

# Control of a Laboratory Helicopter using Switched 2-Step Feedback Linearization

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**Abstract**—In this paper a control structure based on feedback (Input-Output) linearization has been applied to the longitudinal subsystem of a laboratory double-rotor helicopter. This article focuses on the longitudinal subsystem which is underactuated in sense that the number of control variables is less than the number of degrees of freedom. A switching control law between exact and approximate input-output linearization is proposed. The Feedback Linearization has been applied in two steps, first to the nonlinear actuator, and then to the entire system. This law has been tested by simulated and experimental results.

## I. INTRODUCTION

In this paper a study on feedback linearization applied to the longitudinal subsystem of a double-rotor helicopter is made. The double-rotor system is a highly nonlinear, multivariable, underactuated, strongly coupled and non-minimum phase system. The longitudinal system, in turn, is a nonlinear and underactuated system.

In previous works, see [7] and [8] for details, the control structure was based on partial feedback linearization. Particularly, the computed torque technique was used to linearize the slow dynamics of the system (body dynamics)[2]. The rotor dynamics was considered to be fast enough to separate both dynamics. In this way, the angular velocity of the rotor was considered as constant from the point of view of the body dynamics.

A complete linearization for the longitudinal subsystems was searched in [9], taking into account ideas such as approximate linearization, exposed in [5]. It was proved that the input-state linearization was not suitable when the velocity of the rotors were next to zero, since the control law saturated.

In this paper, a switching control between the exact linearization and an approximate linearization is proposed, where the first one is not valid for rotor velocities next to zero, and the second one is only valid for rotor velocities next to zero. The switching law allows the system to be controlled in the whole working range.

Besides, both types of linearization laws have been divided into two steps in order to simplify the practical implementation. The two-step linearization is applied to the longitudinal subsystem: first applying feedback linearization to actuator and then to the entire system.

The remainder of the paper is structured as follows: In Section 2 the system is described and a model is presented. In the third section the two-step input-output linearization carried out is described. The next section shows the switching control between an approximate and the exact linearization law. In section 5 experimental results are presented. And finally, last section summarizes the major conclusions of the paper.

## II. SYSTEM DESCRIPTION AND MODEL

The laboratory helicopter consists of a 2 DOF mechanism thrusted by two rotors resembling a helicopter. The degrees of freedom are the yaw and the pitch angles.



Fig. 1. Double-Rotor Laboratory Helicopter

In this analysis, the orientation angle is fixed ( $\theta = const$ ), and the angular velocity of the tail rotor is null ( $\omega_t = const = 0$ ). The pitch angle will be controlled by the main rotor.

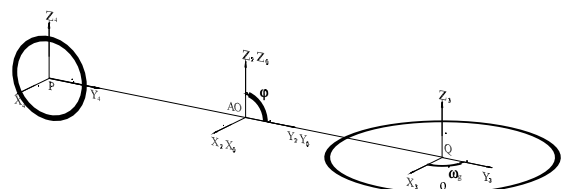


Fig. 2. Longitudinal Subsystem

The equations of the longitudinal dynamics are as follows:

$$I_\varphi \ddot{\varphi} + G_s S(\varphi) + G_c C(\varphi) + K_\varphi \dot{\varphi} = \hat{L}_g |\omega_g| \omega_g \quad (1)$$

$$I_g \dot{\omega}_g = P_m - (B_g + \hat{D}_g |\omega_g|) \omega_g \quad (2)$$

where:

- $\varphi$ : Pitch Angle measured from the horizontal plane.
- $I_\varphi$ : Inertia of the longitudinal system with respect to its rotation axis.
- $\omega_g$ : Angular Velocity of the main rotor.
- $I_g$ : Inertia of the propeller with respect to its rotation axis.
- $\hat{L}_g |\omega_g| \omega_g$ : Torque due to the aerodynamic force of thrust in main rotor.
- $K_\varphi \dot{\varphi}$ : Friction Torque.
- $G_s S(\varphi)$ : Gravity Torque 1. ( $S(\varphi) = \sin(\varphi)$ )
- $G_c C(\varphi)$ : Gravity Torque 2. ( $C(\varphi) = \cos(\varphi)$ )
- $P_m$ : Engine Torque.
- $B_g$ : Friction constant of the engine.
- $\hat{D}_g$ : Drag Constant of the propeller.

