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An Optimization Algorithm for Decentralized Digital Control of Continuous-Time Systems which Accounts for Inter-Sample Ripple

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***Abstract*— In this paper, an algorithm is proposed to design a decentralized digital controller, such that a quadratic performance index is minimized with the property that the inter-sample ripple of the output signal is included in the minimization procedure. The algorithm has the property that it can be applied to either centralized or decentralized systems, which are sampled with either a ZOH or a generalized sampled-data hold function. Numerical examples of the algorithm are included to show the effectiveness of the algorithm.**

I. Introduction

Digital controllers are often used to control continuous-time systems. Such controllers have a simple structure and can be implemented by using a computer, a digital to analog and an analog to digital element as shown in Figure 1. This

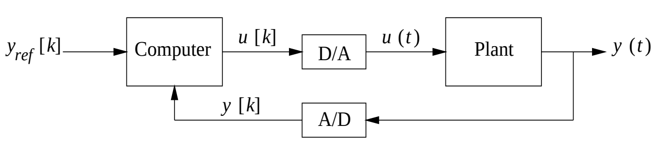

Fig. 1. The structure of a digital controller as a time-varying system.

configuration is equivalent to a time-varying continuous-time controller. Note that the hold function corresponding to the digital to analog block (D/A), which typically is a zero-order hold (ZOH), can in fact be any function defined over one sampling period. Digital control for such systems can be either centralized or decentralized. Application of discrete-time controllers in decentralized systems has been studied in [1], [2], [3], [4], where it was shown that sampling can remove certain types of decentralized fixed modes in the system.

The idea of using generalized sampled-data hold functions (GSHF) instead of a simple ZOH (or first-order hold) in control systems was first introduced by Chammas and Leondes [5]. Kabamba examined the application of GSHFs in control systems, and pointed out that by using GSHF, one can obtain much of the efficiency of state feedback, without the requirement of state estimation [6]; he also showed that GSHFs can significantly improve the performance of the closed-loop system. Application of GSHFs in decentralized systems was investigated in [7], where it was shown that a digital decentralized controller with GSHF can result in a significant improvement compared to a simple ZOH. The application of GSHFs to decentralized control structure modification was also studied in [8] and it was shown that GSHFs can be used to modify the structure of the digraph of the resultant discrete-time plant, by removing certain interconnections in the equivalent discrete-time model. It is to be noted that a disadvantage of generalized sampled-data hold functions is that they are prone to robustness difficulties in the continuous time domain, e.g. see [9], [10].

The optimal decentralized control of a LTI system using GSHFs, which includes ZOH functions, will be considered in this paper. When optimal control methods are used to design such digital controllers, often the performance index chosen ignores "inter-sample ripple effects", which can be significant, particularly if the sampling period is large. There are several related references in measuring inter-sample performance in the context of hold functions, e.g., see [11]. For the special case of a simple ZOH, the quadratic performance optimization problem taking into account the inter-sample behavior has been completely solved in the centralized case [12]. The quantitative and qualitative analysis of inter-sample behavior in a frequency domain setting was given in [9], where it was shown that the generalized hold approach depends upon the generation of high-frequency components in the continuous time output which are folded when the output is sampled. In [13] a method was proposed to find an optimal hold function which minimizes the inter-sample ripple for centralized control case, by solving a linear two-point boundary value

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problem. This paper proposes an algorithm for the optimal design of decentralized digital control using polynomial hold functions, where ripple effects are included. As far as the authors are aware, the optimal control of such systems, taking inter-sample ripple into account, has not been considered before.

