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# Coordinated Decentralized Adaptive Output Feedback Control of Interconnected Systems

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ABSTRACT

A decentralized adaptive output feedback control design is proposed for large-scale interconnected systems. It is assumed that all the controllers share prior information about the system reference models. A linearly parameterized neural network is introduced for each subsystem to partially cancel the effect of the interconnections on tracking performance. Boundedness of error signals is shown through Lyapunov's direct method.

## I. INTRODUCTION

With the advent of complex engineering systems, interest in design of decentralized controllers has especially increased. The problem can be briefly formulated as a control design for a system composed of several *dynamically interconnected* subsystems, such that the output of each subsystem has to track a prescpecified reference trajectory, while no communication is allowed between the controllers. The problem was first introduced in [1] for *weakly* interconnected subsystems having regulated outputs with relative degree 1 or 2. In [2] a framework for model reference adaptive control has been developed under restrictive assumptions, like positive definiteness of an M-matrix involving unknown constants, relative degrees of outputs being one or two, and matched uncertainties. These conditions were further relaxed in [3]–[10]. A detailed review of the cited literature one can find in [11].

Here we formulate and solve the problem of decentralized adaptive output feedback control for a class of nonlinear subsystems with known relative degrees, subject to unknown interconnections with known upper bound. We depart from attempting to obtain global results, and restrict the synthesis approach to a domain over which the interconnections and nonlinearities can be approximated by a linearly parameterized neural network. Similar attempts of incorporating neural networks into decentralized adaptive control have have been reported in [12], [13] for design of *state feedback* controllers. Following [14]–[16], we assume that the desired trajectories are known to all the controllers, i.e. the controllers share prior information about their goals, and we develop an *adaptive output feedback* synthesis approach that achieves ultimate boundedness of tracking errors. As in [14], [15], we will say that the controllers are engaged in *implicit cooperation*. While most of the existing results in decentralized control literature rely on the definition of a robust controller for dominating the interconnections, we show through Lyapunov's direct method that a linearly parameterized neural network, operating over reference model states, can *partially* cancel the interconnection effects. Ultimate boundedness of error signals is shown using Lyapunov's direct method. This paper should be viewed as the extension of adaptive output feedback control approach developed in [17] for centralized control to a decentralized setup, using the viewpoint of [14], [16] for definition of *implicit cooperation*.

The paper is organized as follows. In Section II we state the problem formulation and assumptions about the subsystem dynamics. In Section III we present the approach and define the error dynamics. In Section IV, we define the adaptive controller for each subsystem and derive associated bounds. Section V has a proof on ultimate boundedness of error signals of the large-scale system. In Section VI, we illustrate the theoretical results on non-minimum phase system like three inverted pendulums. Throughout the manuscript bold symbols are used for column vectors, small letters for scalars, capital letters for matrices, $||\cdot||$ denotes 2-norm unless otherwise noted.

## II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Let the large-scale system be composed of $m$ *stabilizable* nonlinear single-input single-output (SISO) subsystems, represented in the following normal form:

$$\begin{aligned}
\dot{\boldsymbol{x}}_i &= A_i\boldsymbol{x}_i + B_i\boldsymbol{z}_i + \boldsymbol{b}_i\left[u_i + f_i(\boldsymbol{x}_1, \boldsymbol{z}_1, \ldots, \boldsymbol{x}_m, \boldsymbol{z}_m)\right] \\
\dot{\boldsymbol{z}}_i &= C_i\boldsymbol{x}_i + D_i\boldsymbol{z}_i + \boldsymbol{g}_i(\boldsymbol{x}_1, \boldsymbol{z}_1, \ldots, \boldsymbol{x}_m, \boldsymbol{z}_m) \qquad (1) \\
y_i &= \boldsymbol{c}_i^T\boldsymbol{x}_i\,, i = 1, \cdots, m,
\end{aligned}$$

where $[\boldsymbol{x}_i^T\ \boldsymbol{z}_i^T]^T \in \mathbb{R}^{r_i+(n_i-r_i)}$ is the state vector of the realization of the $i^{th}$ subsystem in normal form, $r_i$ representing the relative degree, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the control and measurement of the $i^{th}$ subsystem, $f_i : \mathbb{R}^{n_1+\cdots+n_m} \rightarrow \mathbb{R}$ , $\boldsymbol{g}_i : \mathbb{R}^{n_1+\cdots+n_m} \rightarrow \mathbb{R}^{n_i-r_i}$ are sufficiently smooth

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unknown functions, representing the modeling errors and interconnection effects, while $A_i, B_i, \boldsymbol{b}_i, \boldsymbol{c}_i$ are matrices and vectors corresponding to the normal realization:

$$
\begin{aligned}
A_i &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{i_1} & a_{i_2} & \cdots & a_{i_{r_i}} \end{bmatrix}, \\
B_i &= \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & 0 \\ c_{i_1} & \cdots & c_{i_{n_i-r_i}} \end{bmatrix}
\end{aligned}
$$

and $\boldsymbol{b}_i = [\ 0 \quad \cdots \quad b_i\ ]^T,\ \boldsymbol{c}_i = [\ 1 \quad \cdots \quad 0\ ]^T$.

*Assumption 2.1:* The functions $\boldsymbol{g}_i(\boldsymbol{x}_1, \boldsymbol{z}_1, \ldots, \boldsymbol{x}_m, \boldsymbol{z}_m)$ are bounded for all $i = 1, \cdots, m$:

$$\|\boldsymbol{g}_i(\cdot)\| \leq \sum_{j=1}^{m} \alpha_i \left\| \left[\boldsymbol{x}_j^T \ \ \boldsymbol{z}_j^T\right]^T \right\|, \quad \alpha_i > 0 \tag{2}$$

The *objective* is to synthesize *decentralized adaptive output feedback* control laws $u_i$, such that $y_i(t)$ tracks a smooth *bounded* reference trajectory $y_{l_i}(t)$ with bounded errors for all $i = 1, \cdots, m$, under the assumption that the $i^{th}$ controller knows the *desired* states of all the subsystems $j = 1, \cdots, m$, while having access only to its own measurement $y_i(t)$.

As in [14]–[16], we introduce the following assumption.