Rewriting (1) and (2):

$$\begin{bmatrix} I_\varphi & 0 \\ 0 & I_g \end{bmatrix} \begin{bmatrix} \ddot{\varphi} \\ \dot{\omega}_g \end{bmatrix} + \begin{bmatrix} G_s S(\varphi) + G_c C(\varphi) + K_\varphi \dot{\varphi} - \hat{L}_g |\omega_g| \omega_g \\ (B_g + \hat{D}_g |\omega_g|) \omega_g \end{bmatrix} = \begin{bmatrix} 0 \\ P_m \end{bmatrix} \quad (3)$$

It can be seen that there is only an engine ( $P_m$ ) and 2 DOF, the pitch angle ( $\varphi$ ) and the angular velocity of the rotor ( $\omega_g$ ). Therefore it is an underactuated system in the sense that it has less control inputs than degrees of freedom (see [10] for details).

### III. CONTROL STRATEGIES

In this section the control structure is presented, which is based on two control loops (see Fig. 3). The inner one will carry out a feedback input-output linearization in such a way that the resultant system is equivalent to three integrators. The goal of the outer loop is to fulfill the specifications imposed on the system.

With respect to the linearization loop, it was seen in [9] that such a law was not suitable next to the static equilibrium point of the system. In order to control the system in a region around this point, the system model is modified (see [5]) and a new approximate law is obtained, which is suitable only in this region. Next, a switching control based on the two laws is studied and applied depending on the working point.

On the other hand, the outer loop is closed using an LQR controller that is designed to control a chain of three integrators.

To obtain the linearization law, the input-output linearization technique explained in [3] is applied. First, it is applied to the nonlinear actuator dynamics (see Fig.4), and then to the complete set of equations. This is carried out in two steps in order to simplify the practical implementation of the control laws.

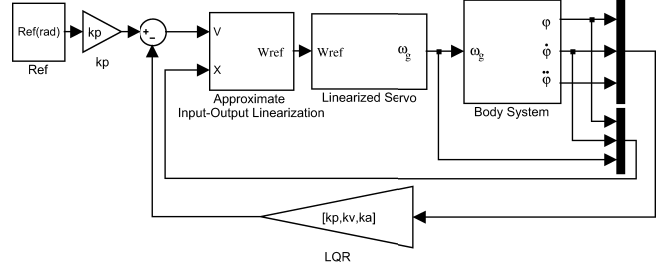


Fig. 3. Controller Scheme

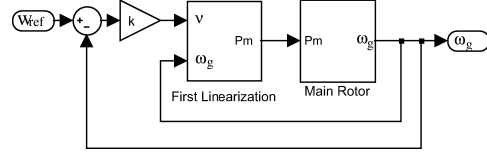


Fig. 4. Linearized Servo

#### Feedback Linearization of the Actuator

Equation 2 represents the dynamics of the actuator. This is a first order equation, but nonlinear in the rotor velocity. It is linearized to a simple integrator using feedback linearization. Then the loop is closed in such a way that it becomes a linear first order system, in which the input is the reference velocity  $W_{ref}$  and the output is the velocity of the rotor  $\omega_g$  (see Fig.4).

