Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Feedforward Systems Linearizable by Coordinate Change

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Abstract. We identify a class of feedforward nonlinear systems that are linearizable by a diffeomorphic coordinate change. This coordinate change has to satisfy a set of partial differential equations (commonly seen in feedback linearization) that are given in the paper. For the class of linearizable feedforward systems we present a control algorithm in the style of the Sepulchre-Jankovic-Kokotovic/Mazenc-Praly integrator forwarding procedure where the need to solve a series of nonlinear ODE's is eliminated and one has to analytically compute only a series of integrals. The results of this paper are further extended in a companion ACC'04 paper [Krstic, "Integrator forwarding control laws for some classes of linearizable feedforward systems"].

1 Introduction

History and Summary of the Literature. In the world of recursive control designs for nonlinear systems, two basic classes of systems are the most easily recognizable—the systems with (strict-)feed*back* structure and the systems with (strict-)feed*forward* structure. The strict-feedback systems, which occupied the attention of the nonlinear control community in the first half of the 1990's, are controlled using *backstepping*, a method that employs aggressive controls[1] necessary to suppress finite escape instabilities inherent (in open loop) to strict-feedback systems. In contrast, the strict-feedforward systems, which were studied intensively in the mid- and late-1990's, can be only marginally unstable in open loop,[2] and permit (and in many cases call for) cautious controllers.

The theoretical foundation of how to exercise 'caution' in the design for feedforward systems was laid by Teel in his 1992 dissertation [34], where he introduced the technique of nested saturations whose parameters are carefully selected to essentially achieve robustness of linear controllers to nonlinearities (of superlinear and other types). Soon after this first design, Teel [36] developed a series of results that, among other things, interpreted and generalized [34] in the light of nonlinear small gain techniques that he developed in [36].

[1] as measured by the growth of their nonlinearities
[2] with solutions growing only polynomially in time

The next major spurt of progress came with the paper of Mazenc and Praly [20], which introduced a Lyapunov approach for stabilization of feedforward systems. This approach, initially conceived in March 1993, has its roots that go further back to Praly's designs for adaptive nonlinear control [24] and output feedback stabilization [25] where he was designing forwarding like coordinate changes involving a stable manifold that can be written as a graph of a function. A related idea was used by Sontag and Sussmann [31] for stabilization of linear systems with saturated controls. Recently, Praly, Ortega, and Kaliora [28] relaxed the conditions under which such manifolds can be found.

Jankovic, Sepulchre, and Kokotovic [9] developed a different Lyapunov solution to the problem of forwarding (and stabilization of a broad class of cascade systems), which, rather than a coordinate change or domination of (certain) 'cross terms' (as Mazenc and Praly), employs an exact cross term in the Lyapunov function. In [30] they presented an algorithmic, inverse optimal design for a class of feedforward systems and provided a detailed insight into the structure of the target system in the forwarding recursion.

