Decentralized Disturbance Attenuation for Large-Scale Nonlinear Systems with Delayed State Interconnections

Yi Guo

Abstract— The problem of decentralized disturbance attenuation is considered for a new class of large-scale nonlinear systems with delayed state interconnections. This class of large-scale time-delay systems broadens most existing classes of large-scale time-delay systems in that the uncertain interconnections are bounded by general nonlinear functions instead of linear or polynomial-type functions. It is shown that by decentralized memoryless state feedback control, the closed-loop system achieves internal global asymptotical stability in the sense of Lyapunov and external stability in the sense of $\mathcal{L}_2$ gain. Nonlinear Lyapunov-Krasovskii functionals are constructed which renders the linear and polynomial-type growth conditions on the interconnections as special cases.

Keywords: **Large-scale systems, decentralized control, time-delay, nonlinear control, backstepping.**

I. INTRODUCTION

Decentralized control has received renewed interests during the past years, motivated by its importance in applications to large complex engineering systems. Advances in robust control and stability analysis for centralized systems have been extended to large-scale interconnected systems. The existence of time delay is frequently a source of instability, see [1], [2]. Among many contributions on time-delay systems (see [3] for a review), robust H_∞ control for *linear* state-delayed systems were studied in [4] using the linear matrix inequality method, and in [5] using backstepping for systems with delay in control. Decentralized control for large-scale time-delay interconnected systems has also received considerable attention. In [6], [7], decentralized stabilization for linear time-invariant large-scale systems with time-delay was discussed. Large-scale linear time-varying systems with delayed state perturbations in the interconnections were considered in [8]. The authors in [9], [10] consider the decentralized stabilization for nonlinear large-scale systems including state delays in the interconnections, where the interconnections are assumed to be bounded by linear functions of states and/or delayed states with matching conditions. It seems that for uncertain large-scale time-delay interconnected systems, decentralized control results for strong interconnections beyond linear bounds have not been reported yet in the control literatures.

In this paper, we extend our earlier result in [11] to large-scale nonlinear systems with delayed state interconnections. The class of large-scale time-delay systems under consideration significantly broadens most existing classes

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of large-scale time-delay systems. Combining the recent development in decentralized nonlinear control, we consider a class of nonlinear systems in their decentralized diagonal form. We design decentralized memoryless state feedback controllers to solve the so-called "Problem of decentralized H_∞ almost disturbance decoupling". On the basis of dissipativity/Lyapunov-stability theory and Lyapunov-Krasovskii functional, we prove the asymptotical stability for the interconnected systems in the absence of disturbance; and in the case of disturbance attenuation, any given $\mathcal{L}_2$ gain from the disturbance to the system output is obtained. The notation used in this paper is standard and is the same as in [11].

II. PROBLEM FORMULATION

We consider in this paper a class of large-scale time-delay nonlinear systems S which are composed of single-input single-output (SISO) subsystems $S_i(1 \leq i \leq N)$ as described in (1), where $z_i(t) = [z_{i1}(t), \ldots, z_{i\kappa}(t)] \in \Re^\kappa$, $z(t) = [z_1^T(t), \ldots, z_N^T(t)]^T$, $\xi_i(t) = [\xi_{i1}(t), \ldots, \xi_{i,n-\kappa}(t)] \in \Re^{n-\kappa}$, and $\xi(t) = [\xi_1^T(t), \ldots, \xi_N^T(t)]^T$, $z(t)$ and $\xi(t)$ are the state vectors, $u_i(t) \in \Re$ is the control input, $\omega_i(t) \in \Re^{m_i}$ is the disturbance input, $y_i(t) \in \Re$ is the to-be-controlled output; the unknown functions γ_{il}, p_{il}, $(0 \leq l \leq n - \kappa)$ are locally Lipschitz in states and piecewise continuous in t, and $\gamma_{il}(t, 0, 0, \ldots, 0) = 0$; $h_{ij}(i, j = 1, 2, \ldots, N)$ are the time delays; A_i, B_i are constant matrices and

$$A_i = \begin{bmatrix} 0 & & \\ \vdots & I_{(\kappa-1) \times (\kappa-1)} \\ 0 & \\ 0 & 0 \ldots 0 \end{bmatrix}, B_i = \begin{bmatrix} 0_{(\kappa-1) \times 1} \\ 1 \end{bmatrix}. \quad (2)$$

The initial condition for z_i-subsystem with time delays is given by

$$z_i(t_0) = \chi_i(t_0), \qquad t_0 \in [-h_i, 0] \quad (3)$$

where $\chi_i(t_0)$ is a uniformly continuous function on $[-h_i, 0]$, and h_i is defined as: $h_i = \max\{h_{ij}, j = 1, 2, \ldots, N\}$.