*Assumption 2.2:* The signals $y_{l_i}(t)$ are assumed to be generated by the following ***stable*** linear closed-loop reference models

$$\dot{\boldsymbol{\xi}}_{l_i} = \bar{A}_i \boldsymbol{\xi}_{l_i} + \boldsymbol{b}_{r_i} y_{c_i}, \quad y_{l_i} = \bar{\boldsymbol{c}}_i^T \boldsymbol{\xi}_{l_i} \tag{3}$$

consisting of an open loop system

$$
\begin{aligned}
\dot{\boldsymbol{x}}_{l_i} &= A_i \boldsymbol{x}_{l_i} + B_i \boldsymbol{z}_{l_i} + \boldsymbol{b}_i u_{l_i} \\
\dot{\boldsymbol{z}}_{l_i} &= C_i \boldsymbol{x}_{l_i} + D_i \boldsymbol{z}_{l_i} \\
y_{l_i} &= \boldsymbol{c}_i^T \boldsymbol{x}_{l_i}, \quad i = 1, \cdots, m
\end{aligned} \tag{4}
$$

and a stabilizing dynamic compensator:

$$
\begin{aligned}
\dot{\boldsymbol{x}}_{c_i} &= A_{c_i} \boldsymbol{x}_{c_i} + \boldsymbol{b}_{c_i}(y_{c_i} - y_{l_i}) \\
u_{l_i} &= \boldsymbol{c}_{c_i}^T \boldsymbol{x}_{c_i} + d_{c_i}(y_{c_i} - y_{l_i}), \quad i = 1, \cdots, m,
\end{aligned} \tag{5}
$$

where

$$\bar{A}_i = \begin{bmatrix} A_i - \boldsymbol{b}_i d_{c_i} \boldsymbol{c}_i^T & B_i & \boldsymbol{b}_i \boldsymbol{c}_{c_i}^T \\ C_i & D_i & 0 \\ -\boldsymbol{b}_{c_i} \boldsymbol{c}_i^T & 0 & A_{c_i} \end{bmatrix} \tag{6}$$

$\boldsymbol{\xi}_{l_i} = [\ \boldsymbol{x}_{l_i}^T \quad \boldsymbol{z}_{l_i}^T \quad \boldsymbol{x}_{c_i}^T\ ]^T$, $\boldsymbol{b}_{r_i} = [\ \boldsymbol{b}_i d_{c_i}^T \quad \mathbf{0} \quad \boldsymbol{b}_{c_i}^T\ ]^T$, $\bar{\boldsymbol{c}}_i = [\ \boldsymbol{c}_i^T \quad \mathbf{0} \quad \mathbf{0}\ ]^T$, and $y_{c_i}$ is a bounded input of interest to track. The matrices $A_i, B_i, \boldsymbol{b}_i, \boldsymbol{c}_i$ are assumed to correspond to the normal realization, as defined in (2), so that $\dim \boldsymbol{x}_{l_i} = \dim \boldsymbol{x}_i$, and $\dim \boldsymbol{z}_{l_i} = \dim \boldsymbol{z}_i$. Notice that this choice of the open loop system in (4) implies that the relative degree of the $i^{th}$ open-loop reference model equals that of the $i^{th}$ open-loop subsystem. The following bounds are assumed to be known

$$\|[\ \boldsymbol{x}_{l_i}^T \quad \boldsymbol{z}_{l_i}^T\ ]^T\| \leq \beta_i, \quad i = 1, \cdots, m \tag{7}$$

*Remark 2.1:* In [14], this problem formulation has been addressed for linear subsystems, and, by a proper choice of robustifying signal, it has been shown that global asymptotic tracking can be achieved if the robustifying gain satisfies a lower bound, depending upon the number of subsystems and the apriori known bound on the interconnection effects. In [15], these results have been extended to nonlinear interconnections, modeled by *known* nonlinear functions. Moreover, output feedback has been formulated and solved for the case of subsystems having regulated outputs with relative degree 1. Our approach is different in two perspectives: i) we formulate the problem in output feedback *for arbitrary relative degree* by extending the results of [17] for centralized control, ii) we use an *adaptive* signal for overcoming the effect of interconnections on the tracking performance. On the other hand it should be understood that, due to results in [18], one cannot expect global results while using dynamic output feedback compensators with the class of nonlinear systems presented here.

## III. Controller Design, Error Dynamics

The control design for each of the subsystems will be based on the logic of combining a linear controller, that stabilizes the nominal linear model in the absence of interconnections, with a neural network (NN) that approximately cancels the interconnection effects in the controllable range. Towards this end, introduce the following control signal $u_i = u_{c_i} - u_{ad_i}$, where $u_{c_i}$ is the output of the following dynamic compensator

$$
\begin{aligned}
\dot{\boldsymbol{\eta}}_{c_i} &= A_{c_i} \boldsymbol{\eta}_{c_i} + \boldsymbol{b}_{c_i}(y_{c_i} - y_i) \\
u_{c_i} &= \boldsymbol{c}_{c_i}^T \boldsymbol{\eta}_{c_i} + d_{c_i}(y_{c_i} - y_i), \quad i = 1, \cdots, m,
\end{aligned} \tag{8}
$$

where $\boldsymbol{\eta}_{c_i} \in \mathbb{R}^{n_{c_i}}$, $A_{c_i}, \boldsymbol{b}_{c_i}, \boldsymbol{c}_{c_i}, d_{c_i}$ are introduced in (5), and the adaptive signal $u_{ad_i}$ will be defined later. This results in the following closed-loop subsystem dynamics:

$$\dot{\boldsymbol{\xi}}_i = \bar{A}_i \boldsymbol{\xi}_i + \boldsymbol{b}_{r_i} y_{c_i} - \bar{\boldsymbol{b}}_i(u_{ad_i} - f_i) + \bar{\boldsymbol{g}}_i \tag{9}$$

$$y_i = \bar{\boldsymbol{c}}_i^T \boldsymbol{\xi}_i, \qquad i = 1, \cdots, m, \tag{10}$$

where $\boldsymbol{\xi}_i = [\boldsymbol{x}_i^T \ \ \boldsymbol{z}_i^T \ \ \boldsymbol{\eta}_{c_i}^T]^T$, $\bar{\boldsymbol{b}}_i = [\boldsymbol{b}_i^T \ \ \mathbf{0} \ \ \mathbf{0}]^{\mathbf{T}}$, $\bar{\boldsymbol{g}}_{\mathbf{i}} = [\mathbf{0} \ \ \boldsymbol{g}_{\mathbf{i}}^{\mathbf{T}} \ \ \mathbf{0}]^{\mathbf{T}}$. Following [15], define the error vector $\boldsymbol{E}_i = \boldsymbol{\xi}_{l_i} - \boldsymbol{\xi}_i$, and write the tracking error dynamics for the $i^{th}$ subsystem:

$$\dot{\boldsymbol{E}}_i = \bar{A}_i \boldsymbol{E}_i + \bar{\boldsymbol{b}}_i(u_{ad_i} - f_i) - \bar{\boldsymbol{g}}_i, \ \bar{\boldsymbol{y}}_i = \bar{C}_i \boldsymbol{E}_i, \tag{11}$$

where $\bar{C}_i = \left[\ \bar{\boldsymbol{c}}_i^T \quad I\ \right]^T$ separates the signals available for feedback.

