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Suppression of Effects of Nonlinearities by Disturbance Observers

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ABSTRACT

In this paper, the application of disturbance observers to suppress chaotic behavior in a class of single-input single-output (SISO) nonlinear systems is studied. A nonlinear system in this class has the property that its output is equal to the summation of the output of a stable SISO linear time-invariant system and a bounded disturbance. The bounded disturbance captures the effects of all nonlinearities in the system. A disturbance observer is designed to estimate the bounded disturbance (equivalently, the effects of nonlinearities in the system) and cancel it subsequently. The disturbance observer is thus able to make the nonlinear system behave linearly and, for instance, be free of chaotic behavior. An example is given to show that chaotic behavior due to a nonlinearity in a Duffing-type system can be effectively suppressed by a disturbance observer.

1. INTRODUCTION

Perhaps all natural and physical systems are governed by nonlinear laws of nature. The dynamics of most of such systems can be mathematically represented by nonlinear differential or integral equations, which can be studied by analytical or numerical techniques. These techniques, in many instances, can successfully explain certain phenomena that are exclusive to nonlinear systems. One such a phenomenon is chaotic behavior of systems. Chaotic behavior can be considered as both desirable and unwanted response of systems. For instance, chaotic systems can be used in secure communication systems to provide chaotic masking and modulation of transmitted messages; see, e.g., references [1-5]. In most engineering systems, however, chaotic behavior is unwanted and should be suppressed. In the past decades, researchers have devised techniques to control chaotic behavior in nonlinear systems; see, e.g., references [1-3, 6-17] and the references therein.

In this paper, it is shown that an effective means of suppressing the effects of nonlinearities, and consequently possible chaotic behavior in a class of nonlinear systems is the application of disturbance observers. Disturbance observers are useful tools that were originally proposed in references [18, 19] as means of estimating disturbances to linear systems and canceling them subsequently. Later, the theory of disturbance observers was advanced in reference therein. It appears that disturbance observers are mostly designed for linear systems. There are, however, some works where the application of disturbance observers to nonlinear systems is reported; see references [28-34]. The present paper illustrates that disturbance observers can make members of a certain class of nonlinear systems behave linearly.

The organization of the paper is as follows. In Section 2, the class of nonlinear systems to be studied is presented. A nonlinear system in this class has the property that its output is equal to the summation of the output of a stable single-input single-output (SISO) linear time-invariant system and a bounded disturbance. The bounded disturbance captures the effects of all nonlinearities in the system. In Section 3, a disturbance observer is designed to estimate the effects of nonlinearities in a system in the class under consideration (equivalently, the bounded disturbance) and cancel them subsequently. Having the nonlinear effects canceled, the system behaves linearly. In Section 4, an example is given to show that chaotic behavior due to a nonlinearity in a Duffing-type system can be effectively suppressed by a disturbance observer.

2. NONLINEAR SYSTEMS

In this section, a class of SISO nonlinear systems is introduced. A member of this class, depicted in Figure 1, is represented by

$$N: \begin{cases} \dot{x}(t) = A\,x(t) + f(x(t), t) + b\,u(t)\,, \quad x(0) =: x_0\,, \\ y(t) = c\,x(t)\,, \end{cases} \tag{1}$$

for all $t \geq 0$, where the state vector $x(t) \in \mathbb{R}^n$, the initial state vector $x_0 \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}$, the output $y(t) \in \mathbb{R}$, the coefficient matrix $A \in \mathbb{R}^{n \times n}$, the input (influence) vector $b \in \mathbb{R}^n$, the output (readout) vector $c = [\, c_{11} \quad c_{12} \ldots c_{1n} \,] \in \mathbb{R}^{1 \times n}$, and the nonlinear function $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is given by

$$f(x(t), t) = [\, f_1(x(t), t) \quad f_2(x(t), t) \ldots f_n(x(t), t) \,]^T \,. \tag{2}$$

It is assumed that:

A1) The matrix A is Hurwitz.

A2) The pairs (A, b) and (A, c) are, respectively, completely controllable and completely observable.

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A3) The nonlinear function f, though not exactly known, is norm bounded. More precisely,

$$\| f \|_\infty := \max_{1 \le i \le n} \sup_{x \in \mathbb{R}^n} \sup_{t \ge 0} | f_i(x, t) | \le k_f < \infty , \quad (3)$$

where $k_f > 0$ is a constant real number.