Remark 1: The class of systems (1) has been motivated by several early papers in the nonlinear control literature and also exists in large power systems (see [12], [13]). Necessary and sufficient geometric conditions are given for the non-delay time-invariant decentralized strict feedback form in [14]. For systems having nonlinear zero dynamics, similar results in the argument of integral-input-to-state stability using output feedback were obtained in [15]. It

$$\dot{z}_i(t) = A_i z_i(t) + B_i \left[\xi_{i1}(t) + \gamma_{i0}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN})) + p_{i0}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}))\omega_i(t) \right]$$

$$\dot{\xi}_{i1}(t) = \xi_{i2}(t) + \gamma_{i1}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_{i1}(t)) + p_{i1}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_{i1}(t))\omega_i(t)$$

$$\dot{\xi}_{i2}(t) = \xi_{i3}(t) + \gamma_{i2}(t, z_1(t-h_{i1}), \ldots, z_N(t-h_{iN}), \xi_{i1}(t), \xi_{i2}(t)) + p_{i2}(t, z_1(t-h_{i1}), \ldots, z_N(t-h_{iN}), \xi_{i1}(t), \xi_{i2}(t))\omega_i(t)$$

$$\vdots \tag{1}$$

$$\dot{\xi}_{i,n-\kappa}(t) = u_i(t) + \gamma_{i,n-\kappa}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_i(t)) + p_{i,n-\kappa}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_i(t))\omega_i(t)$$

$$y_i(t) = z_{i1}(t)$$

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is worth noting that the structure of the interconnections are broadened in (1) by: 1) removing the strict matching conditions for all local and interconnecting uncertainties as in [16], [9], [10]; 2) removing the linear or polynomial-type growth conditions on the interconnections imposed in [14], [9], [10].

The following assumptions will be used to restrict our systems (1):

Assumption 1: There exist known smooth functions $a_{ilj}(\cdot), b_{ilj}(\cdot), \varphi_{ilj}(\cdot), \Phi_{ilj}(\cdot)$, with $\varphi_{ilj}(0) = 0$, $\Phi_{ilj}(0) = 0$, such that for each $0 \leq l \leq n - \kappa$ the uncertain interconnections satisfy

$$\begin{aligned} &|\gamma_{il}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_{i1}(t), \ldots, \xi_{il}(t)) \\ &- \gamma_{il}(t, 0, \ldots, 0, \xi_{i1}(t), \ldots, \xi_{il}(t))| \\ &\leq \sum_{j=1}^{N} a_{ilj}(\xi_{i1}(t), \ldots, \xi_{il}(t))\varphi_{ilj}(|z_j(t - h_{ij})|), \end{aligned} \tag{4}$$

$$\begin{aligned} &|p_{il}(t, z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}), \xi_{i1}(t), \ldots, \xi_{il}(t)) \\ &- p_{il}(t, 0, \ldots, 0, \xi_{i1}(t), \ldots, \xi_{il}(t))| \\ &\leq \sum_{j=1}^{N} b_{ilj}(\xi_{i1}(t), \ldots, \xi_{il}(t))\Phi_{ilj}(|z_j(t - h_{ij})|). \end{aligned} \tag{5}$$

For the sake of simplicity, we let $a_{i0j} = b_{i0j} = 1$.

Assumption 2: There exist known smooth functions $\psi_{il}(\cdot), \Psi_{il}(\cdot)$, with $\psi_{il}(0) = 0$, such that for each $0 \leq l \leq n - \kappa$,

$$\begin{aligned} &|\gamma_{il}(t, 0, \ldots, 0, \xi_{i1}(t), \ldots, \xi_{il}(t))| \\ &\leq \psi_{il}(|(\xi_{i1}(t), \ldots, \xi_{il}(t))|), \end{aligned} \tag{6}$$

$$\begin{aligned} &|p_{il}(t, 0, \ldots, 0, \xi_{i1}(t), \ldots, \xi_{il}(t))| \\ &\leq \Psi_{il}(|(\xi_{i1}(t), \ldots, \xi_{il}(t))|). \end{aligned} \tag{7}$$

Since $\gamma_{i0}(t, 0) = 0$, we can take $\psi_{i0} \equiv 0$; and Ψ_{i0} is a nonnegative constant.