## IV. Neural Network Approximation of Nonlinearities and Adaptive Control

Following [19], given arbitrary $\epsilon^* > 0$ and a continuous function $\boldsymbol{f}(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}^m$, defined on a compact set $\boldsymbol{x} \in \mathcal{D} \subset \mathbb{R}^n$, there exists a set of bounded constant weights $W$, and a set of basis functions $\boldsymbol{\phi}(\boldsymbol{x})$, such that the following representation holds $\forall \boldsymbol{x} \in \mathcal{D}$: $\boldsymbol{f}(\boldsymbol{x}) = W^T\boldsymbol{\phi}(\boldsymbol{x}) + \boldsymbol{\epsilon}(\boldsymbol{x}),\ \|\boldsymbol{\epsilon}(\boldsymbol{x})\| < \epsilon^*$. Thus, one can model the interconnections

$$f_i(\boldsymbol{x}_1, \boldsymbol{z}_1, \cdots, \boldsymbol{x}_m, \boldsymbol{z}_m) = \boldsymbol{W}_i^T\boldsymbol{\phi}_i(\boldsymbol{Y}) + \epsilon_i(\boldsymbol{Y}) \tag{12}$$

using the following input vector $\boldsymbol{Y} = [\boldsymbol{x}_1^T\ \boldsymbol{z}_1^T\ \cdots\ \boldsymbol{x}_m^T\ \boldsymbol{z}_m^T]^T \in \mathcal{D} \subset \mathbb{R}^{n_1+\cdots+n_m}$ and a vector of the radial basis functions $\boldsymbol{\phi}_i(\boldsymbol{Y}) = [\phi_{i_1}(\boldsymbol{Y})\ \cdots\ \phi_{i_{N_i}}(\boldsymbol{Y})]^T$, where $N_i$ is the number of basis functions to be used by the $i^{th}$ subsystem, $\phi_{i_k}(\boldsymbol{Y}) = e^{-\|\boldsymbol{Y}-\boldsymbol{Y}_{i_{c_k}}\|^2/2\sigma_{i_k}}$, $\boldsymbol{Y}_{i_{c_k}}$ is the vector of centers of the basis functions used by the $i^{th}$ subsystem, having the same dimension as $\boldsymbol{Y}$, $\sigma_{i_k}$ specifies the width of the $k^{th}$ basis function in the $i^{th}$ subsystem, and $|\epsilon_i| < \epsilon_i^*,\ \|\boldsymbol{W}_i\| \leq W_i^*$. Since our interest is in decentralized design, the states of other subsystems are not available to individual controllers, therefore the input vector $\boldsymbol{Y}$ cannot be used in designing adaptive elements. Based on the assumption that the controllers share prior information about their reference models, the adaptive control signal for the $i^{th}$ subsystem can be designed following the same logic as in [15],

$$u_{ad_i} = \hat{\boldsymbol{W}}_i^T\boldsymbol{\phi}_i(\boldsymbol{Y}_l) \tag{13}$$

where the vector $\boldsymbol{Y}_l$ is defined as $\boldsymbol{Y}_l = [\boldsymbol{x}_{l_1}^T\ \boldsymbol{z}_{l_1}^T\ \cdots\ \boldsymbol{x}_{l_m}^T\ \boldsymbol{z}_{l_m}^T]^T$ having the states of all the subsystems replaced by their corresponding reference states when compared to $\boldsymbol{Y}$ . Notice that due to boundedness of reference model states there exists a set $\mathcal{D}_l$ in the extended space such that $\boldsymbol{Y}_l \in \mathcal{D}_l$. The adaptive laws for $\hat{\boldsymbol{W}}_i$ are similar to those in [20]:

$$\dot{\hat{\boldsymbol{W}}}_i = -F_i[2\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\hat{\boldsymbol{E}}_i^T P_i\bar{\boldsymbol{b}}_i + k_i\hat{\boldsymbol{W}}_i] \tag{14}$$

in which $P_i$ is the solution of the Lyapunov equation $\bar{A}_i^T P_i + P_i\bar{A}_i = -Q_i$ for some $Q_i > 0$, and $F_i, k_i > 0$ are adaptation gains, while $\hat{\boldsymbol{E}}_i$ propagates according to the following dynamics:

$$\dot{\hat{\boldsymbol{E}}}_i = \bar{A}_i\hat{\boldsymbol{E}}_i + K_i\left(\bar{\boldsymbol{y}}_i - \hat{\boldsymbol{y}}_i\right),\ \hat{\boldsymbol{y}}_i = \bar{C}_i\hat{\boldsymbol{E}}_i, \tag{15}$$

where $K_i$ is a gain matrix, and should be chosen such that $\bar{A}_i - K_i\bar{C}_i$ is asymptotically stable, while $i = 1, \cdots, m$. Let $\tilde{A}_i = \bar{A}_i - K_i\bar{C}_i, \quad \tilde{\boldsymbol{E}}_i = \boldsymbol{E}_i - \hat{\boldsymbol{E}}_i, \quad i = 1, \cdots, m$. Then

$$\dot{\tilde{\boldsymbol{E}}}_i = \tilde{A}_i\tilde{\boldsymbol{E}}_i + \bar{\boldsymbol{b}}_i(u_{ad_i} - f_i) - \bar{\boldsymbol{g}}_i, \quad i = 1, \cdots, m \tag{16}$$