II. Development

Consider a decentralized continuous-time LTI system with m control agents as follows:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} b_i u_i(t), \tag{1a}$$
$$y_i(t) = c_i x(t) + \sum_{j=1}^{m} d_{ij} u_j(t), \tag{1b}$$
$$i \in \bar{\text{m}} = \{1, ..., m\},$$

where $x \in \mathbb{R}^n$ is the state vector, $u_j \in \mathbb{R}^{m_j}$ and $y_j \in \mathbb{R}^{r_j}$ are the input and output of the j^{th} control station, respectively and $j \in \bar{\text{m}}$. A, b_i, c_i, and d_{ij}, $i, j \in \bar{\text{m}}$ are matrices of appropriate dimensions. For simplicity and without loss of generality [14] assume that $m_j = r_j$, $j \in \bar{\text{m}}$. The configuration of the discrete-time equivalent system using GSHFs is shown in Figure 2 and can be formulated as follows:

$$u_j(t) = f_j(t)\tilde{u}_j[k], \tag{2a}$$
$$f_j(t+T) = f_j(t), \tag{2b}$$
$$t \in [kT, (k+1)T), \quad k = 0, 1, 2, ...$$

where $\tilde{u}_j[k]$ and $f_j(t)$, $j \in \bar{\text{m}}$, are the input sequence and

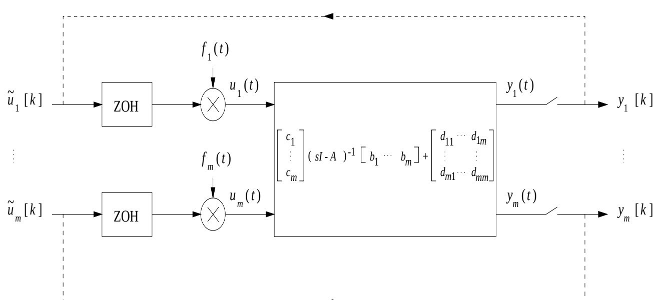

Fig. 2. Generalized sampled-data hold configuration for a control system.

the periodic hold function for control agent j, respectively. The equivalent discrete-time model, corresponding to (1), is represented by:

$$x[k+1] = A_d x[k] + \sum_{i=1}^{m} b_{d_i} u_i[k], \tag{3a}$$
$$y_i[k] = c_{d_i} x[k] + \sum_{j=1}^{m} d_{d_{ij}} u_j[k], \quad i \in \bar{\text{m}}, \tag{3b}$$

where the model parameters can be obtained as follows:

$$A_d = e^{AT}, \tag{4a}$$
$$b_{d_i} = \int_0^T e^{A(T-\tau)} b_i f_i(\tau) d\tau, \quad i \in \bar{\text{m}}, \tag{4b}$$
$$c_{d_i} = c_i, \quad i \in \bar{\text{m}}, \tag{4c}$$
$$d_{d_{ij}} = d_{ij}, \quad i, j \in \bar{\text{m}}, \tag{4d}$$

and where for the feedback control structure of Figure 2 (shown by the dashed line), $\tilde{u}_j[k]$ is equal to $y_j[k]$, $j \in \bar{\text{m}}$.

Definition 1: The closed-loop system obtained by applying the decentralized generalized sampled-data controller (2) to (1) is said to be stable if the equivalent LTI sampled system (3) obtained when the decentralized controller $\tilde{u}_j[k] = y_j[k]$, $j \in \bar{\text{m}}$ is applied, is stable.

Remark 1: In the sampled system discussed above, it is assumed that the system does not posses any processing delay, i.e., samples of input and output signals are taken at the same time instants.

In the next section, an algorithm is proposed which can be used to design an optimal discrete-time controller for system (1), using generalized sampled-data hold functions.