We define our control problem as the following:

Problem of Decentralized H_∞ Almost Disturbance Decoupling: Find decentralized smooth memoryless state feedback controllers $u_i(t) = u_i(z_i(t), \xi_i(t))$ such that, for any given positive constant μ, the closed-loop interconnected system satisfies the following dissipation inequality

$$\int_0^T |y(t)|^2 dt \leq \mu \int_0^T |\omega(t)|^2 dt + \nu(\chi(t_0), \xi(0)),$$

$$\forall \omega \in \mathcal{L}_2(0, T), \ \forall T \geq 0 \tag{8}$$

where ν is a positive semidefinite function, $(\chi(t_0), \xi(0))$ denotes the initial conditions and

$$\chi(t_0) = [\chi_1^T(t_0), \chi_2^T(t_0), \ldots, \chi_N^T(t_0)]^T$$

with $\chi_i(t_0), i = 1, 2, \ldots, N$ are given in (3). Furthermore, the origin is globally uniformly asymptotically stable (GUAS) if $\omega = 0$.

III. DECENTRALIZED STATE FEEDBACK CONTROL DESIGN

In the following, for the ease of presentation, we denote the argument $(z_1(t - h_{i1}), \ldots, z_N(t - h_{iN}))$ as z^h. When omitting the arguments, z_i and ξ_i denote $z_i(t)$ and $\xi_i(t)$ respectively.

Step 0: We start by considering the $z_i(t)$-subsystem with $\xi_{i1}(t)$ as the virtual control input (for $i = 1, \ldots, N$). Choose the following Lyapunov-Krasovskii functional candidate:

$$V_0(z) = \sum_{i=1}^{N} \left\{ W_i(F_i(z_i)) + \sum_{j=1}^{N} \int_{t-h_{ij}}^{t} \phi_{ij}(|z_j(s)|)ds \right\} \tag{9}$$

where $W_i(\cdot), \phi_{ij}(\cdot)$ are both smooth $\mathcal{K}_\infty$ function to be chosen later, $F_i(z_i) = z_i^T P_i z_i$, and P_i is a positive definite symmetric matrix solving the algebric Riccati equation:

$$A_i^T P_i + P_i A_i - 2\varepsilon_i P_i B_i B_i^T P_i + Q_i = 0 \tag{10}$$

where ε_i is a positive constant and Q_i is a positive definite symmetric matrix.

Differentiating (9) along the solution of z_i-subsystem, and applying the interconnection bounds (4), (5) and (7), we have

$$\begin{aligned} \dot{V}_0 = &\sum_{i=1}^{N} \frac{\partial W_i}{\partial F_i} \left\{ 2z_i^T P_i \cdot \{A_i z_i + B_i [\xi_{i1} \right. \\ &+ (\gamma_{i0}(t, z^h) - \gamma_{i0}(t, 0)) \\ &+ (p_{i0}(t, z^h) - p_{i0}(t, 0))\omega_i + p_{i0}(t, 0)\omega_i]\} \\ &+ \left. \sum_{j=1}^{N} [\phi_{ij}(|z_j(t)|) - \phi_{ij}(|z_j(t - h_{ij})|)] \right\} \tag{11} \end{aligned}$$

Repeatedly using the inequality $2ab \leq a^2+b^2$, $(a,b \in \Re)$ for the third to fifth terms in the above equation, we obtain

$$\frac{\partial W_i}{\partial F_i} 2z_i^T P_i B_i(\gamma_{i0}(t,z^h) - \gamma_{i0}(t,0))$$

$$\leq \left|\frac{\partial W_i}{\partial F_i}\right|^2 \cdot |z_i^T P_i B_i|^2 \sum_{j=1}^{N} \delta_{j0}$$

$$+ \sum_{j=1}^{N} \delta_{j0}^{-1} \varphi_{i0j}^2(|z_j(t-h_{ij})|) \ , \qquad (12)$$

$$\frac{\partial W_i}{\partial F_i} 2z_i^T P_i B_i(p_{i0}(t,z^h) - p_{i0}(t,0))\omega_i$$

$$\leq \frac{1}{4\tau_i^2}\left|\frac{\partial W_i}{\partial F_i}\right|^4 \cdot |z_i^T P_i B_i|^4 \sum_{j=1}^{N} \varrho_{j0}$$

$$+ \sum_{j=1}^{N} \varrho_{j0}^{-1} \Phi_{i0j}(|z_j(t-h_{ij})|)^4 + N\tau_i|\omega_i|^2, \quad (13)$$

$$\frac{\partial W_i}{\partial F_i} 2z_i^T P_i B_i p_{i0}(t,0)\omega_i$$

$$\leq \frac{1}{d_{2i}}\left|\frac{\partial W_i}{\partial F_i}\right|^2 |z_i^T P_i B_i|^2 \Psi_{i0}^2 + d_{2i}|\omega_i|^2 \qquad (14)$$

where $d_{2i}, \delta_{j0}, \varrho_{i0}, \tau_i$ are any positive constants.