Further developments on feedforward systems have gone in several directions. The nested saturation ideas have been expanded upon by Arcak, Teel, and Kokotovic [2], Marconi and Isidori [15], and Xudong [39]. Implicit (or explicit) in the first three papers are robustness results with respect to certain classes of unmodeled dynamics. The Lyapunov approach has been developed further by Sepulchre, Jankovic, and Kokotovic [30, 29], Mazenc, Sepulchre, and Jankovic [23], and Mazenc and Praly [22]. In [37] Teel designed $\mathcal{L}_2$ stabilizing controllers for feedforward systems ($\mathcal{L}_\infty$ disturbance attenuation, while impossible in general, remains a problem of interest for subclasses of feedforward systems). Trajectory tracking, while hard to achieve for arbitrary trajectories, has been solved under reasonable conditions by Mazenc and Praly [21] and Mazenc and Bowong [18]. Extensions to nonlinear integrator chains have been proposed by Mazenc [16] and Tsinias and Tzamtzi [38]. Even a generalization to feedforward systems with exponentially unstable linearizations has been reported by Grognard, Sepulchre, and Bastin [6]. Discrete time feedforward systems have also been studied, in a recent paper by Mazenc and Nijmeijer [19]. Linear low-gain semi-global stabilization of feedforward systems was proposed by Grognard, Bastin, Sepulchre, and Praly [7]. Differential geometric characterization of feedforward systems has eluded researchers until recent major progress was reported by Tall and Respondek [33]. Starting with Teel's original interest in the ball-and-beam problem [34] and Mazenc and Praly's design for the pendulum-cart problem [20], the research on forwarding has continuously been driven by applications. The following papers on forwarding are fully (or almost fully) dedicated to applications: Spong and Praly [32] (pole-cart), Barbu, Sepulchre, Lin, and Kokotovic [3] (ball-and-beam), Albouy and Praly [1] (spherical inverted pendulum), Praly, Ortega, and Kaliora [28] (inverted pendulum with disk inertia), and Mazenc and Bowong [17] (pendulum-cart), and Praly [27] (satellite orbit transfer with weak but continuous thrust). For tutorial coverage of forwarding, the reader is referred to the book of Sepulchre, Jankovic, and Kokotovic [29] and to Praly's tutorial [27] (available from his web page). Some coverage of forwarding is also available in the surveys by Coron, Praly, and Teel [5] and Kokotovic and Arcak [11].

Contribution and Organization of the Present Paper. The idea of exact forwarding coordinate transformations as a Lyapunov avenue towards performance improvement relative to the 'cautious' saturation-based approaches first appeared in [20, Section IV]. However, it is not until the result of [30], which considers a special subclass of the systems studied in [20, 9], that this idea crystalized into a conceptually transparent, elegant recursive procedure, which is easy to compare with backstepping. Still, the crucial element that remained lacking in the procedure was computability. In principle, one has to solve (analytically) a series of nonlinear systems and compute (again analytically) a series of integrals. The present paper is dedicated to providing closed-form solutions to these nonlinear systems and integrals. We start in Section 2 by reviewing the Sepulchre-Jankovic-Kokotovic [30] design procedure. While it has been long believed that feedforward systems are "generically not feedback linearizable," in Section 3 we show that many of them are and provide a parametrization of linearizable feedforward systems. For those systems, the SJK procedure provides the needed change of coordinates, which is given explicitly in Section 4. The coordinate change does not require the solution of a series of nonlinear systems (as in the general SJK procedure) but does require analytical computation of a series of integrals. To comply with length restrictions, proofs are omitted.

For two important subclasses of linearizable feedforward systems, those integrals are calculated explicitly in a companion paper [12], providing explicit formulae for control laws, explicit closed-loop solutions, and closed-loop bounds on control effort [12] expressed in terms of initial conditions.

2 The Sepulchre-Jankovic-Kokotovic Algorithm

Consider the class of *strict-feedforward systems*

$$\dot{x}_i = x_{i+1} + \psi_i(\underline{x}_{i+1}) + \phi_i(\underline{x}_{i+1})u\,, \qquad i = 1,2,\ldots,n \tag{1}$$

where $\underline{x}_j = [x_j,\ x_{j+1},\ldots,x_n]^T$, $x_{n+1} = u$, $\phi_n = 1$, $\frac{\partial\psi_i(0)}{\partial x_j} = \phi_i(0) = 0$, $\psi_{n-1}(x_n) \equiv 0$, and

$$\psi_i(x_{i+1},0,\ldots,0) \equiv 0 \tag{2}$$

for $i = 1,2,\ldots,n-1$, $j = i+1,\ldots,n$.

Relative to the class of systems in [30] we make a trade of generality for conceptual clarity by requiring that the drift term be of the form $x_{i+1} + \psi_i(\underline{x}_{i+1})$ with (2).