III. Main Result

Consider the n^{th} order system (1) and assume that it is desired to design a discrete-time decentralized output feedback controller to stabilize the system such that the following performance index is minimized:

$$J = \mathcal{E}\left\{\int_0^{\infty} (y'(t)y(t) + \rho u'(t)u(t))dt\right\}, \tag{5}$$

where $\mathcal{E}$ denotes the expectation operator [15]. Without loss of generality, it can be assumed that $m_j = 1$, $\forall j \in \bar{\text{m}}$ [16] ([16] gives a procedure by which the decentralized control problem for (1) can be converted to the decentralized control problem for a new system in which the control agents of the system have a scalar input and scalar output). Suppose that (3) corresponds to the closed-loop sampled system, obtained by using the decentralized generalized sampled-data controller as follows:

$$u_j(t) = f_j(t)y_j[k], \tag{6a}$$
$$f_j(t+T) = f_j(t), \tag{6b}$$

where $j \in \bar{\text{m}}$, $t \in [kT, (k+1)T)$, $k = 0, 1, 2, ...$ In particular, assume that $f_j(t)$, $j \in \bar{\text{m}}$, are polynomials of the following form:

$$\begin{bmatrix} f_1(t) \\ \vdots \\ f_m(t) \end{bmatrix} = \Gamma_{q-1} \begin{bmatrix} t^0 \\ \vdots \\ t^{q-1} \end{bmatrix}, \tag{7}$$

where Γ_{q-1} is a $m \times q$ matrix whose rows are the coefficients of the corresponding polynomials. The state-space representation of the corresponding closed-loop system with the weighting matrix Γ_{q-1} is given by:

$$x[k+1] = (A_d + B_d(\Gamma_{q-1})(I - D_d)^{-1}C_d)x[k], \tag{8a}$$
$$y[k] = C_d x[k] + D_d u[k], \tag{8b}$$

where A_d is given by (4a),

$$B_d(\Gamma_{q-1}) := \left[\begin{array}{ccc} b_{d_1}(\Gamma_{q-1}) & \dots & b_{d_m}(\Gamma_{q-1}) \end{array}\right],$$
$$b_{d_j}(\Gamma_{q-1}) := \int_0^T e^{A(t-\tau)} b_j S_j(\tau) d\tau,$$

and:

$$S_j(t) := e_j \Gamma_{q-1} \left[\begin{array}{c} t^0 \\ \vdots \\ t^{q-1} \end{array}\right], \quad e_j \in \mathbb{R}^{1\times m},$$

$$e_j(i) = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}, \quad i, j \in \bar{\text{m}}$$

$$C_d = \left[\begin{array}{c} c_{d_1} \\ \vdots \\ c_{d_m} \end{array}\right], \quad D_d = \left[\begin{array}{ccc} d_{d_{11}} & \dots & d_{d_{1m}} \\ \vdots & & \vdots \\ d_{d_{m1}} & \dots & d_{d_{mm}} \end{array}\right],$$

$$u[k] = \left[\begin{array}{c} u_1[k] \\ \vdots \\ u_m[k] \end{array}\right], \quad y[k] = \left[\begin{array}{c} y_1[k] \\ \vdots \\ y_m[k] \end{array}\right].$$

Assumption 1: Consider system (1) and assume that the discrete-time equivalent model obtained by using a ZOH with a sampling period $T_0 > 0$ can be stabilized by applying a static decentralized LTI output feedback. This means that there exists a $\Gamma_0 \in \mathbb{R}^{m\times 1}$ such that all eigenvalues of the matrix $A_d + B_d(\Gamma_0)(I - D_d)^{-1} C_d$ lie inside the unit disc. In the special case of an open-loop stable system, one can always choose $\Gamma_0 = 0$.

It is desired now to determine the optimal matrix Γ_{q-1}^{op} and the optimal sampling period T_{q-1}^{op}, which minimize the performance index (5) (corresponding to a continuous-time system representation). The following algorithm is proposed to do this.