Choose our virtual control $\xi_{i1} = \xi_{i1}^*(z_i)$ as

$$\xi_{i1}^*(z_i) = -\left\{ \varepsilon_i B_i^T P_i z_i + \frac{1}{2}\frac{\partial W_i}{\partial F_i}(z_i^T P_i B_i)\left(\sum_{j=1}^{N}\delta_{j0}\right) \right.$$

$$+ \frac{1}{8\tau_i^2}\left(\frac{\partial W_i}{\partial F_i}\right)^3 (z_i^T P_i B_i)^3 \left(\sum_{j=1}^{N}\varrho_{j0}\right)$$

$$\left. + \frac{1}{2d_{2i}}\frac{\partial W_i}{\partial F_i}(z_i^T P_i B_i)\ \Psi_{i0}^2 \right\} \qquad (15)$$

Notice $\xi_{i1}^*(0) = 0$.

When substituting (15) into (14), we get:

$$\dot{V}_0 \leq \sum_{i=1}^{N}\left\{ \frac{\partial W_i}{\partial F_i}(-z_i^T Q_i z_i) + \sum_{j=1}^{N}[\phi_{ij}(|z_j|)] \right.$$

$$+ \sum_{j=1}^{N}\left[\delta_{i0}^{-1}\varphi_{i0j}^2(|z_j(t-h_{ij})|)\right.$$

$$\left.+ \varrho_{j0}^{-1}\Phi_{i0j}^4(|z_j(t-h_{ij})|) - \phi_{ij}(|z_j(t-h_{ij})|)\right]$$

$$\left. + (N\tau_i + d_{2i})|\omega_i|^2 \right\}$$

$$\triangleq \sum_{i=1}^{N}\left\{ -\alpha_{i0}(|z_i|) + \sum_{j=1}^{N}\beta_{i0j}(|z_j|) \right.$$

$$\left. + \sum_{j=1}^{N}\beta_{i0j}^h(|z_j(t-h_{ij})|) + \gamma_{i0}|\omega_i|^2 \right\} \qquad (16)$$

The choice of W_i, ϕ_{ij} are left till the final stage of the design problem. Note that the inequality (16) is only valid when $\xi_{i1} = \xi_{i1}^*$; otherwise, additional terms must be included. From here, we start our recursive backstepping procedure.

Step 1: Augment the z_i-subsystem with the ξ_{i1}-subsystem (for $i = 1, \ldots, N$), and choose a storage function as

$$V_1(z,\xi_1) = V_0(z) + \sum_{i=1}^{N}(\xi_{i1} - \xi_{i1}^*)^2 \qquad (17)$$

Denote

$$-\dot{\xi}_{i1}^* = -\frac{\partial \xi_{i1}^*}{\partial z_i}(A_i z_i + B_i \xi_{i1})$$

$$-\frac{\partial \xi_{i1}^*}{\partial z_i}B_i[\gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i]$$

$$\triangleq \vartheta_{i1}(z_i,\xi_{i1}) + \sigma_{i0}(z_i)[\gamma_{i0}(t,z^h) + p_{i0}(t,z^h)\omega_i]$$

Differentiating $V_1(z,\xi_1)$ along the solutions of the (z_i,ξ_{i1})-subsystem yields (18), where we denote $\widetilde{\xi}_{i1} = \xi_{i1} - \xi_{i1}^*(z_i)$ and $\sigma_{i1} = 1$.

Since $\xi_{i1}^*(z_i)$ is smooth and vanishes at the origin, there exists a smooth function $\widetilde{\zeta}_i(z_i)$ such that

$$|\xi_{i1}| \leq \widetilde{\zeta}_i(z_i) \cdot \left(|\widetilde{\xi}_{i1}| + |z_i|\right). \qquad (19)$$

Applying the inequalities in Assumptions 1 and 2, and after some calculations, we have:

$$2\widetilde{\xi}_{i1}\gamma_{i1}(t,0,\xi_{i1})$$

$$\leq 2|\widetilde{\xi}_{i1}|^2 \widetilde{\psi}_{i1}(z_i,\xi_{i1}) + \frac{1}{d_{1i}}|\widetilde{\xi}_{i1}|^2 \widetilde{\psi}_{i1}^2(z_i,\xi_{i1}) + d_{1i}|z_i|^2, \qquad (20)$$

$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}\ p_{ik}(t,0,\xi_{i1})\omega_i$$

$$\leq \frac{1}{d_{2i}}|\widetilde{\xi}_{i1}|^2\left[\sum_{k=0}^{1}\sigma_{ik}\Psi_{ik}(|\xi_{ik}|)\right]^2 + d_{2i}|\omega_i|^2 \ , \qquad (21)$$