We immediately note that for arbitrary positive definite $\tilde{Q}_i > 0$ there exists a unique solution $\tilde{P}_i = \tilde{P}_i^T > 0$ such that $\tilde{A}_i^T\tilde{P}_i + \tilde{P}_i\tilde{A}_i = -\tilde{Q}_i$. The error dynamics in (11) can be expressed as:

$$\begin{aligned} \dot{\boldsymbol{E}}_i &= \bar{A}_i\boldsymbol{E}_i + \bar{\boldsymbol{b}}_i\left[\hat{\boldsymbol{W}}_i^T\boldsymbol{\phi}_i(\boldsymbol{Y}_l) - \boldsymbol{W}_i^T\boldsymbol{\phi}_i(\boldsymbol{Y}) - \epsilon_i\right] - \bar{\boldsymbol{g}}_i \\ \bar{\boldsymbol{y}}_i &= \bar{C}_i\boldsymbol{E}_i. \end{aligned} \tag{17}$$

Through several algebraic manipulations and, using the mean value theorem, one can obtain

$$u_{ad_i} - f_i = \tilde{\boldsymbol{W}}_i^T\boldsymbol{\phi}_i(\boldsymbol{Y}_l) + \boldsymbol{W}_i^T\boldsymbol{\phi}_i'(\boldsymbol{Y}^*)\tilde{\boldsymbol{Y}} - \epsilon_i \tag{18}$$

where $\tilde{\boldsymbol{Y}} = [\tilde{\boldsymbol{x}}_1^T\ \tilde{\boldsymbol{z}}_1^T\ \cdots\ \tilde{\boldsymbol{x}}_m^T\ \tilde{\boldsymbol{z}}_m^T]^T$ is comprised of the tracking errors of all the subsystems ($\tilde{\boldsymbol{x}}_i = \boldsymbol{x}_{l_i} - \boldsymbol{x}_i,\ \tilde{\boldsymbol{z}}_i = \boldsymbol{z}_{l_i} - \boldsymbol{z}_i$), $\boldsymbol{\phi}_i'(\boldsymbol{Y}^*)$ is the bounded derivative of the basis function in an intermediate point $\boldsymbol{Y}^* = \boldsymbol{Y}_l + (1-\lambda)\boldsymbol{Y}$, $0 < \lambda < 1$, and $\tilde{\boldsymbol{W}}_i \triangleq \hat{\boldsymbol{W}}_i - \boldsymbol{W}_i$ is the parameter error vector. From the definition of $\boldsymbol{E}_j$ and $\tilde{\boldsymbol{Y}}$ it follows that

$$\|\tilde{\boldsymbol{Y}}\| \leq \sum_{j=1}^{j=m}\|\boldsymbol{E}_j\| \tag{19}$$

## V. Stability Analysis

In this section we show through Lyapunov's direct method that the error signals $\boldsymbol{E}_i, \tilde{\boldsymbol{E}}_i, \tilde{\boldsymbol{W}}_i$, $i = 1, \cdots, m$, are ultimately bounded. To this end, introduce the composite error vector $\boldsymbol{\zeta} \triangleq \left[\boldsymbol{E}_1^T\ \cdots\ \boldsymbol{E}_m^T\ \tilde{\boldsymbol{E}}_1^T\ \cdots\ \tilde{\boldsymbol{E}}_m^T\ \tilde{\boldsymbol{W}}_1^T\ \cdots\ \tilde{\boldsymbol{W}}_m^T\right]^T \in \mathbb{R}^{2(n_1+\cdots+n_m)} \times \mathbb{R}^{N_1+\cdots+N_m}$, and consider the following positive definite function $V(\boldsymbol{\zeta}) \triangleq \boldsymbol{\zeta}^T T\boldsymbol{\zeta}$, where $T = blockdiag[P_1\ \cdots\ P_m\ \tilde{P}_1\ \cdots\ \tilde{P}_m\ \frac{1}{2}F_1^{-1}\ \cdots\ \frac{1}{2}F_m^{-1}]$. Further, notice that the RBF network approximation can be defined over arbitrarily large compact set $\mathcal{D}$. Based on the definition of the compact set $\mathcal{D}_l$, and the boundedness of $\boldsymbol{x}_{c_i}$ and $\boldsymbol{\eta}_{c_i}$, in the subspace of the error variables consider the following compact set $\Omega_{\boldsymbol{E}}$ of possible initial errors: $\Omega_{\boldsymbol{E}} = \Big\{[\ \boldsymbol{E}_1^T\ \ \cdots\ \ \boldsymbol{E}_m^T\ ]^T \in \mathbb{R}^{n_1+\cdots+n_m}\ :\ \boldsymbol{Y} \in \mathcal{D},\ \boldsymbol{Y}_l \in \mathcal{D}_l,\ \boldsymbol{x}_c \in \mathcal{D}_{\boldsymbol{x}_c},\ \boldsymbol{\eta}_c \in \mathcal{D}_{\boldsymbol{\eta}_c}\Big\}$. In the expanded space of the error variable $\boldsymbol{\zeta} \in \mathbb{R}^{2(n_1+\cdots+n_m)} \times \mathbb{R}^{N_1+\cdots+N_m}$, consider the largest level set of

$$V(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^T T\boldsymbol{\zeta} \tag{20}$$

corresponding to $[\ \boldsymbol{E}_1^T\ \ \cdots\ \ \boldsymbol{E}_m^T\ ]^T \in \Omega_{\boldsymbol{E}}$ and introduce the largest ball that lies inside this level set:

$$\mathcal{B}_R = \{\boldsymbol{\zeta}\ |\ \|\boldsymbol{\zeta}\| \leq R\} \tag{21}$$

Let $\alpha$ be the minimum value of $V(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^T T\boldsymbol{\zeta}$ on the boundary of $\mathcal{B}_R$:

$$\alpha \triangleq \min_{\|\boldsymbol{\zeta}\|=R} V(\boldsymbol{\zeta}) = R^2\lambda_{\min}(T) \tag{22}$$

where $\lambda_{\min}(T)$ is introduced for the minimum eigenvalue of $T$. Introduce the set

$$\Omega_\alpha \triangleq \{\boldsymbol{\zeta} \in \mathcal{B}_R\ |\ V(\boldsymbol{\zeta}) \leq \alpha\} \tag{23}$$