Suppose that the system N exhibits a behavior exclusive to nonlinear systems, such as chaotic behavior. Moreover, suppose that this behavior is deemed undesirable. Thus, the goal would be to suppress the nonlinear behavior. This goal can be achieved by a disturbance observer as it will be shown later.

Before presenting the design of disturbance observes, some mathematical results are established.

From equations (1), it follows that the output of the system N can be written as

$$y(s) = H(s)\, u(s) + c\, (sI_n - A)^{-1}\, x_0 + d(s) , \quad (4)$$

where $y(s)$, $u(s)$, and $d(s)$ are, respectively, the Laplace transforms of $y(\cdot)$, $u(\cdot)$, and the time function

$$d(t) = c \int_0^t \exp(A(t - \tau))\; f(x(\tau), \tau)\; d\tau \;\in \mathbb{R} , \quad (5)$$

for all $t \ge 0$, I_n denotes the $n \times n$ identity matrix, and

$$H(s) = c(sI_n - A)^{-1}\, b . \quad (6)$$

The time function $t \longmapsto d(t)$ has a useful property established as follows. Since by assumption (A1), the matrix A is Hurwitz, there exist constant real numbers $M > 0$ and $\sigma > 0$, such that

$$\| \exp(At) \|_\infty \le M \exp(-\sigma t) , \quad (7)$$

for all $t \ge 0$ (see, e.g., reference [35, p. 195]). Using inequalities (3) and (7) in equation (5), it is concluded that $t \longmapsto d(t)$ is a bounded function of time. More precisely,

$$\| d \|_\infty := \sup_{t \ge 0} | d(t) | \le \sum_{j=1}^{n} | c_{1j} |\, M\, k_f / \sigma < \infty . \quad (8)$$

From equations (4)-(6) and inequalities (7) and (8), it is concluded that the output of the nonlinear system N is equal to the summation of the output of the stable SISO linear time-invariant system

$$H: \begin{cases} \dot{\bar{x}}(t) = A\, \bar{x}(t) + b\, u(t) , \quad \bar{x}(0) =: \bar{x}_0 = x_0 , \\ y_L(t) = c\, \bar{x}(t) , \end{cases} \quad (9)$$

and the bounded function of time $d(t)$ for all $t \ge 0$, where the state vector $\bar{x}(t) \in \mathbb{R}^n$ and the output $y_L(t) \in \mathbb{R}$. By assumption (A2), the representation of the system H is minimal. The transfer function corresponding to H is irreducible and is that given in equation (6).

A conclusion to be drawn is that the nonlinear system N can be equivalently represented by the linear system in Figure 2. This system is denoted by H_{+d} and has a useful property to be exploited in the next section.

3. LINEAR BEHAVIOR BY DISTURBANCE OBSERVERS

Representing the nonlinear system N by the equivalent linear system H_{+d} in Figure 2 is of great advantage, because the effects of nonlinearities in N appear as the bounded disturbance $d(\cdot)$ in H_{+d}. Therefore, if one seeks to suppress the effects of nonlinearities in N, then one should design a control law that suppresses the effect of $d(\cdot)$ in H_{+d}. The latter can be achieved by a disturbance observer that estimates $d(\cdot)$ and cancels it subsequently. Therefore, the goal of this section is to design a disturbance observer to make N behave linearly and, for instance, be free of chaotic behavior.

A disturbance observer added to the system H_{+d} is shown in Figure 3. In this figure, $H_n(s)$ represents the nominal transfer function (mathematical model) corresponding to $H(s)$ in equation (6). In order to implement a disturbance observer, the filter $Q(s)$ is added to the system to make $Q(s)\, H_n^{-1}(s)$ a realizable (at least a proper) transfer function, because $H_n^{-1}(s)$ is often unrealizable. A successful design of a disturbance observer crucially depends on the design of $Q(s)$. Due to its important role, the design of $Q(s)$ has been extensively studied; see, e.g., references [19, 20, 24, 25]. It turns out that $Q(s)$ should be a low-pass filter of unity DC-gain. A typical form of $Q(s)$ is

$$Q(s) = \frac{\sum_{k=1}^{m-\rho} a_k\, (\tau s)^k + 1}{\sum_{k=1}^{m} a_k\, (\tau s)^k + 1} , \quad (10)$$

where ρ is at least equal to the relative degree of $H_n(s)$ and $a_k > 0$ and $\tau > 0$ are constant real numbers.