$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}\ [\gamma_{ik}(t,z^h,\xi_{i1}) - \gamma_{ik}(t,0,\xi_{i1})]$$

$$\leq |\widetilde{\xi}_{i1}|^2 \sum_{k=0}^{1}|\sigma_{ik}|^2 \sum_{j=1}^{N}\delta_{jk}a_{ikj}^2(\xi_{i1})$$

$$+ \sum_{k=0}^{1}\sum_{j=1}^{N}\delta_{jk}^{-1}\varphi_{ikj}^2(|z_j(t-h_{ij})|) \ , \qquad (22)$$

$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}\ [p_{ik}(t,z^h,\xi_{i1}) - p_{ik}(t,0,\xi_{i1})]\omega_i$$

$$\leq \frac{1}{4\tau_i^2}|\widetilde{\xi}_{i1}|^4 \sum_{k=0}^{1}|\sigma_{ik}|^4 \sum_{j=1}^{N}\varrho_{jk}b_{ikj}^4(\xi_{i1})$$

$$+ \sum_{k=0}^{1}\sum_{j=1}^{N}\varrho_{jk}^{-1}\Phi_{ikj}^4(|z_j(t-h_{ij})|) + 2N\tau_i|\omega_i|^2, \quad (23)$$

$$\dot{V}_1 \le \sum_{i=1}^{N} \left\{ -\alpha_{i0}(|z_i|) + \sum_{j=1}^{N} \beta_{i0j}(|z_j|) + \sum_{j=1}^{N} \beta_{i0j}^{h}(|z_j(t-h_{ij})|) + \gamma_{i0}|\omega_i|^2 + 2\widetilde{\xi}_{i1} \left\{ \xi_{i2} + \left(\frac{\partial W_i}{\partial F_i} z_i^T P_i B_i + \vartheta_{i1} \right) \right. \right.$$

$$\left. \left. + \sum_{k=0}^{1} \sigma_{ik} \left[\gamma_{ik}(t, z^h, \xi_{i1}) - \gamma_{ik}(t, 0, \xi_{i1}) \right] + \sum_{k=0}^{1} \sigma_{ik} \left[p_{ik}(t, z^h, \xi_{i1}) - p_{ik}(t, 0, \xi_{i1}) \right] \omega_i + \gamma_{i1}(t, 0, \xi_{i1}) + \sum_{k=0}^{1} \sigma_{ik} p_{ik}(t, 0, \xi_{i1}) \omega_i \right\} \right\} \tag{18}$$

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where c_{i1} is a positive constant, $d_{1i}, d_{i2}, \delta_{ik}, \varrho_{ik}, c_{i1}$ are positive constants, and $\widetilde{\psi}_{i1}$ is a smooth nonnegative function.

Choose the virtual controller $\xi_{i2} = \xi_{i2}^*$ as in (24).

Substitute (20)-(23) into (18). After simplifications, we obtain

$$\dot{V}_1 \le \sum_{i=1}^{N} \left\{ -\alpha_{i1}(|z_i|) - C_{i11}\widetilde{\xi}_{i1}^2 + \sum_{j=1}^{N} \beta_{i0j}(|z_j|) \right.$$
$$\left. + \sum_{j=1}^{N} \beta_{i1j}^{h}(|z_j(t-h_{ij})|) + \gamma_{i1}|\omega_i|^2 \right\} \tag{25}$$

where $\alpha_{i1}(|z_i|), \beta_{i1j}^{h}(|z_j(t-h_{ij})|), \gamma_{i1}, C_{i11}$ are appropriate functions and constants.

Note that the inequality (25) is only valid when $\xi_{i2} = \xi_{i2}^*$, as in the standard backstepping design process.

From *Step* 2, we can establish the following Claim. The proof of the Claim is not difficult using the same approach as in [11], and is omitted here due to space limitations.

Claim: Given any index $2 \le \iota \le n - \kappa$, for the system $S_{i\iota}, i = 1, \ldots, N$

$$\dot{z}_i = A_i z_i + B_i \left[\xi_{i1} + \gamma_{i0}(t, z^h) + p_{i0}(t, z^h)\omega_i \right]$$
$$\dot{\xi}_{i1} = \xi_{i2} + \gamma_{i1}(t, z^h, \xi_{i1}) + p_{i1}(t, z^h, \xi_{i1})\omega_i$$
$$\vdots$$
$$\dot{\xi}_{i\iota} = \xi_{i,\iota+1} + \gamma_{i\iota}(t, z^h, \xi_{i1}, \ldots, \xi_{i\iota})$$
$$+ p_{i\iota}(t, z^h, \xi_{i1}, \ldots, \xi_{i\iota})\omega_i \tag{26}$$