*Assumption 5.1:* Let

$$R > \gamma\sqrt{\lambda_{\max}(T)/\lambda_{\min}(T)} \tag{24}$$

where $\lambda_{\max}(T)$ is introduced for the maximum eigenvalue of $T$, while $\gamma = \max\left(\sqrt{\frac{\omega}{\lambda_{\min}(D)}}, \sqrt{\frac{\omega}{\lambda_{\min}(\tilde{D})}}, \sqrt{\frac{\omega}{\lambda_{\min}(\Lambda)}}\right)$, in which $D \triangleq \mathrm{diag}\left[\begin{array}{ccc}\theta_1 & \cdots & \theta_m\end{array}\right]$, $\theta_i = \lambda_{\min}(Q_i) - 2m\alpha_i\lambda_{\max}(P_i) - (m\phi_i^* + 1)\|P_i\bar{\boldsymbol{b}}_i\| - \sum_{j=1}^{j=m}(2\|P_j\bar{\boldsymbol{b}}_j\| + \Theta_j)\phi_j^* - \alpha_i\left(\lambda_{\max}(P_i) + \lambda_{\max}(\tilde{P}_i)\right) > 0$, $\tilde{D} \triangleq \mathrm{diag}\left[\begin{array}{ccc}\tilde{\theta}_1 & \cdots & \tilde{\theta}_m\end{array}\right]$, $\tilde{\theta}_i = \lambda_{\min}(\tilde{Q}_i) - 2m\alpha_i\lambda_{\max}(\tilde{P}_i) - (m\phi_i^* + 1)\left(\|P_i\bar{\boldsymbol{b}}_i\| + \mu_i\right) - \mu_i\|\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\| > 0$, $\Lambda \triangleq \mathrm{diag}\left[\begin{array}{ccc}\lambda_1 & \cdots & \lambda_m\end{array}\right]$, $\lambda_i = \frac{k_i}{2} - \mu_i\|\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\| > 0$, $\omega \triangleq \sum_{i=1}^{m}(\frac{k_i}{2}(W_i^*)^2 + (2\|P_i\bar{\boldsymbol{b}}_i\| + \mu_i)(\epsilon_i^*)^2 + \alpha_i(\lambda_{\max}(P_i) + \lambda_{\max}(\tilde{P}_i))\sum_{j=1}^{j=m}\beta_j^2)$, $\phi_i^* \triangleq W_i^*\|\boldsymbol{\phi}_i'(\boldsymbol{Y}^*)\|$, $\mu_i \triangleq \|\tilde{P}_i\bar{\boldsymbol{b}}_i + P_i\bar{\boldsymbol{b}}_i\|$.

*Theorem 5.1:* Let Assumptions 2.1, 2.2 and 5.1 hold. If the initial errors lie in $\Omega_\alpha$, defined in (23), then all the signals $\boldsymbol{E}_i, \tilde{\boldsymbol{E}}_i, \tilde{\boldsymbol{W}}_i, i = 1, \cdots, m$, in the closed loop system are ultimately bounded.

**Proof.** Consider the following Lyapunov function candidate for each of the subsystems:

$$V_i(\boldsymbol{E}_i, \tilde{\boldsymbol{E}}_i, \tilde{\boldsymbol{W}}_i) = \boldsymbol{E}_i^T P_i \boldsymbol{E}_i + \tilde{\boldsymbol{E}}_i^T \tilde{P}_i \tilde{\boldsymbol{E}}_i + \frac{1}{2}\tilde{\boldsymbol{W}}_i^T F_i^{-1}\tilde{\boldsymbol{W}}_i$$

Substituting the adaptive laws from (14) implies:

$$\begin{aligned}\dot{V}_i \quad = \quad & -\boldsymbol{E}_i^T Q_i \boldsymbol{E}_i - \tilde{\boldsymbol{E}}_i^T \tilde{Q}_i \tilde{\boldsymbol{E}}_i \\ & +2\hat{\boldsymbol{E}}_i^T P_i \bar{\boldsymbol{b}}_i\Big[\boldsymbol{W}_i^T \boldsymbol{\phi}_i'(\boldsymbol{Y}^*)\tilde{\boldsymbol{Y}} - \epsilon_i\Big] + 2\boldsymbol{E}_i^T P_i \bar{\boldsymbol{g}}_i \\ & +2\tilde{\boldsymbol{E}}_i^T(\tilde{P}_i\bar{\boldsymbol{b}}_i + P_i\bar{\boldsymbol{b}}_i)\Big[\tilde{\boldsymbol{W}}_i^T \boldsymbol{\phi}_i(\boldsymbol{Y}_l) \\ & +\boldsymbol{W}_i^T \boldsymbol{\phi}_i'(\boldsymbol{Y}^*)\tilde{\boldsymbol{Y}} - \epsilon_i\Big] + 2\tilde{\boldsymbol{E}}_i^T \tilde{P}_i \bar{\boldsymbol{g}}_i - k_i\Big[\tilde{\boldsymbol{W}}_i\hat{\boldsymbol{W}}_i\Big]\end{aligned}$$

Notice that using the bound in (7), the upperbound in (2) can be represented:

$$\|\boldsymbol{g}_i\| \leq \sum_{j=1}^{j=m}\alpha_i[\|\boldsymbol{E}_j\| + \beta_j] \tag{25}$$

Then using (19), the following upper bound can be derived:

$$\begin{gathered}\dot{V}_i \leq -\lambda_{\min}(Q_i)\|\boldsymbol{E}_i\|^2 - \lambda_{\min}(\tilde{Q}_i)\|\tilde{\boldsymbol{E}}_i\|^2 \\ +2\left(\|\tilde{\boldsymbol{E}}_i\| + \|\boldsymbol{E}_i\|\right)\|P_i\bar{\boldsymbol{b}}_i\|\left[\phi_i^*\sum_{j=1}^{j=m}\|\boldsymbol{E}_j\| + \epsilon_i^*\right] \\ +2\alpha_i\lambda_{\max}(P_i)\|\boldsymbol{E}_i\|\sum_{j=1}^{j=m}[\|\boldsymbol{E}_j\| + \beta_j] \\ +2\|\tilde{\boldsymbol{E}}_i\|\mu_i\left[\|\tilde{\boldsymbol{W}}_i\|\|\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\| + \phi_i^*\sum_{j=1}^{j=m}\|\boldsymbol{E}_j\| + \epsilon_i^*\right] \\ +2\alpha_i\lambda_{\max}(\tilde{P}_i)\|\tilde{\boldsymbol{E}}_i\|\sum_{j=1}^{j=m}[\|\boldsymbol{E}_j\| + \beta_j] - k_i\left[\tilde{\boldsymbol{W}}_i\hat{\boldsymbol{W}}_i\right]\end{gathered}$$