Algorithm 1: A modified performance index for the original system (1) with the decentralized digital feedback controller (6) will now be defined. Since the optimal output feedback law (corresponding to any quadratic performance index) depends on the initial conditions of the plant (1), we will use an approach similar to [15] to design the digital controller. Consider the following set of initial states:

$$x_1 = \left[\begin{array}{c} 1 \\ 0 \\ \vdots \\ 0 \end{array}\right],\ x_2 = \left[\begin{array}{c} 0 \\ 1 \\ \vdots \\ 0 \end{array}\right],\ \dots,\ x_n = \left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ 1 \end{array}\right], \tag{9}$$

which are uniformly distributed on a unit sphere and span the whole space. Now let N be a positive integer, and define the modified performance index for the system (1) corresponding to the initial state vector $x(0) = x_i$ given in (9) as:

$$\tilde{J}_{x_i}(NT) = \int_0^{NT} \rho u'(t) u(t) dt + \int_0^{NT} y'(t) y(t) dt, \tag{10}$$

where $i = 1, 2, ..., n$, and the sampling period T is a positive real number. From (6) the first right hand term of (10) simplifies to become:

$$\int_0^{NT} \rho u'(t) u(t) dt = T \sum_{k=0}^{N-1} \rho y'(kT) F y(kT), \tag{11}$$

where $F = \text{diag}(F_1, ..., F_m)$, and:

$$F_j = \int_0^T f_j^2(t) dt, \quad j \in \bar{\text{m}}.$$

Let:

$$\tilde{J}(NT) := \frac{1}{n} \sum_{i=1}^{n} \tilde{J}_{x_i}(NT),$$

and consider the following parameter optimization problem:

Parameter Optimization Problem

$$\min_{\substack{T,\ \Gamma_{q-1} \\ \text{given by (6), (7)}}} \quad \tilde{J}(NT)$$

subject to the constraint that (8) is stable, using as the starting point $T_{q-1} = T_0$ for a ZOH and $\Gamma_{q-1} = \left[\begin{array}{cc} \Gamma_0 & \mathbf{0}_{m\times q-1} \end{array}\right]$. In this case, for sufficiently large NT, the optimal value of the performance index $\tilde{J}(NT)$ will approach the minimum of the performance index (5).

One can use any multidimensional constrained nonlinear optimization method to solve this problem. In the numerical examples of the next section, the Nelder-Mead simplex (direct search) method [17] is used, with a penalty function to impose the stability constraint. This can be accomplished by using "fminsearch" on MATLAB 6.1. Note that this is a nonconvex optimization problem and there is no guarantee that the result obtained is a global minimum.

The optimal solution for Γ_{q-1} and T_{q-1} can be obtained step by step as follows:

1) using as starting point Γ_0 and T_0 of Assumption 1, find the optimal GSHF parameters Γ_0^{op} and the corresponding optimal sampling period T_0^{op} for a ZOH.
2) use the optimal parameters of the previous step as the initial parameters of this step to find the optimal values for a first-order polynomial GSHF. In other words, use $\Gamma_1 = \left[\begin{array}{cc} \Gamma_0^{op} & \mathbf{0}_{m\times 1} \end{array}\right]$ and $T = T_0^{op}$ as the initial values in the optimization procedure to find the optimal GSHF parameters Γ_1^{op} and the corresponding optimal sampling period T_1^{op}.

⋮

q) use the optimal parameters of step (q-1) to form the initial parameters $\Gamma_{q-1} = \left[\begin{array}{cc} \Gamma_{q-2}^{op} & \mathbf{0}_{m\times 1} \end{array}\right]$ and $T_{q-1} = T_{q-2}^{op}$ to find the optimal parameters Γ_{q-1}^{op} and T_{q-1}^{op}.

In this case, since the starting point used corresponds to an optimal static output controller with a ZOH, it can be concluded that the optimal controller obtained will, in general, outperform the controller obtained using a ZOH.

It is to be noted that since Algorithm 1 minimizes the quadratic performance index (10), this implies that the optimization algorithm directly takes inter-sample ripple effects into account.

From equation (11) it can be observed that the performance index given by (10) is a combination of energy of the output in the sampling instants and the energy of the output between samples. For large values of ρ, the performance index will be mainly equal to the weighted values of the samples of output, where the weighting matrix is given by F.