there exist $\iota + 1$ smooth functions

$$\xi_{ik}^* = \xi_{ik}^*(z_i, \xi_{i1}, \ldots, \xi_{i,k-1}),$$

$$\xi_{ik}^*(0, 0, \ldots, 0) = 0, \ 1 \le k \le \iota + 1$$

such that in new coordinates

$$\widetilde{z}_i = z_i, \ \widetilde{\xi}_{ik} = \xi_{ik} - \xi_{ik}^*(z_i, \xi_{i1}, \ldots, \xi_{i,k-1}),$$

$$1 \le k \le \iota$$

the storage function

$$V_\iota = V_0 + \sum_{i=1}^{N} \sum_{k=1}^{\iota} \widetilde{\xi}_{ik}^2 \tag{27}$$

has time derivative, with $\xi_{i,\iota+1} = \xi_{i,\iota+1}^*$, satisfying the dissipation inequality

$$\dot{V}_\iota \le \sum_{i=1}^{N} \left\{ -\alpha_{i\iota}(|z_i|) - \sum_{k=1}^{\iota} C_{i\iota k}\widetilde{\xi}_{ik}^2 + \sum_{j=1}^{N} [\beta_{i0j}(|z_j|) \right.$$
$$\left. + \beta_{i\iota j}^{h}(|z_j(t-h_{ij})|)] + \gamma_{i\iota}|\omega_i|^2 \right\} \tag{28}$$

where

$$\alpha_{i\iota}(|z_i|) = \alpha_{i,\iota-1}(|z_i|) - d_{1i}|z_i|^2 , \tag{29}$$
$$\beta_{i\iota j}^{h}(|z_j(t-h_{ij})|) = \beta_{i,\iota-1,j}^{h}(|z_j(t-h_{ij})|)$$
$$+ \sum_{k=0}^{\iota} \left\{ \delta_{jk}^{-1} \varphi_{ikj}^2(|z_j(t-h_{ij})|) \right.$$
$$\left. + \varrho_{jk}^{-1} \Phi_{ikj}^4(|z_j(t-h_{ij})|) \right\} , \tag{30}$$
$$\gamma_{i\iota} = \gamma_{i,\iota-1} + (\iota+1)N\tau_i + d_{2i} , \tag{31}$$
$$C_{i\iota k} = \begin{cases} C_{i,\iota-1,k} - d_{3i} & \text{for} \quad k = 1, \ldots, \iota - 1 \\ c_{ik} & \text{for} \quad k = \iota \end{cases} \tag{32}$$

IV. MAIN RESULT

The following theorem states our main result:

Theorem 1: Under Assumptions 1 and 2, the Problem of Decentralized H_∞ Almost Disturbance Decoupling is solvable for system (1).

Proof: As shown by the Claim, at the last step, a decentralized memoryless state feedback control

$$u_i(t) = \xi_{i,n-\kappa+1}^*(z_i(t), \xi_i(t)) \tag{33}$$

is iteratively built. For the closed-loop system S, for $V = V_{n-\kappa}$, we have

$$\dot{V} \le \sum_{i=1}^{N} \left\{ -\alpha_i(|z_i|) - \sum_{l=1}^{n-\kappa} C_{il}\widetilde{\xi}_{il}^2 \right.$$
$$\left. + \sum_{j=1}^{N} \left[\beta_{i0j}(|z_j|) + \beta_{ij}^{h}(|z_j(t-h_{ij})|) \right] + \gamma_i|\omega_i|^2 \right\} \tag{34}$$

To eliminate the term $\beta_{ij}^{h}(|z_j(t-h_{ij})|)$, we choose the design function ϕ_{ij} in (9) as:

$$\phi_{ij}(|s|) = \sum_{l=0}^{n-\kappa} \left\{ (n-\kappa-l+1)\delta_{jl}^{-1}\varphi_{ilj}^2(|s|) \right.$$
$$\left. + (n-\kappa-l+1)\varrho_{jl}^{-1}\Phi_{ilj}^4(|s|) \right\} \tag{35}$$

$$\xi_{i2}^* = -\frac{1}{2}\left\{ c_{i1}\widetilde{\xi}_{i1} + \frac{\partial W_i}{\partial F_i}2z_i^T P_i B_i + 2\vartheta_{i1} + 2\widetilde{\xi}_{i1}\widetilde{\psi}_{i1}(z_i,\xi_{i1}) + \frac{1}{d_{1i}}\widetilde{\xi}_{i1}\widetilde{\psi}_{i1}^2(z_i,\xi_{i1}) \right.$$
$$\left. +\frac{1}{d_{2i}}\widetilde{\xi}_{i1}\left[\sum_{k=0}^{1}\sigma_{ik}\Psi_{i1}(|\xi_{ik}|)\right]^2 + \widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}^2\sum_{j=1}^{N}\delta_{jk}a_{ikj}^2(\xi_{i1}) + \frac{1}{4\tau_i^2}\widetilde{\xi}_{i1}^3\sum_{k=0}^{1}\sigma_{ik}^4\sum_{j=1}^{N}\varrho_{jk}b_{ikj}^4(\xi_{i1})\right\} \quad (24)$$