The control law for this class of systems is designed as follows. Let

$$\beta_{n+1} = \alpha_{n+1} = 0\,. \tag{3}$$

For $i = n, n-1, \ldots, 2, 1$

$$z_i = x_i - \beta_{i+1} \tag{4}$$

$$w_i(\underline{x}_{i+1}) = \phi_i - \sum_{j=i+1}^{n-1} \frac{\partial\beta_{i+1}}{\partial x_j}\phi_j - \frac{\partial\beta_{i+1}}{\partial x_n} \tag{5}$$

$$\alpha_i(\underline{x}_i) = \alpha_{i+1} - w_i z_i \tag{6}$$

$$\begin{aligned}\beta_i(\underline{x}_i) = -\int_0^\infty \Big[&\xi_i^{[i]}(\tau,\underline{x}_i) + \psi_{i-1}\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right) \\ &+\phi_{i-1}\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right)\alpha_i\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right)\Big]\,d\tau\,,\end{aligned} \tag{7}$$

where the notation in the integrand of (7) refers to the solutions of the (sub)system(s)

$$\frac{d}{d\tau}\xi_j^{[i]} = \xi_{j+1}^{[i]} + \psi_j\left(\underline{\xi}_{j+1}^{[i]}\right) + \phi_j\left(\underline{\xi}_{j+1}^{[i]}\right)\alpha_i\left(\underline{\xi}_i^{[i]}\right) \tag{8}$$

for $j = i, i+1, \ldots, n$, at time τ, starting from the initial condition $\underline{x}_i$. The control law is

$$u = \alpha_1\,. \tag{9}$$

It is important to first understand the meaning of the integral in (7). Clearly, the solution $\underline{\xi}_i(\tau,\underline{x}_i)$ is impossible to obtain analytically in general. Dealing with this issue is the main subject of this paper. Note that the last of the β_i's that need to be computed is β_2 (β_1 is not defined).

The stability analysis of the closed-loop system is straightforward. It is easy to verify that $\dot{z}_i =$

$w_i\left(u+\sum_{j=i+1}^{n} w_j z_j\right)$. Noting from (9) and (6) that $u=-\sum_{i=1}^{n} w_i z_i$, and taking the Lyapunov function $V=\frac{1}{2}\sum_{i=1}^{n} z_i^2$, one obtains

$$\dot{V}=-\frac{1}{2}\sum_{i=1}^{n} w_i^2 z_i^2-\frac{1}{2}\left(\sum_{i=1}^{n} z_i w_i\right)^2 . \tag{10}$$

Theorem 1 [30] *The feedback system (1), (9) is globally asymptotically stable at the origin.*

The Lyapunov-function equipped SJK algorithm not only shows good performance in simulations, this performance can be quantified. This is already implicit in the inverse optimality result in [30] for $u^*=2\alpha_1$, but is actually true even for $u=\alpha_1$.

Theorem 2 *The control effort for the feedback system (1), (9) satisfies the following bound:*

$$\|u\|_{\mathcal{L}_2}\leq\sqrt{\sum_{j=1}^{n}\left(x_j(0)-\beta_{j+1}(\underline{x}_{j+1}(0))\right)^2} . \tag{11}$$

3 Linearizability of Feedforward Systems

The main interest in this paper is making the computation of the integral in (7) tractable. Towards that end, let us start by noting that the system (8), which needs to be solved analytically, can be written in the z-coordinates[3] as

$$\frac{d}{d\tau}\zeta_j^{[i]}=-w_j^2\zeta_j^{[i]}-\sum_{l=1}^{j-1} w_j w_l \zeta_l^{[i]}, \qquad j=i,i+1,\ldots,n, \tag{12}$$

which is obtained with $\dot{\zeta}_j^{[i]}=w_j\alpha_i$. Suppose now that (somehow, miraculously,...) all of the w_l's were equal to 1 (for all x, rather than just $w_l(0)=1$). We would have a lower triangular linear system

$$\frac{d}{d\tau}\zeta_j^{[i]}=-\zeta_j^{[i]}-\sum_{l=1}^{j-1}\zeta_l^{[i]}, \qquad j=i,i+1,\ldots,n, \tag{13}$$

which is easily solvable in closed form. Then, the only difficulty remaining would be the integration with respect to τ of the integral (7) (using an appropriate coordinate change from $\underline{\zeta}_i^{[i]}$ to $\underline{\xi}_i^{[i]}$). Calculating the integral is by no means trivial, but it is a much easier task than solving the nonlinear ODE (8) *and* calculating the integral.