Remark 2: One can also use the method given in [7] to find a stabilizing starting point for Γ_{q-1} in the optimization problem. It is to be noted that each starting point may lead to a local optimal value for Γ_{q-1} which is not necessarily the global optimal solution.

Remark 3: A special case of GSHF occurs when the GSHF is assumed to be a constant. If $f_j(t),\ j \in \bar{\mathrm{m}}$ are all constants (zero-order polynomials), then the output feedback law for each control agent will be equivalent to a digital controller with a simple ZOH and a constant feedback gain given by the corresponding constant value. This is illustrated in the next section.

Remark 4: Since a centralized control system is a special case of a decentralized control system, then the proposed algorithm can also be directly applied to centralized systems.

Remark 5: From an implementation point of view, it may be simpler to have a piecewise constant hold. The authors are currently investigating the pro's and con's of this approach versus that of a polynomial with the same number of degrees of freedom.

The following example compares the decentralized control of a system under the following conditions:

(i) using a ZOH with the proposed performance index given by Algorithm 1
(ii) using a first-order polynomial GSHF with the proposed performance index given by Algorithm 1
(iii) using a first-order polynomial GSHF with a conventional performance index

It will be shown that the controller for case (ii) is superior to the controller for case (i), and that the controller for case (i) is superior to the controller for case (iii).

IV. Numerical Examples

Consider a controllable, observable, non-minimum phase, unstable system described by the following state space matrices:

$$A = \begin{bmatrix} -1 & 0 & -3 \\ 0 & 0.1 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \tag{12a}$$

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \tag{12b}$$

$$c_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, c_2 = \begin{bmatrix} -1.1 & 0.005 & 0.1 \end{bmatrix}, \tag{12c}$$

$$d_{11} = d_{12} = d_{21} = d_{22} = 0. \tag{12d}$$

It is desired to find a decentralized digital controller to stabilize the unstable system (12), such that the following performance index is minimized:

$$J = \mathcal{E}\left\{\int_0^\infty (y'(t)y(t) + u'(t)u(t))dt\right\}. \tag{13}$$

The following different cases for decentralized digital control are examined, where Example 1, 2, 3 correspond to case (i), (ii), (iii) above, respectively.

Example 1: GSHFs in the form of zero-order polynomials. Consider applying the digital controller:

$$u_1(t) = f_1.y_1[k], \tag{14}$$

$$u_2(t) = f_2.y_2[k], \quad t \in [kT, (k+1)T), \quad k = 0, 1, 2, \ldots \tag{15}$$

to (12), with a sampling period of $T > 0$ and zero-order polynomials for the hold functions f_1, f_2. Then on minimizing the performance index (13) using Algorithm 1 with respect to $\Gamma = [f_1\ \ f_2]'$ and T, the optimal performance index of $J^{op} = 4896$ is obtained. The corresponding optimal GSHFs are given by the following weighting matrix:

$$\Gamma_0^{op} = \begin{bmatrix} 9.996 \\ -0.7069 \end{bmatrix}, \tag{16}$$

and the optimal sampling period is $T_0^{op} = 3.085\ sec.$ The eigenvalues of the resultant closed-loop system for the equivalent discrete-time system (8) are given by:

$$\mathrm{sp}(A_d + B_d(\Gamma_0)C_d) = \{0.7704 \pm 0.2336i, -0.8942\}$$

which shows that the unstable system (12) has been stabilized. The corresponding performance index:

$$J_{x(0)} = \int_0^\infty (y'(t)y(t) + u'(t)u(t))dt \tag{17}$$

for $x(0) = [1\ \ 1\ \ 1]'$ is 2.116×10^4 and Figure 3 gives the resultant input and output signals for this initial value. The controller obtained in this case is in fact a combination of a ZOH and a constant gain for each control agent.