Since $\varphi_{ilj}(\cdot), \Phi_{ilj}(\cdot)$ are smooth functions and vanish at origin, there exists smooth nondecreasing functions $\widetilde{\varphi}_{jli}(|z_i|), \widetilde{\Phi}_{jli}(|z_i|)$ such that

$$\varphi_{jli}(|z_i|) \leq |z_i|\widetilde{\varphi}_{jli}(|z_i|) , \quad (36)$$
$$\Phi_{jli}(|z_i|) \leq |z_i|\widetilde{\Phi}_{jli}(|z_i|) . \quad (37)$$

Now we can choose our design function W_i in (15). Notice that

$$|z_i| \leq \sqrt{\frac{F_i(z_i)+1}{\lambda_{\min}(P_i)}} \triangleq \eta_i(F_i) \quad (38)$$

where the functions η_i are smooth positive functions.

Construct the derivative of W_i as

$$\frac{\partial W_i}{\partial F_i} = k_i + \frac{1}{\lambda_{\min}(Q_i)}\sum_{j=1}^{N}\sum_{l=0}^{n-\kappa}(n-\kappa-l+1)\delta_{il}^{-1}$$

$$\cdot [\widetilde{\varphi}_{jli}\circ\eta_i(F_i)]^2 + \frac{F_i}{\lambda_{\min}(Q_i)\cdot\lambda_{\min}(P_i)}$$

$$\cdot\sum_{j=1}^{N}\sum_{l=0}^{n-\kappa}(n-\kappa-l+1)\varrho_{il}^{-1}\left[\widetilde{\Phi}_{jli}\circ\eta_i(F_i)\right]^4 \quad (39)$$

where $k_i's$ are positive constants. It can be checked that by the above construction and $W_i(0) = 0$, W_i is a smooth $\mathcal{K}_\infty$ function.

In view of the above, we can check that

$$\sum_{i=1}^{N}\frac{\partial W_i}{\partial F_i}z_i^T Q_i z_i \geq \sum_{i=1}^{N}k_i\lambda_{\min}(Q_i)|z_i|^2$$
$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\beta_{i0j}(|z_j|) \quad (40)$$

Therefore, we obtain

$$\sum_{i=1}^{N}\left\{-\alpha_i(|z_i|) + \sum_{j=1}^{N}\beta_{i0j}(|z_j|)\right\} \leq \sum_{i=1}^{N}\left(-D_i|z_i|^2\right) \quad (41)$$

where $D_i = k_i\lambda_{\min}(Q_i) - (n-\kappa)d_{1i}$. Choose the design parameter d_{1i} as

$$d_{1i} < \frac{k_i\lambda_{\min}(Q_i)}{n-\kappa} \quad (42)$$

so that $D_i > 0$.

Choose the design parameter d_{3i} as

$$d_{3i} < \min_{1\leq l\leq n-\kappa-1}\left\{\frac{c_{il}}{n-\kappa-l}\right\} \quad (43)$$

so that $C_{il} > 0$.

Therefore we obtain:

$$\dot{V} \leq \sum_{i=1}^{N}\left\{-D_i|z_i|^2 - \sum_{l=1}^{n-\kappa}C_{il}\widetilde{\xi}_{il}^2 + \gamma_i|\omega_i|^2\right\}, \quad (44)$$

and it follows

$$\dot{V} \leq \sum_{i=1}^{N}\left\{-D_i|z_{i1}|^2 + \gamma_i|\omega_i|^2\right\}. \quad (45)$$

Taking the integral along time t, the $\mathcal{L}_2$ gain from ω to y of the closed-loop system is obtained as

$$\int_0^T |y(t)|^2 dt \leq \mu\int_0^T |w(t)|^2 dt + \nu(z(t_0),\xi(0)) \quad (46)$$

where

$$\mu = \min_{1\leq i\leq N}\{\gamma_i\}/\max_{1\leq i\leq N}\{D_i\},$$

$$\nu(z(t_0),\xi(0)) = \left\{\sum_{i=1}^{N}z_i(0)^T P_i z_i(0)\right.$$

$$\left.+\sum_{j=1}^{N}\int_{-h_{ij}}^{0}\phi_{ij}(|z_j(s)|)ds + \sum_{l=1}^{n-\kappa}\widetilde{\xi}_{il}^2(0)\right\}/\max_{1\leq i\leq N}\{D_i\}. \quad (47)$$

In the absence of the disturbance, i.e. when $\omega_i = 0$, from (44), we have

$$\dot{V} \leq \sum_{i=1}^{N}\left\{-D_i|z_i|^2 - \sum_{l=1}^{n-\kappa}C_{il}\widetilde{\xi}_{il}^2\right\} \triangleq -V_a(z,\xi) \leq 0 \quad (48)$$

From (9) and (27), $V(z,\xi)$ is a continuously differentiable, positive definite and radially unbounded function and its derivative is negative definite. From the Lyapunov stability theorems for equations with bounded delays ([1, page 103]), we obtain the GUAS of the closed-loop system when $\omega_i = 0$.