In order to linearize this subsystem, the linearization law, denoted by  $\nu$  (see Fig.4), will be taken as  $\nu = \dot{\omega}_g$ , which corresponds to a simple integrator. Therefore,  $\nu$  can be expressed as follows:

$$\nu = \dot{\omega}_g = \frac{1}{I_g} (P_m - (B_g + \hat{D}_g |\omega_g|) \omega_g) \quad (4)$$

The resultant system can be expressed in terms of a transfer function as  $\frac{\omega_g}{\nu} = \frac{1}{s}$ . The equation that is intended to be introduced corresponds to a linear first order system. Therefore, the loop of the integrator will be closed using a proportional controller. The resultant transfer function can be expressed as:

$$\frac{\omega_g}{W_{ref}} = \frac{k}{k+s} = \frac{1}{1 + \frac{1}{k}s} = \frac{1}{1 + \tau s}$$

which expressed in the time domain gives rise to

$$\dot{\omega}_g = kW_{ref} - k\omega_g \quad (5)$$

#### Input-Output Linearization

Once the feedback linearization of the actuator has been applied, the system dynamics can be expressed with this new set of equations:

$$I_\varphi \ddot{\varphi} + G_s S(\varphi) + G_c C(\varphi) + K_\varphi \dot{\varphi} = \hat{L}_g |\omega_g| \omega_g \quad (6)$$

$$\dot{\omega}_g = kW_{ref} - k\omega_g$$

Next, Feedback linearization is applied again to this set of equations. First of all, the system is expressed in the state space defining the state vector as follows:

$$X = \begin{bmatrix} \varphi - \varphi_{eq} \\ \dot{\varphi} \\ \omega_g \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (7)$$

Using the above definition, (6) can be expressed as follows:

$$\dot{X} = f(X) + g(X) \cdot u_e$$

where:

$$f(X) = \begin{bmatrix} x_2 \\ \frac{-G_s S(\varphi_{eq} + x_1) - G_c C(\varphi_{eq} + x_1) - K_\varphi \cdot x_2 + \hat{L}_g |x_3| x_3}{I_\varphi} \\ -k x_3 \end{bmatrix} \quad (8)$$

$$g(X) = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} \quad (9)$$

$$u_e = W_{ref} \quad (10)$$

(Note that  $W_{ref}$  is denoted by  $u_e$ , which represents the exact linearization law (Fig.3)).

The longitudinal system is controllable if the rotor velocity is not next to zero, in fact, in [9] a non global diffeomorphism was obtained.

The Input-Output Linearization is based on expressing the output or a derivative output as a function of the control signal. After this, the control signal will be isolated and the derivative output will be made equivalent to the input signal of the resultant linearized system.

Thus, choosing

$$y = h(x) = x_1 = \varphi - \varphi_{eq}$$

the first derivative output yields:

$$\dot{y} = \mathbf{L}_f h + \mathbf{L}_g h u_e$$

where  $u_e$  only can be isolated if  $\mathbf{L}_g h$  is non-null. (Note that  $\mathbf{L}$  represents the directional derivative, also known as Lie derivative). Computing this value, it gives rise to:

$$\mathbf{L}_g h = \nabla h \cdot g = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = 0$$

Since this is not the case, a second derivative output has to be obtained, yielding:

$$\begin{aligned} \ddot{y} &= \mathbf{L}_f^2 h + \mathbf{L}_g \mathbf{L}_f h u_e = \mathbf{L}_f(\nabla h \cdot f) + \mathbf{L}_g(\nabla h \cdot f) u_e \\ &= \nabla(\nabla h \cdot f) \cdot f + \nabla(\nabla h \cdot f) \cdot g u_e \end{aligned} \quad (11)$$

where,

$$\mathbf{L}_f h = \nabla h \cdot f = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = f_1 = x_2$$

$$\mathbf{L}_g \mathbf{L}_f h = \nabla(\nabla h \cdot f) \cdot g = \nabla x_2 \cdot g = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = 0$$

Since  $u_e$  cannot be isolated, it is necessary to derive again.