Completing the squares twice and regroupong, the following upper bound can be derived:

$$\begin{gathered}\dot{V}_i \leq -\Big(\lambda_{\min}(Q_i) - 2m\alpha_i\lambda_{\max}(P_i) - (m\phi_i^* \\ +1)\|P_i\bar{\boldsymbol{b}}_i\|\Big)\|\boldsymbol{E}_i\|^2 - \Big(\lambda_{\min}(\tilde{Q}_i) - 2m\alpha_i\lambda_{\max}(\tilde{P}_i) \\ -(m\phi_i^* + 1)\Big(\|P_i\bar{\boldsymbol{b}}_i\| + \mu_i\Big) - \mu_i\|\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\|\Big)\|\tilde{\boldsymbol{E}}_i\|^2 \\ -\left(\tfrac{k_i}{2} - \mu_i\|\boldsymbol{\phi}_i(\boldsymbol{Y}_l)\|\right)\|\tilde{\boldsymbol{W}}_i\|^2 + \sum_{j=1}^{j=m}\Big[(2\|P_i\bar{\boldsymbol{b}}_i\| \\ +\mu_i)\phi_i^* + \alpha_i\left(\lambda_{\max}(P_i) + \lambda_{\max}(\tilde{P}_i)\right)\Big]\|\boldsymbol{E}_j\|^2 \\ +\tfrac{k_i}{2}(W_i^*)^2 + \left(2\|P_i\bar{\boldsymbol{b}}_i\| + \mu_i\right)(\epsilon_i^*)^2 \\ +\alpha_i\Big(\lambda_{\max}(P_i) + \lambda_{\max}(\tilde{P}_i)\Big)\sum_{j=1}^{j=m}\beta_j^2\end{gathered}$$

Now introduce the following Lyapunov function for the whole system:

$$\dot{V} \quad = \quad \sum_{i=1}^{m}\dot{V}_i \tag{26}$$

Then, using the notations from Assumption 5.1, the upper bound reduces to:

$$\begin{gathered}\dot{V} = \sum_{i=1}^{m}\dot{V}_i \leq \\ \sum_{i=1}^{m}\Big[-\Big(\lambda_{\min}(Q_i) - 2m\alpha_i\lambda_{\max}(P_i) \\ -(m\phi_i^* + 1)\|P_i\bar{\boldsymbol{b}}_i\|\Big)\|\boldsymbol{E}_i\|^2 - \tilde{\theta}_i\|\tilde{\boldsymbol{E}}_i\|^2 \\ -\lambda_i\|\tilde{\boldsymbol{W}}_i\|^2 + \sum_{j=1}^{j=m}\Big[(2\|P_i\bar{\boldsymbol{b}}_i\| + \mu_i)\phi_i^* \\ +\alpha_i\left(\lambda_{\max}(P_i) + \lambda_{\max}(\tilde{P}_i)\right)\Big]\|\boldsymbol{E}_j\|^2\Big] + \omega\end{gathered}$$

Regrouping, this can be written:

$$\begin{aligned}\dot{V} \quad = \quad & \sum_{i=1}^{m}\dot{V}_i \leq \\ & \sum_{i=1}^{m}\Big[-\theta_i\|\boldsymbol{E}_i\|^2 - \tilde{\theta}_i\|\tilde{\boldsymbol{E}}_i\|^2 - \lambda_i\|\tilde{\boldsymbol{W}}_i\|^2\Big] + \omega\end{aligned} \tag{27}$$

Following an argument similar to that in [12], define the vectors $\boldsymbol{E} \triangleq \left[\begin{array}{ccc}\|\boldsymbol{E}_1\| & \cdots & \|\boldsymbol{E}_m\|\end{array}\right]^T$, $\tilde{\boldsymbol{E}} \triangleq \left[\begin{array}{ccc}\|\tilde{\boldsymbol{E}}_1\| & \cdots & \|\tilde{\boldsymbol{E}}_m\|\end{array}\right]^T$, $\tilde{\boldsymbol{W}} \triangleq \left[\begin{array}{ccc}\|\tilde{\boldsymbol{W}}_1\| & \cdots & \|\tilde{\boldsymbol{W}}_m\|\end{array}\right]^T$. Then the expression in (27) can be put into the following form:

$$\dot{V} \quad \leq \quad -\boldsymbol{E}^T D\boldsymbol{E} - \tilde{\boldsymbol{E}}^T\tilde{D}\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{W}}^T\Lambda\tilde{\boldsymbol{W}} + \omega$$

The following upper bound

$$\begin{gathered}\dot{V} \leq -\lambda_{\min}(D)\|\boldsymbol{E}\|^2 - \lambda_{\min}(\tilde{D})\|\tilde{\boldsymbol{E}}\|^2 \\ -\lambda_{\min}(\Lambda)\|\tilde{\boldsymbol{W}}\|^2 + \omega\end{gathered}$$

implies that either of the following conditions

$$\begin{gathered}\|\boldsymbol{E}\| > \sqrt{\frac{\omega}{\lambda_{\min}(D)}} \\ \left\|\tilde{\boldsymbol{E}}\right\| > \sqrt{\frac{\omega}{\lambda_{\min}(\tilde{D})}} \\ \|\tilde{\boldsymbol{W}}\| > \sqrt{\frac{\omega}{\lambda_{\min}(\Lambda)}}\end{gathered}$$

will render $\dot{V} < 0$ outside the compact set

$$\mathcal{B}_\gamma = \{\boldsymbol{\zeta} \mid \|\boldsymbol{\zeta}\| \leq \gamma\} \tag{28}$$

Let $\Gamma$ be the maximum value of the function $V(\boldsymbol{\zeta})$ on the boundary of $\mathcal{B}_\gamma$:

$$\Gamma \triangleq \max_{\|\boldsymbol{\zeta}\|=\gamma} V = \gamma^2\lambda_{\max}(T) \tag{29}$$

Assumption 5.1 ensures

$$\Omega_\gamma \triangleq \{\boldsymbol{\zeta} | \quad V = \Gamma\} \subset \Omega_\alpha \tag{30}$$

Thus, if the initial error $\boldsymbol{\zeta}_0 = \boldsymbol{\zeta}(0)$ belongs to $\Omega_\alpha$, then there exists a time instant $t_{\boldsymbol{\zeta}}(\boldsymbol{\zeta}_0)$, such that $\boldsymbol{\zeta}(t)$ will enter the set $\Omega_\gamma$ at $t_{\boldsymbol{\zeta}}$ and remain inside it for all $t > t_{\boldsymbol{\zeta}}$. This implies ultimate boundedness of $\boldsymbol{\zeta}$, completing the proof.