[3] We point out that, analogous to (8), we use ζ, a Greek version of z, to denote the solution of the $\underline{z}_i$ subsystem, under the control α_i, starting from initial condition $\underline{z}_i$. It should be also self understood that w_j stands for $w_j\left(\underline{\xi}_{j+1}^{[i]}\right)$, where $\xi_k^{[i]}=\zeta_k^{[i]}+\beta_{k+1}\left(\underline{\xi}_{k+1}^{[i]}\right)$, and so on (i.e., expressing w_j as a function of $\underline{\zeta}_{j+1}^{[i]}$).

Before we start exploring the conditions under which one would get

$$w_i(\underline{x}_{i+1})=\phi_i-\sum_{j=i+1}^{n-1}\frac{\partial\beta_{i+1}}{\partial x_j}\phi_j-\frac{\partial\beta_{i+1}}{\partial x_n}=1 , \tag{14}$$

let us note another consequence of this. In this case, the coordinate change, before applying the feedback, would yield

$$\dot{z}=\begin{bmatrix} 0 & 1 & 1 & \cdots & 1\\ 0 & 0 & 1 & & \vdots\\ \vdots & 0 & 0 & \ddots & \vdots\\ \vdots & & \ddots & \ddots & 1\\ 0 & \cdots & \cdots & 0 & 0\end{bmatrix} z+\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix} u . \tag{15}$$

We refer to this as the Teel [35] canonical form. This is a completely controllable linear system. Hence the systems that satisfy (14) are linearizable (into this form, and, ultimately, into the Brunovsky form).

Thus, the exploration of analytical computability of control laws for strict-feedforward systems that we undertake in this paper amounts to a study of linearizability. Clearly, merely checking the coordinate-free conditions for linearizability [8] won't get us any closer to actually finding the control laws. Such a test would lead to conditions on the ϕ_i's in the form of partial differential equations that they have to satisfy (these conditions would arise from the involutivity test).

Up until now we have used the word "linearizable" loosely. Next we make this notion precise.

Definition 1 *If there exists a diffeomorphism*

$$\begin{aligned} y_i &= x_i-\theta_{i+1}(\underline{x}_{i+1}), \qquad i=1,\ldots,n-1 && (16)\\ y_n &= x_n , && (17)\end{aligned}$$

where

$$\theta_i(0)=\frac{\partial\theta_i(0)}{\partial x_j}=0, \qquad i=2,\ldots,n, \quad j=i,\ldots,n, \tag{18}$$

transforming the strict-feedforward system (1) into a system of the form

$$\begin{aligned} \dot{y}_i &= y_{i+1}, \qquad i=1,2,\ldots,n-1 && (19)\\ \dot{y}_n &= u && (20)\end{aligned}$$

the system (1) is said to be diffeomorphically equivalent to a chain of integrators (DECI).

We point out that the term DECI does not reflect that (16), (17) restrict the class of diffeomorphisims to a "triangular" form. Next, we give sufficient conditions for characterizing DECI strict-feedforward systems.

Theorem 3 *All strict-feedforward systems (1) with* $\psi_i(\underline{x}_{i+1}), \phi_i(\underline{x}_{i+1})$ *that can be written as*

$$\begin{aligned}
\phi_{n-1}(x_n) &= \theta_n'(x_n) && (21)\\
\psi_{n-1}(x_n) &= 0 && (22)\\
\phi_i(\underline{x}_{i+1}) &= \sum_{j=i+1}^{n-1} \frac{\partial\theta_{i+1}(\underline{x}_{i+1})}{\partial x_j}\phi_j(\underline{x}_{j+1}) \\
&\quad + \frac{\partial\theta_{i+1}(\underline{x}_{i+1})}{\partial x_n} && (23)\\
\psi_i(\underline{x}_{i+1}) &= \sum_{j=i+1}^{n-1} \frac{\partial\theta_{i+1}(\underline{x}_{i+1})}{\partial x_j}\left(x_{j+1} + \psi_j(\underline{x}_{j+1})\right) \\
&\quad - \theta_{i+2}(\underline{x}_{i+2}) && (24)\\
i &= n-2, \ldots, 1
\end{aligned}$$

using some C^1 scalar-valued functions $\theta_i(\underline{x}_i)$ satisfying (18), are DECI.