Example 2: GSHFs in the form of first-order polynomials. Consider now applying sampled-data hold functions $f_1(t) = a_1 t + b_1$ and $f_2(t) = a_2 t + b_2$ to each control agent. On minimizing the performance index (13)using Algorithm 1, the following results are obtained for the optimal sampled-data hold functions and optimal sampling period:

$$\Gamma_1^{op} = \begin{bmatrix} 2.923 & -4.907 \\ -1.980 & 1.779 \end{bmatrix}, \tag{18a}$$

$$T_1^{op} = 1.151\ sec. \tag{18b}$$

The resultant minimum performance index obtained in this case is $J^{op} = 14.92$. Note that although only one parameter has been added to the control design of the decentralized controller, as compared to Example 1, a significant improvement in performance is achieved. The corresponding input and output signals of control agent 1 and control agent 2 for $x(0) = [1\ \ 1\ \ 1]'$ are depicted in Figure 4 where (a) and (b) give the output and input signals of control agent 1, and (c) and (d) give the corresponding signals of control

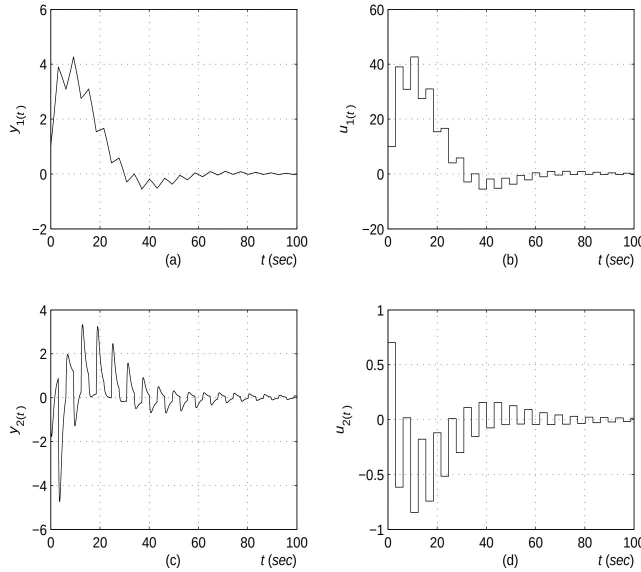

Fig. 3. Closed-loop simulation results for Example 1, using optimal decentralized digital controller with zero-order GSHFs given by (16). (a) Output signal of control agent 1; (b) input signal of control agent 1; (c) output signal of control agent 2; (d) input signal of control agent 2.

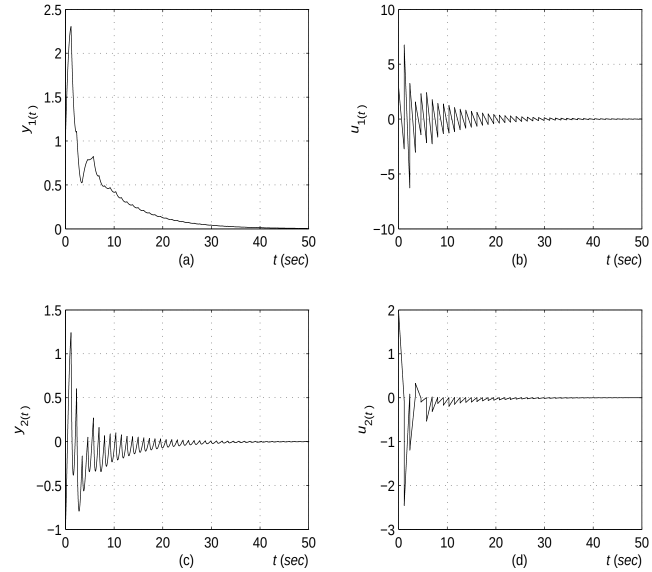

Fig. 4. Closed-loop simulation results for Example 2, using optimal decentralized digital controller with first-order GSHFs (18). (a) Output signal of control agent 1; (b) input signal of control agent 1; (c) output signal of control agent 2; (d) input signal of control agent 2.