From Figure 3, it is concluded that in the absence of the measurement noise ($w \equiv 0$)

$$\tilde{d}(s) = [\; H(s) - H_n(s)\;]\; v(s) + c\, (sI_n - A)^{-1}\, x_0 + d(s) , \quad (11a)$$

$$\begin{aligned} y_{DOB}(s) = {} & [\; 1 + H(s)\, (1 - Q(s))^{-1}\, Q(s)\, H_n^{-1}(s)\;]^{-1} \\ & \times\; H(s)\, (1 - Q(s))^{-1}\, u(s) \\ & + [\; 1 + H(s)\, (1 - Q(s))^{-1}\, Q(s)\, H_n^{-1}(s)\;]^{-1} \\ & \times\; [\; c\, (sI_n - A)^{-1}\, x_0 + d(s)\;] , \end{aligned} \quad (11b)$$

where the output of the system is denoted by y_{DOB} to indicate a disturbance observer is implemented. Several comments regarding equations (11) are made:

(i) The filter $Q(s)$ should be designed such that the transfer function

$$[\ 1 + H(s)\ (1 - Q(s))^{-1}\ Q(s)\ H_n^{-1}(s)\ ]^{-1}\ , \qquad (12)$$

is stable.

(ii) By assumption (A1), the matrix A is Hurwitz. Thus, when equation (11a) is considered in the time domain, the effect of the initial state vector x_0 in this equation decays to zero. Moreover, $H(s) \approx H_n(s)$. Thus, from equation (11a), it is concluded that $\tilde{d}(\cdot)$ is an estimate of the bounded disturbance $d(\cdot)$.

(iii) The filter $Q(s)$ is a low pass filter of unity DC-gain. Thus, from equation (11b), it follows that

$$y_{DOB}(s) \approx H_n(s)\ u(s)\ . \qquad (13)$$

That is, the effect of the bounded disturbance $d(\cdot)$ (as well as the decaying effect of the initial state vector x_0) in the system in Figure 3 is suppressed, and the output of the system is approximately equal to that of the *linear* nominal system.

An implementation of the disturbance observer on the system N is shown in Figure 4. The system in this figure is denoted by N_{DOB} to indicate a disturbance observer is added to N. The equivalence of N_{DOB} and the system in Figure 3 asserts that the effects of nonlinearities in N_{DOB} can be suppressed. That is, N_{DOB} would behave linearly.

Next, the performance of N_{DOB} is examined.

4. EXAMPLE

In this section, an example is presented to illustrate the efficacy of disturbance observers in suppressing the effects of nonlinearities and possible chaotic behavior in a nonlinear system in the class of systems considered in this paper.

Consider a Duffing-type system represented by

$$\ddot{\xi}(t) + 0.1\ \dot{\xi}(t) + \xi(t) - \tan h(2\xi(t)) = 0.5\ \cos t\ ,$$

$$\xi(0) = 0\ , \quad \dot{\xi}(0) = 0\ , \qquad (14)$$

for all $t \geq 0$, where $\xi(t) \in \mathbb{R}$. Considering the first two terms in the expansion $\tan h(2\xi) = 2\xi - 8\xi^3/3 + \ldots$, the system (14) should behave like a Duffing system (see, e.g., references [36-38]). This fact is shown in the following.

By letting $x_1(t) = \xi(t)$ and $x_2(t) = \dot{\xi}(t)$ for all $t \geq 0$, the system (14) can be written as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \tan h(2x_1(t)) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} 0.5\ \cos t\ , \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad (15a)$$

$$y(t) = [\ 1 \quad 0\ ] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \qquad (15b)$$

It is straightforward to verify that assumptions (A1)-(A3) hold for the system (15). The output the system (15) is depicted in Figure 5 and is denoted by y. It is evident that the system (15) exhibits a chaotic behavior typical of Duffing systems. The output of the linear system H, that is, the system (15) in the absence of the nonlinearity $\tan h(2x_1(\cdot))$, is depicted in Figure 5 and is denoted by y_L. The steady-state of y_L is the periodic function of time $t \mapsto 5 \sin t$, which is obtained by applying results from the theory of linear oscillations.

The difference between y and y_L is due to the nonlinearity in the system (15). It is now shown that the effect of this nonlinearity, and consequently chaotic behavior, can be effectively suppressed by a disturbance observer. It is remarked that the control of chaotic behavior in Duffing systems is of great interest; see, e.g., references [39-43].