$$y^{(3)} = \mathbf{L}_f^3 h + \mathbf{L}_g \mathbf{L}_f^2 h u_e$$

Taking into account that:

$$\mathbf{L}_g \mathbf{L}_f^2 h = \mathbf{L}_g(\mathbf{L}_f x_2)$$

$$\mathbf{L}_f x_2 = \nabla x_2 \cdot f = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot f = f_2$$

$$\begin{aligned} \mathbf{L}_g(f_2) &= \nabla(f_2) \cdot g = \begin{bmatrix} \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} \\ &= \frac{\partial f_2}{\partial x_3} k = \frac{2k \hat{L}_g}{I_\varphi} |x_3| \neq 0 \iff |x_3| \neq 0 \end{aligned} \quad (12)$$

Since  $\mathbf{L}_g \mathbf{L}_f^2 h \neq 0$  is non null, the system is said to have *relative degree three*. Next, the control signal  $u_e$  will be calculated taking the input of the linearized system (see Fig. 3) as  $V = y^{(3)}$  which corresponds to a chain of three integrators.

In this way, the linearization law can be expressed as follows:

$$V = \mathbf{L}_f^3 h + \mathbf{L}_g \mathbf{L}_f^2 h u_e \quad (13)$$

Isolating  $u_e$ , the control signal to apply to the actuator is obtained as follows:

$$u_e = W_{ref} = \frac{V - \mathbf{L}_f^3 h}{\mathbf{L}_g \mathbf{L}_f^2 h}$$

where  $V$  is computed by the external LQR controller and

$$\mathbf{L}_g \mathbf{L}_f^2 h = \frac{I_g \hat{L}_g}{I_\varphi} 2|x_3| \neq 0 \iff |x_3| \neq 0 \quad (14)$$

$$\begin{aligned} \mathbf{L}_f^3 h &= \mathbf{L}_f^2 \mathbf{L}_h = \mathbf{L}_f^2(\nabla h \cdot f) = \mathbf{L}_f^2 x_2 \\ &= \mathbf{L}_f \mathbf{L}_f x_2 = \mathbf{L}_f(f_2) = \nabla(f_2) \cdot f \end{aligned} \quad (15)$$

$$\nabla(f_2) = \left[ \frac{\partial f_2}{\partial x_1}, \frac{\partial f_2}{\partial x_2}, \frac{\partial f_2}{\partial x_3} \right]$$

$$\begin{aligned} \mathbf{L}_f^3 h &= \left( \frac{K_\varphi (G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1))}{I_\varphi^2} \right) + \\ &+ \left( \frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left( \frac{K_\varphi}{I_\varphi} \right)^2 \right) \cdot x_2 - \\ &- \left( \frac{\hat{L}_g}{I_\varphi} \left( \frac{K_\varphi}{I_\varphi} + 2k \right) \right) \cdot x_3 \cdot |x_3| \end{aligned} \quad (16)$$

It can be seen that the obtained law is well-defined for all values of the state,  $X$ , except for the value  $x_3 = 0$ . Therefore, the control law will be well-defined if and only if  $\mathbf{L}_g \mathbf{L}_f^2 h$  is well-defined for all values of the state  $X$ , which is not satisfied when  $x_3 = 0$ . In this way, the attained linearization is not global for all  $X$ . Thus, a region around the  $x_3 = 0$  has to be studied in order to find the set of values of  $x_3 = \omega_g$  makes  $\mathbf{L}_g \mathbf{L}_f^2 h$  be too small and  $u_e$  be too high. This set causes a saturation phenomenon in the actuator without having null velocity.

Furthermore, the minimum velocity of the rotor that do not cause saturation of the law have to be determined. This velocity will be denoted as  $\omega_s$ .

### Approximate Input-Output Linearization

In this section a simplified model of the aerodynamic forces applied to the system has been used. It is well-known that these forces vary proportionally with the square of the angular velocity. The simplification consists of linearizing the aerodynamic force in a region that contains  $x_3 = 0$ , that is, linearizing the force when the angular velocity is next to zero. Therefore, this approximate model is valid only for small forces. In this region, the constant  $L_g$  has a value proportional to the angular velocity when switching between both laws. This fact will be demonstrated in next section.