Remark 2: As an extension to the result in [11], the nett energy dissipation for the interconnected system S is obtained by constructing decentralized memoryless state feedback control laws to dominate other subsystem interactions by the subsystem stability margins. This is done by choosing the Lyapunov/storage function as the sum of subsystem Lyapunov-Krasovskii functionals. The novelty of handling nonlinear bounds of interconnections is the careful selection of the nonlinear storage function W_i, as adopted in [11], [16]. The results render the linear ([6], [8], [7]) and polynomial-type ([14]) growth conditions on the
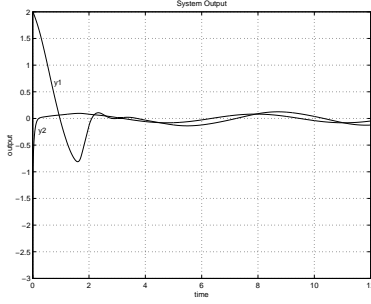
Fig. 1. Output of the closed-loop system

interconnections as special cases where W_i is chosen to be linear and higher-order polynomial respectively.

V. An Illustrative Example

We consider the following interconnected time-delay system composed of two subsystems:

$$S_1 : \begin{cases} \dot{z}_{11}(t) = z_{12}(t) \\ \dot{z}_{12}(t) = \xi_{11}(t) + \sin(2t)z_{21}(t-2.0) \\ \quad \cdot e^{(z_{21}(t-2.0))} + (1 + z_{11}(t-1.0)z_{21}(t))\omega_1(t) \\ \dot{\xi}_{11}(t) = u_1(t) + \xi_{11}(t)z_{21}(t)\omega_1(t) \\ y_1(t) = z_{11}(t) \end{cases}$$

$$S_2 : \begin{cases} \dot{z}_{21}(t) = \xi_{21}(t) + z_{11}(t)z_{12}(t-1.0)\sin(z_{21}(t)) \\ \quad + \omega_2(t) \\ \dot{\xi}_{21}(t) = u_2(t) + \omega_2(t) \\ y_2(t) = z_{21}(t) \end{cases}$$

The initial conditions for the two subsystems are as follows:

$$[z_{11}(t), z_{12}(t), \xi_{11}(t)]^T = [2.0, 0.5, 1.0]^T, \ t \in [-2.0, 0];$$
$$[z_{21}(t), \xi_{21}(t)]^T = [-3.0, 1.7]^T, \ t \in [-2.0, 0].$$

It can be checked that system S is in form (1) and satisfies Assumptions 1 and 2. We choose the design parameters $\varepsilon_1 = 1, \varepsilon_2 = 1, Q_1 = 2I, Q_2 = 2$ to get

$$P_1 = \begin{bmatrix} 2.8284 & 1.0000 \\ 1.0000 & 1.4142 \end{bmatrix}, \quad P_2 = 1.$$

Other parameters are chosen as follows, for $i, l = 1, 2$,

$$k_i = 8, \quad \delta_{il} = \varrho_{il} = 0.1, \quad \tau_i = 1, \quad c_i = 0.1, \quad d_{2i} = 2.$$

Construct the control law following that described in Steps 0 and 1 in Section 3. The system output is shown in Figure 1 which demonstrate a satisfactory disturbance attenuation performance of our closed-loop system. The disturbances applied are $\omega_1 = \omega_2 = \sin(t)$.

VI. Conclusions

We considered a class of large-scale nonlinear systems with delayed-state interconnections in this paper. The problem of decentralized H_∞ almost disturbance decoupling has been solved for such systems. Decentralized memoryless state feedback control laws are constructed iteratively applying backstepping. The results broaden the large-scale delay systems in previous literatures to nonlinear systems with uncertain interconnections to be time-varying and bounded by general nonlinear functions. Nonlinear Lyapunov-Krasovskii functionals are constructed to prove the internal global asymptotical stability in the sense of Lyapunov and external stability in the sense of $\mathcal{L}_2$ gain. Simulations of a two-subsystem example show a satisfactory performance. The results are expected to be applied to practical decentralized control problems of large-scale nonlinear systems with delayed state perturbations in the interconnections.

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