alpha_1 = \sin(\theta) \text{ and } \alpha_2 = \cos(\theta) \tag{31}$$

With Eq.(29), we have:

$$|v_j| = \sqrt{q_{1j}^2 + q_{2j}^2}\left|\sin(\theta + arctg(\frac{q_{2j}}{q_{1j}}))\right| \tag{32}$$

Therefore, according to Eqs.(21) and (25), the max-min criterion becomes:

$$\max_{\theta}\left[\frac{1}{\sqrt{\sin^2(\theta)\lambda_{k+1} + \cos^2(\theta)\lambda_{k+2}}}\cdot \min_j\left(\sqrt{q_{1j}^2 + q_{2j}^2}\left|\sin(\theta + arctg(\frac{q_{2j}}{q_{1j}}))\right|\right)\right] \tag{33}$$

θ is set between $-\pi/2$ and $\pi/2$ for a complete search. For each row $\mathbf{q}_j^T = [q_{1j} \quad q_{2j}]$ of $\mathbf{Q}_2$, (33) represents a distorted absolute sine function, each with the same period but different amplitudes and shifts. By visualizing those curves, one can easily search the max-min value.

For an example in Figure 1, four curves are plotted in one picture. The thick line represents the overall minimal values along α_1 (or θ). We locate the maximal value, marked by "o", along it. The four points at the optimal α_1 (or θ) on the respective curves are the absolute values of the fault-to-noise ratios for a row.

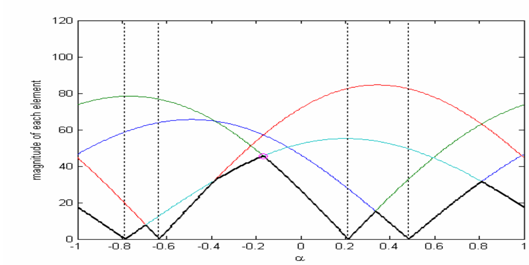

Figure 1: Max-min search based on a cluster of sine curves

Note that this picture also provides information about the maximized zero design. Where one of the curves touches zero, the other three points (along the dotted lines) are the fault-to-noise ratios with the maximized zeros design.

6. SIMULATION STUDY

Two 1-input-3-output systems will be investigated in this section. This type of systems has the simplest structure allowing the max-min optimization design. Through the studies with these systems, the key idea and procedure of the above max-min optimization design will be demonstrated.

The general format of such 1-input-3-output systems is:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \boldsymbol{\pi} u = \begin{bmatrix} a \\ b \\ c \end{bmatrix} u \tag{34}$$

In the simulation, a sequence of 1000 data with Gaussian distribution has been utilized as the input. There is no noise in the system.

Considering all the variable set $[u, \ y_1, \ y_2, \ y_3]$, we may develop a maximized zeros structure as:

$$\begin{array}{l} r_1\!: \ 1 \ \ 1 \ \ 0 \ \ 0 \\ r_2\!: \ 1 \ \ 0 \ \ 1 \ \ 0 \\ r_3\!: \ 1 \ \ 0 \ \ 0 \ \ 1 \\ r_4\!: \ 0 \ \ 1 \ \ 1 \ \ 0 \\ r_5\!: \ 0 \ \ 1 \ \ 0 \ \ 1 \\ r_6\!: \ 0 \ \ 0 \ \ 1 \ \ 1 \end{array} \tag{35}$$

and the four "1"s structure, which allows one additional freedom to optimize sensitivity in the max-min sense, as:

$$\begin{array}{l} q_1\!: \ 1 \ \ 1 \ \ 1 \ \ 0 \\ q_2\!: \ 1 \ \ 1 \ \ 0 \ \ 1 \\ q_3\!: \ 1 \ \ 0 \ \ 1 \ \ 1 \\ q_4\!: \ 0 \ \ 1 \ \ 1 \ \ 1 \end{array} \tag{36}$$

Note that replacing one "1" with "0" in a line of structure (36), which is equivalent to deleting one variable in the partial PCA model set, we obtain a new line that belongs to set (35). Every q_i line is related to three r_j lines, e.g q_1 to r_1, r_2, r_4.

Case 1: $\pi = [1.5 \quad 7.5 \quad 6.0]^T$.

The maximized zero design as per structure (35) results in the transformation matrix as follows:

$$\mathbf{Q}^{(1)} = \begin{bmatrix} -0.8322 & 0.5545 & 0 & 0 \\ -0.9912 & 0 & 0.1321 & 0 \\ -0.9864 & 0 & 0 & 0.1643 \\ 0 & -0.9806 & 0.1961 & 0 \\ 0 & -0.9701 & 0 & 0.2426 \\ 0 & 0 & -0.6247 & 0.7809 \end{bmatrix}.$$

For each column, that is for each fault, the minimal gain magnitudes are:

$$[0.8322 \quad 0.5545 \quad 0.1321 \quad 0.1643]$$

The visualization of the max-min optimization search is shown in Figure 2. Each plot corresponds to a $\mathbf{q}_i$. Note that the intersections of a dashed line with the curves represent the magnitude of the elements in the row vectors of $\mathbf{Q}^{(1)}$. The optimization results in:

$$\mathbf{Q}^{(2)} = \begin{bmatrix} -0.5442 & -0.8058 & 0.2337 & 0 \\ -0.5306 & -0.7973 & 0 & 0.2877 \\ -0.5625 & 0 & 0.5609 & -0.6074 \\ 0 & 0.5771 & -0.5775 & 0.5775 \end{bmatrix}$$

For each column, the minimal gain magnitudes are:

$$[0.5306 \quad 0.5771 \quad 0.2337 \quad 0.2877]$$

Except for the first gain, the gain magnitudes in this design are greater than with the first design. The minimal value in $\mathbf{Q}^{(1)}$, 0.1321, is smaller than the minimum in $\mathbf{Q}^{(2)}$, 0.2337. Accordingly, we choose $\mathbf{Q}^{(2)}$ as the final residual transformation matrix.