The equations of the longitudinal dynamics are now changed by the following ones:

$$I_\varphi \ddot{\varphi} + G_s S(\varphi) + G_c C(\varphi) + K_\varphi \cdot \dot{\varphi} = L_g \omega_g \quad (17)$$

$$\dot{\omega}_g = \hat{k} W_{ref} - \hat{k} \omega_g \quad (18)$$

where the constant  $L_g$  and  $\hat{k}$  are different from those of the quadratic forces  $\hat{L}_g$  and  $k$ .

Rewriting the equations in the state space, yields:

$$\dot{X} = f(x) + g(x) \cdot u_a$$

where:

$$f(x) = \begin{bmatrix} x_2 \\ \frac{-G_s S(\varphi_{eq} + x_1) - G_c C(\varphi_{eq} + x_1) - K_\varphi \cdot x_2 + \hat{L}_g x_3}{I_\varphi} \\ -\hat{k} x_3 \end{bmatrix} \quad (19)$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} \quad (20)$$

$$u_a = W_{ref} \quad (21)$$

(Note that  $W_{ref}$  is denoted by  $u_a$ , which represents the approximate linearization law (Fig.3)).

Applying the same procedure as before to this set of equations, it is necessary to calculate the third derivative output to obtain a function of the signal input.

Thus, choosing

$$y = h(x) = x_1 = \varphi - \varphi_{eq}$$

and computing the first, the second and the third derivative output, the following terms are obtained:

$$\mathbf{L}_g h = \nabla h \cdot g = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = 0$$

$$\mathbf{L}_f h = \nabla h \cdot f = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = f_1 = x_2$$

$$\mathbf{L}_g \mathbf{L}_f h = \nabla(\nabla h \cdot f) \cdot g = \nabla x_2 \cdot g = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = 0$$

$$\mathbf{L}_g \mathbf{L}_f^2 h = \mathbf{L}_g(\mathbf{L}_f x_2)$$

$$\mathbf{L}_f x_2 = \nabla x_2 \cdot f = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot f = f_2$$

$$\begin{aligned} \mathbf{L}_g(f_2) &= \nabla(f_2) \cdot g = \begin{bmatrix} \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} \\ &= \frac{\partial f_2}{\partial x_3} \hat{k} = \frac{\hat{k} \hat{L}_g}{I_\varphi} \neq 0 \end{aligned} \quad (22)$$

Since  $\mathbf{L}_g \mathbf{L}_f^2 h$  is non-null, the system is said to have relative degree three, and  $u_a$  can be isolated from

$$y^{(3)} = \mathbf{L}_f^3 h + \mathbf{L}_g \mathbf{L}_f^2 h u_a$$

Thus, taking

$$V = y^{(3)}$$

as the input to the linearized system, this will be equivalent to a chain of three integrators.

The control signal to apply to the actuator is obtained, isolating  $u_a$ .

$$u_a = W_{ref} = \frac{V - \mathbf{L}_f^3 h}{\mathbf{L}_g \mathbf{L}_f^2 h}$$

where  $V$  is computed by the external LQR controller and

$$\begin{aligned} \mathbf{L}_f^3 h &= \mathbf{L}_f^2 \mathbf{L}_f h = \mathbf{L}_f^2(\nabla h \cdot f) = \mathbf{L}_f^2 x_2 \\ &= \mathbf{L}_f \mathbf{L}_f x_2 = \mathbf{L}_f(f_2) = \nabla(f_2) \cdot f \end{aligned} \quad (23)$$

$$\nabla(f_2) = \left[ \frac{\partial f_2}{\partial x_1}, \frac{\partial f_2}{\partial x_2}, \frac{\partial f_2}{\partial x_3} \right]$$

$$\begin{aligned} \mathbf{L}_f^3 h &= \left( \frac{K_\varphi (G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1))}{I_\varphi^2} \right) + \quad (24) \\ &+ \left( \frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left( \frac{K_\varphi}{I_\varphi} \right)^2 \right) \cdot x_2 - \\ &- \left( \frac{\hat{L}_g}{I_\varphi} \left( \frac{K_\varphi}{I_\varphi} + \hat{k} \right) \right) \cdot x_3 \end{aligned}$$