*Remark 5.1:* The results obtained above can be extended to the case where the modeling errors also depend upon the control signal, i.e. in (1) one can have $f_i(u_i, \boldsymbol{x}_1, \boldsymbol{z}_1, \ldots, \boldsymbol{x}_m, \boldsymbol{z}_m)$, subject to $\partial f_i/\partial u_i \neq 0$. Notice then that the adaptive signal will be introduced to cancel a function $f_i(u_i(u_{ad_i}(\cdot), \cdot)$ of itself. To avoid this algebraic loop, one way of implementing this is to use a one step delayed value of the control signal $u_i(t-d)$, where $d > 0$ is sufficiently small.

*Remark 5.2:* Assumption 5.1 may be interpreted as placing both upper and lower bounds on the adaptation gains. Let $\bar{\gamma} \triangleq \max(\lambda_{\max}(F_i))$, $\underline{\gamma} \triangleq \min(\lambda_{\min}(F_i))$, $\bar{\lambda} \triangleq \max(\lambda_{\max}(P_i), \lambda_{\max}(\tilde{P}_i))$, $\underline{\lambda} \triangleq \min(\lambda_{\min}(P_i), \lambda_{\min}(\tilde{P}_i))$, $i = 1, \cdots, m$. Then an upper bound for the adaptation gains results when $2\bar{\lambda}\underline{\gamma} > 1$ and $2\underline{\lambda}\bar{\gamma} > 1$, for which the relation in (24) reduces to $\bar{\gamma} < R^2/(2\gamma^2\bar{\lambda})$. A lower bound for the adaptation gains results when $2\bar{\lambda}\underline{\gamma} < 1$ and $2\underline{\lambda}\bar{\gamma} < 1$, for which the relation in (24) reduces to $\underline{\gamma} > \gamma^2/(2R^2\underline{\lambda})$. Notice that the upper bound for the adaptation gain has $R$ in the numerator, while the lower bound has $R$ in the denominator. Therefore, $R$ can be selected sufficiently large to ensure that $\underline{\gamma} < \bar{\gamma}$.

## VI. SIMULATIONS

We consider three inverted pendulums mounted on carts, as depicted in Figure 1. The carts are connected by springs and dampers. In each subsystem, we assume that the position of the cart($x_i$) and the angle of the pendulum($\theta_i$) are measured and the cart is regulated by input forces($u_i$). The equations of motion for the system are described as follows:

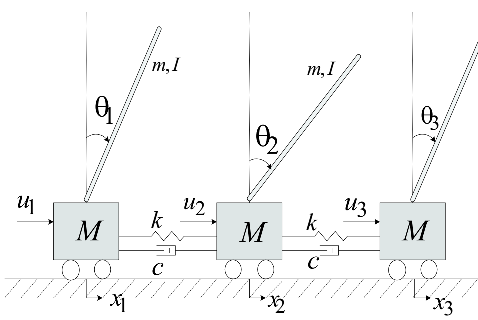



Fig. 1. Three inverted pendulums on three carts

$$(M+m)\ddot{x}_1 + ml_p\ddot{\theta}_1\cos\theta_1 - ml_p\dot{\theta}_1^2\sin\theta_1 = u_1 + s_1$$

$$ml_p\cos\theta_1\ddot{x}_1 + (I + ml_p^2)\ddot{\theta}_1 - mgl_p\sin\theta_1 = 0$$

$$(M+m)\ddot{x}_2 + ml_p\ddot{\theta}_2\cos\theta_2 - ml_p\dot{\theta}_2^2\sin\theta_2 = u_2 - s_1 + s_2$$

$$ml_p\cos\theta_2\ddot{x}_2 + (I + ml_p^2)\ddot{\theta}_2 - mgl_p\sin\theta_2 = 0$$

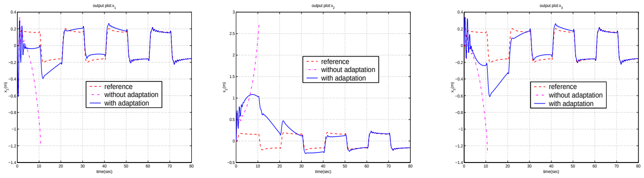
$$(M+m)\ddot{x}_3 + ml_p\ddot{\theta}_3\cos\theta_3 - ml_p\dot{\theta}_3^2\sin\theta_3 = u_3 - s_2$$

$$ml_p\cos\theta_3\ddot{x}_3 + (I + ml_p^2)\ddot{\theta}_3 - mgl_p\sin\theta_3 = 0$$

where $u_1, u_2, u_3$ are input forces to the carts(N), $M$ is the mass of the cart (kg), $m$ is the mass of the rod(kg), $l_p$ is the distance from the pivot on the cart to the center of gravity of the rod(half of full length)(m), $I(=\frac{1}{3}ml_p^2)$ is the moment of inertia of the rod with respect to its center of mass (kg·m$^2$), $g$ is the gravitational acceleration (kg·m/sec$^2$), $k$ is the spring constant (N/m), $c$ is the damping constant (N·sec/m), $s_1 = k(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1), s_2 = k(x_3 - x_2) + c(\dot{x}_3 - \dot{x}_2)$ are interconnection forces due to springs and dampers. The parameter values are: $M = 0.9, m = 0.18, l_p = 0.305, g = 9.8, k = 1, c = 2 \times 10^{-5}$. Our control objective is to regulate the displacements of the carts $\boldsymbol{x}_i$ while balancing the inverted rods on the carts without velocity measurements. The open loop subsystem in (4) is derived after the dynamics are first linearized with respect to equilibrium position $x_i = \theta_i = 0$, and then put into a normal form by the transformation: $x_{l_{i_1}} = x_i, x_{l_{i_2}} = \dot{x}_i, z_{l_{i_1}} = \theta_i, z_{l_{i_2}} = \frac{\dot{x}_i}{l_p} + \dot{\theta}_i$. The linear subsystems, for $i = 1, 2, 3$, is described by the following system matrices:

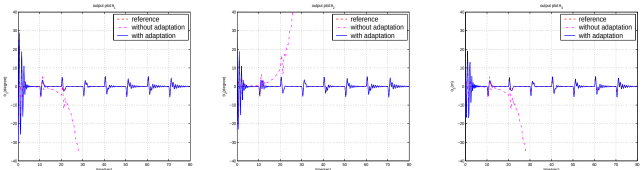
$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B_i = \begin{bmatrix} 0 & 0 \\ -\frac{\hat{m}}{\hat{M}} & 0 \end{bmatrix}, \; \boldsymbol{b}_i = \begin{bmatrix} 0 \\ \frac{1}{\hat{M}} \end{bmatrix}$$
$$C_i = \begin{bmatrix} 0 & -\frac{1}{l_p} \\ 0 & 0 \end{bmatrix}, \; D_i = \begin{bmatrix} 0 & 1 \\ \frac{g}{l_p} & 0 \end{bmatrix}, \; \boldsymbol{c}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{31}$$

The constants $\hat{M} = 0.815, \hat{m} = 0.21$ represent parameter estimates for $M, m$ respectively. Further, in this linear model, the inverted rod is treated as a lumped mass located on its center of mass, i.e., $I = 0$. Putting each subsystem into normal form leads to the following modelling errors and interconnection effects defined in (1): $f_i = \frac{\hat{M}}{M + m(1 - 3/4\cos\theta_i^2)}(u_i + ml_p\dot{\theta}_i^2\sin\theta_i - \frac{3}{4}mg\sin\theta_i\cos\theta_i + \tau_i) + \hat{m}g\theta_i - u_i$, $\boldsymbol{g}_i = \begin{bmatrix} -\frac{\cos\theta_i}{l_p}x_{i_2} - \frac{1}{3}\dot{\theta}_i + \frac{1}{l_p}x_{i_2} \\ \frac{g}{l_p}\sin\theta_i - \frac{1}{l_p}x_{i_2}\dot{\theta}_i\sin\theta_i - \frac{g}{l_p}\theta_i \end{bmatrix}$ where $\tau_1 = s_1, \; \tau_2 = -s_1 + s_2, \; \tau_3 = -s_2$. The $\tau_i$ terms imply that the spring and the damper are not considered in the open loop model. The term $u_i$ means that the modelling error also depends on the control signal as in Remark 5.1. Note that the interconnections between the two carts and the modelling errors contain velocity terms which are not measured. This implies that the existing adaptive output feedback approaches in the decentralized control literature, such as the ones developed in [8], [21] and many others, although establishing global results, cannot be applied. Moreover, regulation of $x_i$ using $u_i$ to the carts renders the control problem *nonminimum* phase -linearization of each subsystem about vertical-up position leads to unstable zero $\sqrt{\frac{g}{L}}$. These issues make the control problem even more challenging. The dynamic compensator in (8) for each subsystem is designed as a LQG controller based on the open loop model in (31), in which two measured outputs




(a) $x_1$ (b) $x_2$ (c) $x_3$

Fig. 2. Comparison of the cart displacements without and with adaptive signal $u_{ad_i}$.




(a) $\theta_1$ (b) $\theta_2$ (c) $\theta_3$

Fig. 3. Comparison of the rod angles without and with adaptive signal $u_{ad_i}$.

$x_i, \theta_i$ are available for control design. The error observer in (15) is designed to have the smallest eigenvalue of $\tilde{A}_i$ equal approximately five times the smallest eigenvalue of $\bar{A}_i$. The basis functions have the following structure for three subsystems: $\phi_{i_k}(\boldsymbol{Y}_l) = e^{-\|\boldsymbol{Y}_l - \boldsymbol{Y}_{i_{c_k}}\|^2/2\sigma_{i_k}}, \sigma_{i_k} = 1, i = 1,2,3,\ k = 1,\ldots,N_i$, where $N_1 = N_3 = 7, N_2 = 9$. The centers $\boldsymbol{Y}_{i_{c_k}}$ are randomly selected over a grid of possible values for the vector $\boldsymbol{Y}_l$. All of the NN inputs are normalized using an estimate for their maximum values. Since the dynamics of the first and third carts are coupled only through the dynamics of the middle cart, and the modelling error contains a control signal, we choose the NN input vectors as: $\boldsymbol{Y}_{l_1}^T = [u_1\ x_{l_{1_1}}\ x_{l_{1_2}}\ z_{l_{1_1}}\ z_{l_{1_2}}\ x_{l_{2_1}}\ x_{l_{2_2}}\ 0\,0\,0\,0\,0\,0]^T$, $\boldsymbol{Y}_{l_2}^T = [u_2\ x_{l_{1_1}}\ x_{l_{1_2}}\ 0\ 0\ x_{l_{2_1}}\ x_{l_{2_2}}\ z_{l_{2_1}}\ z_{l_{2_2}}\ x_{l_{3_1}}\ x_{l_{3_2}}\ 0\ 0]^T$, $\boldsymbol{Y}_{l_3}^T = [u_3\ 0\,0\,0\,0\ x_{l_{2_1}}\ x_{l_{2_2}}\ 0\,0\ x_{l_{3_1}}\ x_{l_{3_2}}\ z_{l_{3_1}}\ z_{l_{3_2}}]^T$, where $\boldsymbol{Y}_{l_i}$ represents the NN input vector for the ith subsystem. Adaptation gains are chosen as: $F_i = 0.5I, k_i = 0.05$. Figure 2 compares output tracking performances when the reference command $y_{c_i}, i = 1,2,3$ is a square wave signal of magnitude 0.15m and 0.05 Hz. The pendulum angles are shown in Figure 3. The initial conditions are: $x_1(0) = \dot{x}_1(0) = 0,\ \theta_1(0) = -30^o,\ \dot{\theta}_1(0) = -10^o/sec, x_2(0) = \dot{x}_2(0) = 0,\ \theta_2(0) = 30^o,\ \dot{\theta}_2(0) = 10^o/sec, x_3(0) = \dot{x}_3(0) = 0,\ \theta_3(0) = 20^o,\ \dot{\theta}_2(0) = -10^o/sec$. Without adaptive control compensation, the system goes unstable. When each control is augmented with an adaptive term, the three carts are in synchronous motion with the pendulums balanced, implying implicit cooperation for output tracking.

## VII. CONCLUSIONS

A methodology is presented for adaptive output feedback decentralized control design under the assumption that the reference trajectories are known to all subsystems. A linearly parameterized neural network is used to model the interconnection effects on-line. Boundedness of error signals is shown using Lyapunov's direct method. The methodology is applicable to non-minimum phase subsystems.

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