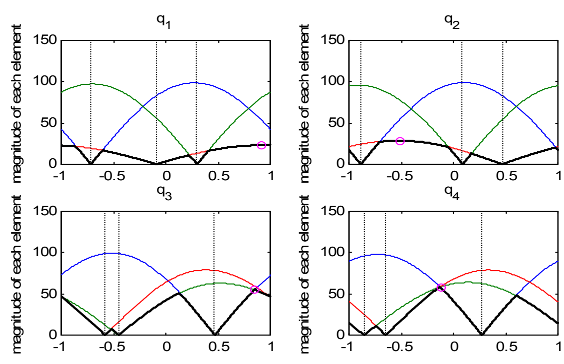

Figure 2: Visualization of max-min search for Case 1 model

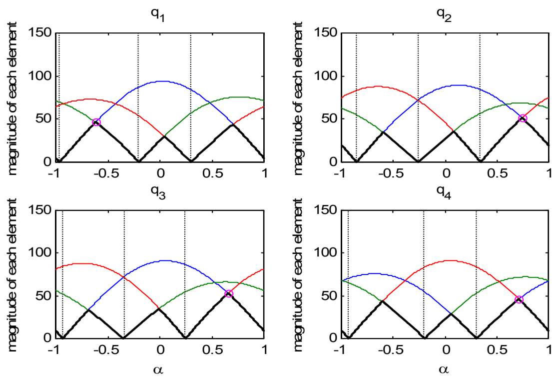

Figure 3: Visualization of max-min search for Case 2 model

Case 2: $\pi = [1.5 \quad 1.6 \quad 1.0]^T$.
Following the same procedure, we obtain:

$$\mathbf{Q}^{(1)} = \begin{bmatrix} -0.8319 & 0.5549 & 0 & 0 \\ -0.8480 & 0 & 0.5300 & 0 \\ -0.7070 & 0 & 0 & 0.7072 \\ 0 & -0.7297 & 0.6838 & 0 \\ 0 & -0.5548 & 0 & 0.8320 \\ 0 & 0 & -0.5299 & 0.8481 \end{bmatrix}$$

The minimal gains for each column are:

$$[0.7070 \quad 0.5548 \quad 0.5299 \quad 0.7072]$$

The optimization results in:

$$\mathbf{Q}^{(2)} = \begin{bmatrix} -0.4747 & -0.4744 & 0.7413 & 0 \\ -0.5158 & 0.6861 & 0 & -0.5130 \\ -0.5307 & 0 & 0.6622 & -0.5290 \\ 0 & -0.4735 & 0.7411 & -0.4760 \end{bmatrix}$$

Now the minimal gains are:

$$[0.4747 \quad 0.4735 \quad 0.6622 \quad 0.4760]$$

The max-min optimization search is shown in Figure 3. Except for the third gain, the magnitudes in the first design are greater than in the second. The minimal value in $\mathbf{Q}^{(1)}$, 0.5299, is greater than the minimum in $\mathbf{Q}^{(2)}$, 0.4735. Accordingly, we choose $\mathbf{Q}^{(1)}$ as the better design.

7. CONCLUSION

We have considered structured residuals, generated in static linear systems by partial principal component modeling. Structured design has been combined with optimization for fault sensitivity, by giving up zeros in the residual structure in order to gain additional design freedom for optimization. A max-min criterion has been applied to the fault-to-noise response ratio in the residual. While this approach makes a lot of practical sense, it also leads to a procedure easy to visualize and to execute, especially if the dimension of optimization is just one.

8. REFERENCES

Gertler, J. and Singer, D., "A New Structural Framework for Parity Equation Based Fault Detection and Isolation, " *Automatica*, **26**, 381 (1990).

Gertler, J. and McAvoy, T. "Principal Component Analysis and Parity relations – A Strong Duality." IFAC Safeprocess Symp. (Hull, UK), Prep. **2**, 837 (1997).

Gertler, J., "Fault Detection and Diagnosis in Engineering Systems, " *Marcel Dekker Inc.,* NY, (1998)

Gertler, J., Li, W., Huang, Y., and McAvoy, T., "Isolation-enhanced Principal Component Analysis," *AIChE Journal*, **45**, 323 (1999)

Kourti, T. and MacGregor, J: "Process analysis, monitoring and diagnosis using multivariate projection methods." Chemometrics and Intell. Lab. Systems, **28**, 3 (1995).

Qin, S. J. and Li, W., "Detection, Identification, and Reconstruction of Faulty Sensors with Maximized Sensitivity, " *AIChE Journal*, **45**, No.9, 1963 (1999)

Raich, A. and Cinar, A., "Diagnosis of Process Disturbances by Statistical Distance and Angle Measures", *Comput. And Chem. Eng.*, **21**, 661 (1997).

Willsky, A.S. "A Survey of Several Failure Detection Methods". Automatica, **12**, 601 (1976)

Xu, R. and Kwan, C., "Robust Isolation of Sensor Failures", will appear in *Asian Journal of Control*, (2003).