It can be noticed that this law is well-defined for all value of the state vector  $X$ . Furthermore, the control law is also well-defined due to the fact that the value of  $\mathbf{L}_g \mathbf{L}_f^2 h = \frac{\hat{k} \hat{L}_g}{I_\varphi}$  is a non-null constant. Therefore, the obtained linearization is global for all  $X$ , although its applicability is limited to small thrust forces, where the model is valid.

### IV. SWITCHING BETWEEN EXACT AND APPROXIMATE INPUT-OUTPUT LINEARIZATION

In previous sections, exact feedback linearization has been demonstrated to be valid far from the point of static equilibrium of the system, in which engine saturation ensues. It has been also shown that in a region next to the static equilibrium of the system, an approximate linearization law is valid using a linear model of thrust.

To follow up, in this section, a switching law between the exact and the approximate laws will be carried out. The choice of the switching velocity depends only on the saturation phenomenon ( $\omega_s$ ), taking into account a non-abrupt switching.

Next, the conditions to ensure a soft switching will be developed.

### Approximate Linear Thrust Model

To sum up, the control law applied to the actuator will follow the law

$$u_a = \frac{V - \mathbf{L}_f^3 h}{\mathbf{L}_g \mathbf{L}_f^2 h} = \frac{V - \alpha_1}{\beta_1} = \frac{V}{\beta_1} - \xi_1$$

where

$$\beta_1 = \frac{\hat{L}_g \hat{K}}{I_\varphi} \neq 0 \quad (25)$$

$$\alpha_1 = K_1 + K_2 - \left[ \frac{\hat{L}_g}{I_\varphi} \left( \frac{K_\varphi}{I_\varphi} + \hat{K} \right) \right] x_3 \quad (26)$$

$$\xi_1 = \frac{(K_1 + K_2) I_\varphi}{\hat{L}_g \hat{K}} - \left( \frac{K_\varphi}{I_\varphi \hat{K}} + 1 \right) x_3 \quad (27)$$

and

$$K_1 = \frac{K_\varphi}{I_\varphi^2} [G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1)] \quad (28)$$

$$K_2 = \left[ \frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left( \frac{K_\varphi}{I_\varphi} \right)^2 \right] x_2 \quad (29)$$

### Quadratic Thrust Model

$$u_e = \frac{V - \mathbf{L}_f^3 h}{\mathbf{L}_g \mathbf{L}_f^2 h} = \frac{V - \alpha_2}{\beta_2} = \frac{V}{\beta_2} - \xi_2$$

where

$$\beta_2 = \frac{2L_g K |x_3|}{I_\varphi} \quad (30)$$

$$\alpha_2 = K_1 + K_2 - \left[ \frac{L_g}{I_\varphi} \left( \frac{K_\varphi}{I_\varphi} + 2K \right) \right] |x_3| x_3 \quad (31)$$

$$\xi_2 = \frac{(K_1 + K_2) I_\varphi}{2L_g K |x_3|} - \left( \frac{K_\varphi}{2I_\varphi K} + 1 \right) x_3 \quad (32)$$

and  $K_1$  y  $K_2$  are the same as in the fore mentioned case.