The first step to the design of a disturbance observer is to obtain the transfer function $H(s)$ corresponding to the system H. This transfer function is readily determined from equation (6) and is given by

$$H(s) = \frac{1}{s^2 + 0.1\ s + 1}\ . \qquad (16)$$

Having $H(s)$, a disturbance observer is implemented on the system (15). The resulting system is N_{DOB} in Figure 4, where $H_n(s) = H(s)$, the system N is that in equations (15), and

$$Q(s) = \frac{700}{s^2 + 9s + 700}\ . \qquad (17)$$

The output of N_{DOB} in the absence of the measurement noise ($w \equiv 0$) is shown in Figure 5 and is denoted by y_{DOB}. It is evident that y_{DOB} and y_L almost overlap, except that the former has a slightly larger amplitude. That is, the disturbance observer has successfully suppressed the effect of the nonlinearity in the system (15).

The effect of the measurement noise $w(\cdot)$ on the performance of the system N_{DOB} is studied next. Let $w(\cdot)$ be a band-limited white noise. The output of the system in the presence of $w(\cdot)$ is depicted in Figure 6 and is denoted by y_{DOB}. This output is compared to that of the system H, denoted by y_L. It is evident that y_{DOB} and y_L almost overlap, except that the former has a slightly larger amplitude. That is, the disturbance observer is able to suppress the effect of the nonlinearity in the system (15) even in the presence of the measurement noise.

5. CONCLUSIONS

In this paper, the application of disturbance observers to suppress chaotic behavior in a class of single-input single-output nonlinear systems was studied. A nonlinear system in this class has the property that the effects of all nonlinearities in the system can be captured in a bounded disturbance. Knowing this fact, it was shown how a disturbance observer can be designed to estimate the bounded disturbance (equivalently, the effects of nonlinearities in the system) and cancel it subsequently. The disturbance observer is thus able to make the nonlinear system behave linearly and, for instance, be free of chaotic behavior. The results of the paper were corroborated by using a disturbance observer to suppress chaotic behavior due to a nonlinearity in a Duffing-type system.

Three remarks are made regarding disturbance observers applied to the class of nonlinear systems: (i) disturbance observers are linear systems, but yet they are able to suppress the effects of nonlinearities in the systems; (ii) disturbance observers can suppress the effects of nonlinearities that are not exactly known; (iii) the application of disturbance observers is not exclusive to chaotic systems. If nonlinearities in a system cause an undesirable behavior, say limit cycle behavior, then disturbance observers can be used to suppress such a behavior. For instance, flutter in aircraft wings can be considered as limit cycle behavior. Thus, disturbance observers can be used to suppress flutter; work in this area is in progress and will be reported elsewhere.

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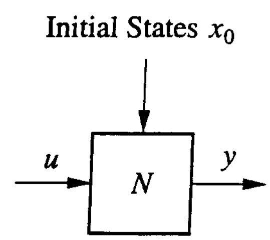

Figure 1. The nonlinear system N represented by equation (1).

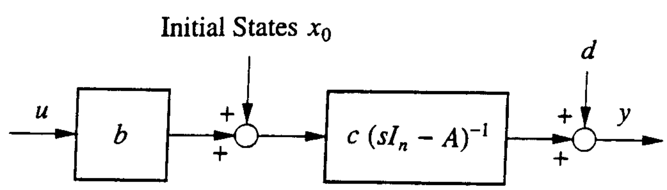

Figure 2. The linear system H_{+d}. This system is an equivalent representation of the nonlinear system N.

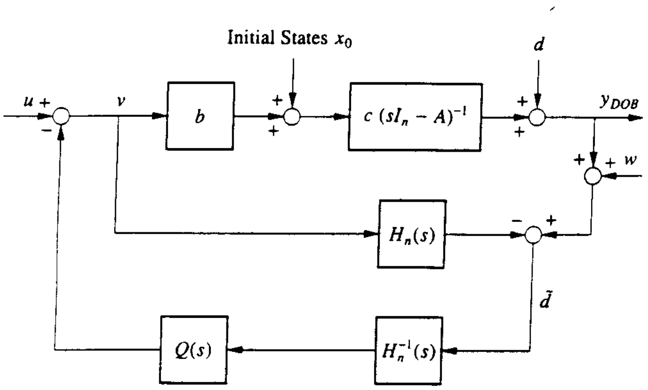

Figure 3. A disturbance observer added to the system H_{+d} (equivalently N) to estimate d which captures the effects of nonlinearities in N. An estimate of d is $\tilde{d}$ which is canceled subsequently.