Theorem 3 is neither a geometric test of linearizability, nor a design tool. It is just a *parametrization* of a subclass of strict-feedforward systems that are DECI.

For instance, all third-order strict-feedforward systems of the form

$$\begin{aligned}
\dot{x}_1 &= x_2 + \frac{\partial\theta_2(x_2,x_3)}{x_2}x_3 - \theta_3(x_3) \\
&\quad + \left(\frac{\partial\theta_2(x_2,x_3)}{x_2}\theta_3'(x_3) + \frac{\partial\theta_2(x_2,x_3)}{x_3}\right)u && (25)\\
\dot{x}_2 &= x_3 + \theta_3'(x_3)u && (26)\\
\dot{x}_3 &= u && (27)
\end{aligned}$$

are linearizable, where any two locally quadratic C^1 functions $\theta_2(x_2,x_3)$ and $\theta_3(x_3)$ are the "parameters." Take, for instance, $\theta_2(x_2,x_3) \equiv 0$ and $\theta_3(x_3) = \cosh(x_3) - 1$, which is locally quadratic. We get that the strict-feedforward system

$$\begin{aligned}
\dot{x}_1 &= x_2 + \cosh(x_3) - 1 && (28)\\
\dot{x}_2 &= x_3 + \sinh(x_3)u && (29)\\
\dot{x}_3 &= u && (30)
\end{aligned}$$

is linearizable using the coordinate change

$$y_1 = x_1\,, \quad y_2 = x_2 + \cosh(x_3) - 1\,, \quad y_3 = x_3\,. \quad (31)$$

Unfortunately, there is no easy systematic way to obtain this coordinate change (we know what it is because we started with $\theta_3(x_3) = \cosh(x_3) - 1$ and constructed the system). The only systematic way to arrive at it is the SJK procedure. In the next section we show that the SJK procedure greatly simplifies for DECI strict-feedroward systems, and, in particular, directly leads to (31) for (28)–(30) without having to solve nonlinear ODEs of the form (8).

4 A Control Algorithm for All Linearizable Feedforward Systems

For linearizable strict-feedforward systems we present the following design algorithm. Let

$$\beta_{n+1} = \alpha_{n+1} = 0\,. \quad (32)$$

For $i = n, n-1, \ldots, 2, 1$

$$\begin{aligned}
\alpha_i(\underline{x}_i) &= -\sum_{j=i}^{n}\left(x_j - \beta_{j+1}(\underline{x}_{j+1})\right) && (33)\\
\xi_n^{[i]}(\tau,\underline{x}_i) &= \mathrm{e}^{-\tau}\sum_{k=0}^{n-i}\frac{(-\tau)^k}{k!}\left(x_{n-k} - \beta_{n-k+1}(\underline{x}_{n-k+1})\right) && (34)\\
\xi_j^{[i]}(\tau,\underline{x}_i) &= \mathrm{e}^{-\tau}\sum_{k=0}^{j-i}\frac{(-\tau)^k}{k!}\left(x_{j-k} - \beta_{j-k+1}(\underline{x}_{j-k+1})\right) \\
&\quad + \beta_{j+1}\left(\underline{\xi}_{j+1}^{[i]}(\tau,\underline{x}_i)\right), \\
&\qquad j = n-1, \ldots, i+1, i && (35)\\
\beta_i(\underline{x}_i) &= -\int_0^\infty \Big[\xi_i^{[i]}(\tau,\underline{x}_i) + \psi_{i-1}\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right) && (36)\\
&\quad + \phi_{i-1}\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right)\alpha_i\left(\underline{\xi}_i^{[i]}(\tau,\underline{x}_i)\right)\Big]d\tau\,. && (37)
\end{aligned}$$