### Non-Abrupt Switching Conditions

In order to ensure a soft switching between both laws, the condition  $u_a = u_e$  has to be imposed at the switching instant. Similarly  $V$  will be imposed to be the same at the switching instant. Due to this, it can be obtained that

$$u_a = \frac{V}{\beta_1} - \xi_1 = u_e = \frac{V}{\beta_2} - \xi_2$$

and then

$$\frac{\beta_2 - \beta_1}{\beta_1 \beta_2} V = \xi_1 - \xi_2$$

Since this equality has to be valid for all  $V$ , the following two conditions are obtained.

$$\beta_1 = \beta_2 \quad (33)$$

$$\implies \xi_1 = \xi_2 \quad (34)$$

The analysis of these conditions gives the following relations. From the first condition, (33):

$$\beta_1 = \beta_2 \implies \frac{\hat{L}_g \hat{K}}{I_\varphi} = \frac{2L_g K |x_3|}{I_\varphi} \implies \hat{L}_g \hat{K} = 2L_g K |x_3| \quad (35)$$

From the second condition, (34):

$$\frac{(K_1 + K_2) I_\varphi}{\hat{L}_g \hat{K}} - \left( \frac{K_\varphi}{I_\varphi \hat{K}} + 1 \right) x_3 = \frac{(K_1 + K_2) I_\varphi}{2L_g K |x_3|} - \left( \frac{K_\varphi}{2I_\varphi K} + 1 \right) x_3$$

Substituting (35) in the above equation:

$$\left( \frac{K_\varphi}{I_\varphi \hat{K}} + 1 \right) = \left( \frac{K_\varphi}{2I_\varphi K} + 1 \right) \quad (36)$$

and finally, isolating parameters:

$$\hat{K} = 2K \quad (37)$$

$$\hat{L}_g = L_g |x_3| \quad (38)$$

where  $x_3$  will be the rotor velocity at the switching instant denoted as  $\omega_s$ . This velocity has to be chosen in such a way that the engine does not saturate when the velocity is next to zero.

## V. EXPERIMENTAL RESULTS

### Switching linearization laws with an external LQR

Taking into account the non-abrupt switching conditions, an appropriate value of  $\omega_s$  has to be chosen in such a way that the exact linearization law does not generate a control signal  $u$  that makes the engine saturate. The value of  $\omega_s = 0.08 \cdot \omega_{max}$  has been chosen, where  $\omega_{max}$  is the maximum velocity of the rotor.

Figs. 5, 6, 7 and 8 show respectively the external controller signal  $V$ , the reference velocity of the rotor  $W_{ref}$ , the rotor velocity  $\omega_g$ , the control signal applied to the engine  $Pm$  and the pitch angle  $\varphi$ , when applying this controller.

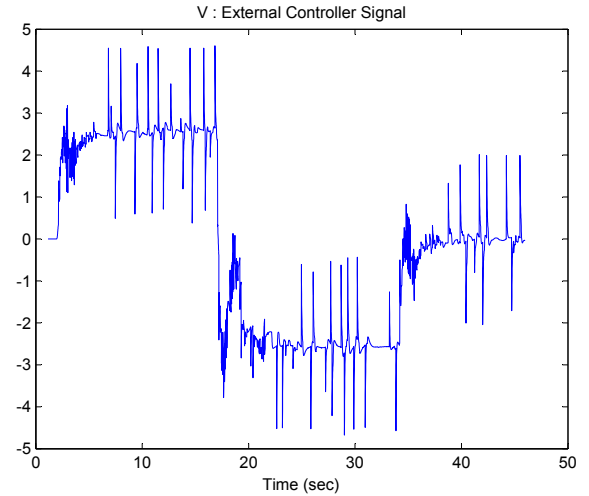


Fig. 5. Control signal generated by the external controller,  $V$

The noise in the external control signal,  $V$ , is due to the quantizer effect introduced by the encoder that reads the pitch angle. This measurement is used to estimate the angular velocity and the angular acceleration, which are used by the external LQR controller to compute  $V$ . This noise also affects the other signals,  $W_{ref}$  and  $Pm$  (see Figs. 6 and 7 respectively). In order to check the quality of