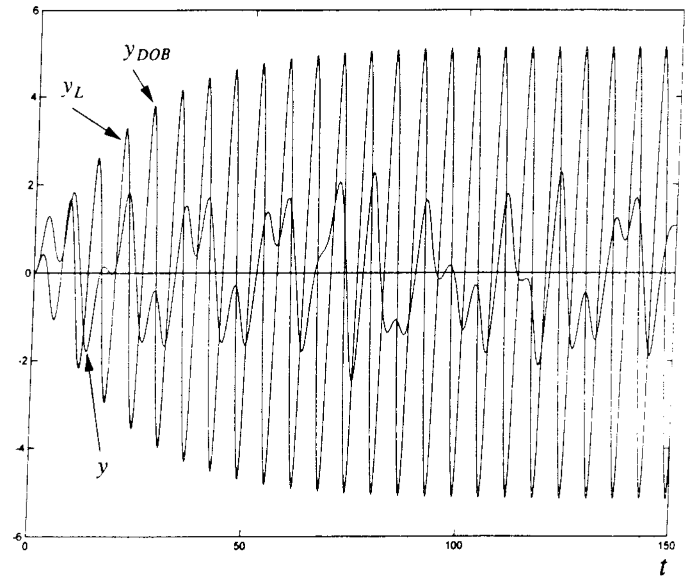

Figure 5. Responses of the systems N, H (the nonlinearity-free N), and N_{DOB}, denoted by y, y_L, and y_{DOB}, respectively, in the absence of the measurement noise w. It is evident that y is chaotic. Moreover, it is evident that y_{DOB} and y_L almost overlap. That is, the disturbance observer has suppressed the effect of the nonlinearity.

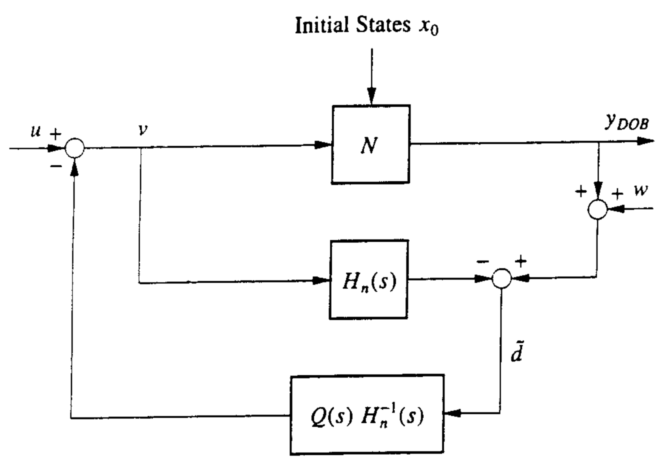

Figure 4. The system N_{DOB}. This system is N to which a disturbance observer is added.

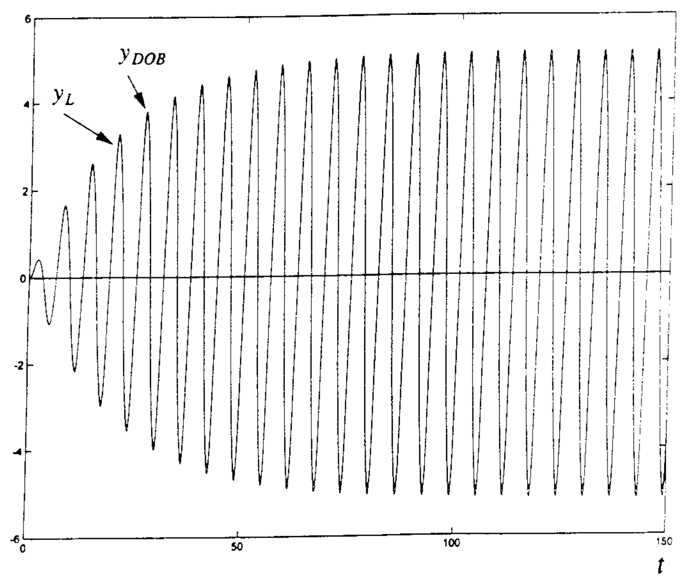

Figure 6. Responses of the systems H and N_{DOB}, denoted by y_L and y_{DOB}, respectively, in the presence of the measurement noise w. It is evident that y_{DOB} and y_L almost overlap.