agent 2, respectively. The resultant performance index (17) for $x(0) = [1 \quad 1 \quad 1]'$ in this case is 49.63. Intermediate results obtained in the optimization procedure which start at $J = 4896$ (corresponding to $\Gamma_1 = \begin{bmatrix} 9.996 & 0 \\ -0.7069 & 0 \end{bmatrix}$ and $T_1 = 3.085\ sec$) and converge to $J = 14.92$ (corresponding to $\Gamma_1 = \begin{bmatrix} 2.923 & -4.907 \\ -1.980 & 1.779 \end{bmatrix}$ and $T_1 = 1.151\ sec$) are shown in Figure 5.

In the previous two examples, the controller design was based on using the continuous-time system performance index (13). For completeness, the case of decentralized digital control for(12) using the same class of sampled data hold functions as used in Example 2 with a conventional discrete performance index, will now be examined for comparison purposes.

Example 3: Decentralized digital controller design using conventional performance index. Consider now applying a decentralized digital controller of the form (6) to (12) to minimize the conventional performance index given by:

$$J_d = \mathcal{E}\left\{\sum_{k=0}^{\infty}(y'[k]y[k] + u'[k]u[k])dt\right\}. \qquad (19)$$

Then on using first order polynomials for the GSHF of each agent with the sampling period $T = 1.151\ sec$ obtained in Example 2, the following optimal hold functions are obtained:

$$\text{Optimal } f_1(t): \ f_1(t) = -9.064t + 0.1531, \qquad (20a)$$

$$\text{Optimal } f_2(t): \ f_2(t) = -0.4279t - 0.1467. \qquad (20b)$$

The resultant performance index for the original continuous-time system given by (17) for $x(0) = [1 \quad 1 \quad 1]'$ is $J_{x(0)} = 107.6$, which is significantly greater than the performance

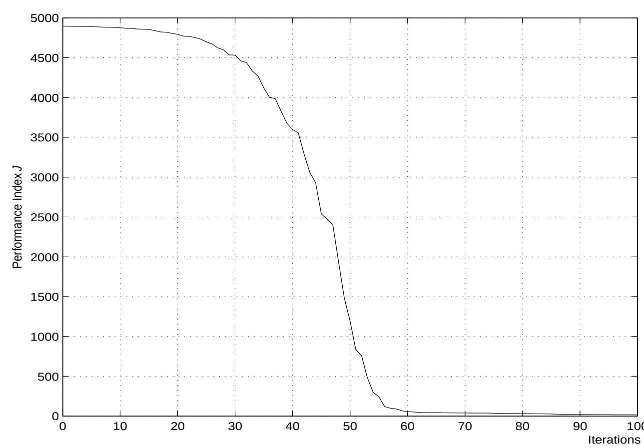

Fig. 5. Intermediate results obtained in the optimization procedure for Example 2.

index of 49.68 obtained in Example 2. This is due to the fact that the controller obtained in this example does not take inter-sample effects into account. Figure 6 gives the resultant input and output signals obtained for this case.

So far, various decentralized digital output feedback controllers have been considered in this example. It is to be noted for this problem, that the minimum achievable performance index for (13) using any type of controller is obtained by using the centralized continuous-time state feedback law:

$$u(t) = \begin{bmatrix} -0.1237 & 0.06193 & 0.001431 \\ 0.1997 & -1.134 & -0.1363 \end{bmatrix} x(t), \qquad (21)$$

and the corresponding performance index for $x(0) = [1 \quad 1 \quad 1]'$ in this case is given by $J^{op}_{x(0)} = 1.448$.