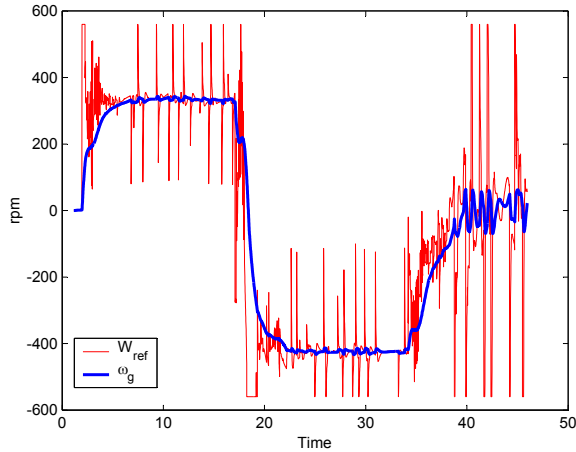


Fig. 6. Switched Linearization Signal  $W_{ref}$  versus Rotor Velocity  $\omega_g$  (rpm)

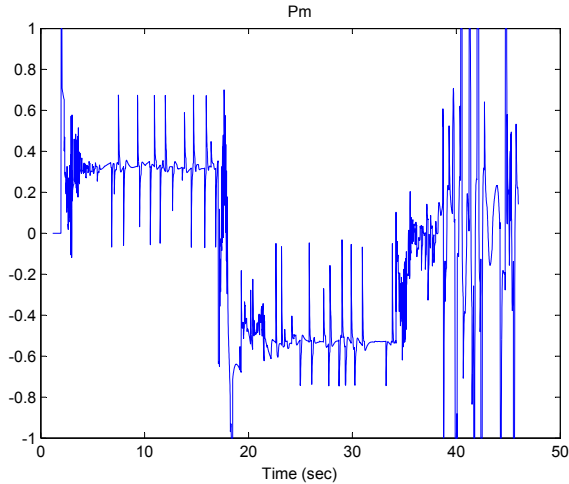


Fig. 7. Control signal applied to the engine,  $P_m$

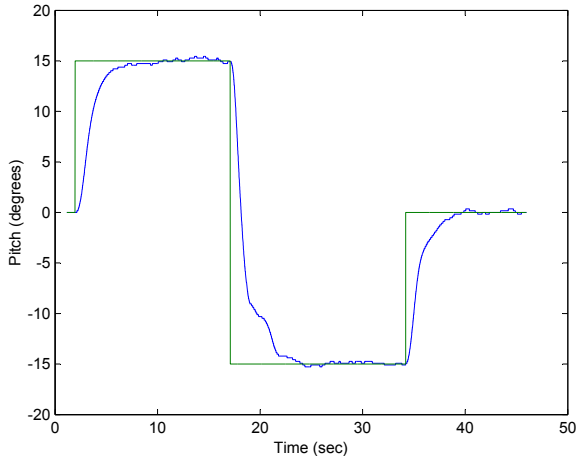


Fig. 8. Pitch angle of the system  $\varphi$  (degrees)

the control performance achieved, the response of the pitch angle, using a square wave as reference, is shown in Fig.7.

Notice that pitch oscillation values, when the reference angle is zero, are less than 0.4 degrees, which represents twice the resolution of the encoder.

## VI. CONCLUSIONS

In this paper an input-output linearization law has been applied to the longitudinal subsystem of a laboratory double-rotor helicopter. As the exact input-output linearization provides a law that cannot be applied in the whole operating range, a switching law has been developed. The second law applied, has been obtained using an approximation of the model in the working range, in which the exact law made the engine saturate. In order to make the practical implementation easier the linearization has been carried out in two steps. One to linearize the engine-rotor system and the other one to linearize the complete system.

Both laws have been experimentally tested using external LQR controllers designed for a chain of three integrators, which bring forth the improvements achieved with the proposed switching law.

As a possible future development, a suitable linearization law will be worked out for the complete laboratory helicopter, using new results for non-minimum phase MIMO systems and taking into account [3] and [4].

## VII. ACKNOWLEDGEMENTS

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