The control law is

$$u = \alpha_1\,. \quad (38)$$

We stress that, due to linearizability, the ODEs (8) are solved in closed form, and the only calculation remaining is the integrals (37), which can be obtained with symbolic software (coded in Mathematica or Maple/Matlab). This calculation is particularly straightforward (and can be done, in principle, by hand) when the nonlinearities $\psi_i(\cdot), \phi_i(\cdot)$ are polynomial. In that case, the following identity is useful:

$$\int_0^\infty \tau^p \mathrm{e}^{-q\tau}d\tau = \frac{p!}{q^{p+1}}\,, \qquad \forall p,q \in N\,. \quad (39)$$

Theorem 4 *If the strict-feedforward plant (1) is DECI, then the feedback system (1), (38) is globally asymptotically stable at the origin.*

Proof. One can verify that in the coordinates

$$z_i = x_i - \beta_{i+1}(\underline{x}_{i+1}) \quad (40)$$

the control system becomes (15), and under the feedback control (38), the resulting system is

$$\dot{z} = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ -1 & -1 & 0 & & \vdots \\ \vdots & -1 & -1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ -1 & \cdots & \cdots & -1 & -1 \end{bmatrix} z\,. \tag{41}$$

The rest of the proof is as in Theorem 1. ◇

As we indicated in Section 3, checking the geometric conditions for linearizability is easy, whereas actually constructing the linearizing coordinates is not. The algorithm (33)–(37) constructs the coordinate change into the (non-Brunovsky) Teel canonical form (15). The next theorem gives the coordinate change into the Brunovsky/chain-of-integrators form.

Theorem 5 *If the strict-feedforward plant (1) is DECI, it has a relative degree n with respect to the output*

$$y_1 = \sum_{j=1}^{n} \begin{pmatrix} n-1 \\ j-1 \end{pmatrix} (-1)^{j-1} \left(x_j - \beta_{j+1}(\underline{x}_{j+1})\right) . \tag{42}$$

Furthermore, the coordinate change (33)–(37), (40), and

$$y_i = \sum_{j=i}^{n} \begin{pmatrix} n-i \\ j-i \end{pmatrix} (-1)^{j-i} z_j\,, \qquad i = 1,2,\ldots,n \tag{43}$$

converts system (1) into the chain of integrators (19)–(20).

Inverse optimality, proved for the general case in [30], becomes particularly meaningful in the linearizable case.

Theorem 6 *The control law*

$$u^* = 2\alpha_1(x) = -2\sum_{j=1}^{n} \left(x_j - \beta_{j+1}(\underline{x}_{j+1})\right) , \tag{44}$$

where $\alpha_1(x)$ is defined via (33)–(37), minimizes the cost functional

$$J = \int_0^\infty \left(l(x(t)) + u(t)^2\right) dt \tag{45}$$

along the solutions of (1), where

$$l(x) = \sum_{j=1}^{n} \left(x_j - \beta_{j+1}(\underline{x}_{j+1})\right)^2 \tag{46}$$

$$+ \left(\sum_{j=1}^{n} \left(x_j - \beta_{j+1}(\underline{x}_{j+1})\right)\right)^2 \tag{47}$$

is a positive definite, radially unbounded function. Furthermore, the control law (44) remains globally asymptotically stabilizing at the origin in the presence of input unmodeled dynamics of the form

$$a(I + \mathcal{P})\,, \tag{48}$$

where $a \geq \frac{1}{2}$ is a constant, $\mathcal{P}u$ is the output of any strictly passive nonlinear system[4] with u as its input, and I denotes the identity operator.

Proof. It follows from Theorem 2.8, Theorem 2.17, and Corollary 2.18 in [13]. ◇

The main result of this section was a control algorithm that eliminates the requirement to solve the ODEs (8) and reduces the problem to calculating only the integrals (37). In the companion paper [12] we present algorithms that eliminate even the need to calculate the integrals (37) for two subclasses of DECI strict-feedforward systems.

Acknowledgements I would like to thank Laurent Praly and Andy Teel for their comments on the first draft of this paper and for bringing nearly a dozen references to my attention. This work was supported by grants from NSF and AFOSR.

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