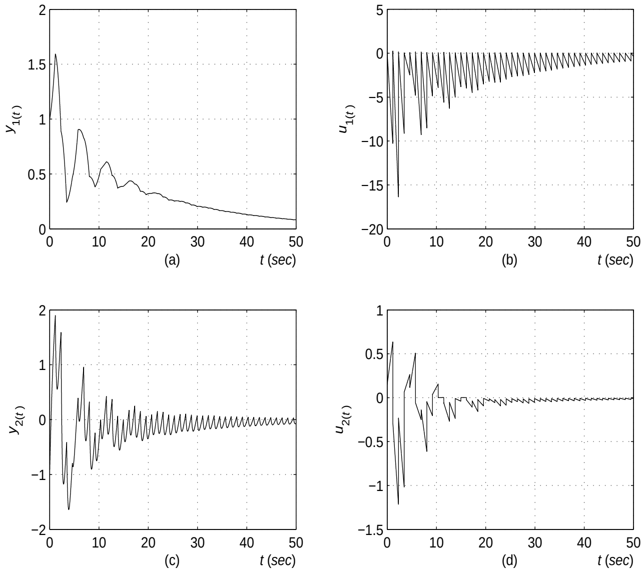

Fig. 6. Closed-loop simulation results for Example 3, using optimal decentralized digital controller with first-order GSHFs (20a) and (20b). (a) Output signal of control agent 1; (b) input signal of control agent 1; (c) output signal of control agent 2; (d) input signal of control agent 2.

V. Conclusions

In this paper, a class of LTV decentralized controllers has been introduced for continuous LTI plants, using generalized sampled-data hold functions (GSHF), and an optimization algorithm (Algorithm 1) is proposed to design the corresponding discrete-time controllers, which account for inter-sample ripple. In this case, the generalized sampled-data hold functions chosen consist of linear combinations of polynomials of various degrees; clearly other basis functions could also have been used. Since zero-order hold functions (ZOH) are special cases of GSHF, this implies that the proposed optimization algorithm can also be used to design conventional discrete centralized or decentralized controllers, taking inter-sample ripple into account, and this was demonstrated in some of the numerical examples studied. The examples in the paper show the effectiveness of the proposed algorithm.

References

[1] S. H. Wang, "Stabilization of decentralized control systems via time-varying controllers," *IEEE Transactions on Automatic Control*, vol. AC-27, no. 3, pp. 741–744, 1982.

[2] Ü. Özgüner and E. J. Davison, "Sampling and decentralized fixed modes," in *Proceedings of the 1985 American Control Conference*, pp. 257–262, 1985.

[3] J. L. Willems, "Time-varying feedback for the stabilization of fixed modes in decentralized control systems," *Automatica*, vol. 25, no. 1, pp. 127–131, 1989.

[4] P. P. Khargonekar and A. B. Özgüler, "Decentralized control and periodic feedback," *IEEE Transactions on Automatic Control*, vol. 39, no. 4, pp. 877–882, 1994.

[5] A. B. Chammas and C. T. Leondes, "On the design of linear time invariant systems by periodic output feedback, part i and ii," *International Journal of Control*, vol. 27, pp. 885–903, 1978.

[6] P. T. Kabamba, "Control of linear systems using generalized sampled-data hold functions," *IEEE Transactions on Automatic Control*, vol. AC-32, no. 9, pp. 772–783, 1987.

[7] A. G. Aghdam and E. J. Davison, "Decentralized control of systems, using generalized sampled-data hold functions," in *Proceedings of the 38'th IEEE Conference on Decision and Control*, pp. 3912–3913, 1999.

[8] A. G. Aghdam and E. J. Davison, "Application of generalized sampled-data hold functions to decentralized control structure modification," in *Proceedings of IFAC Symposium in Large Scale Systems*, Barcelona, Spain, vol. 1, Area Code 5a, Session Slot T-Th-A13.

[9] A. Feuer and G. C. Goodwin, "Generalized sample hold functions-frequency domain analysis of robustness, sensitivity, and intersample difficulties," *IEEE Transactions on Automatic Control*, vol. AC-39, no. 5, pp. 1042–1047, 1994.

[10] J. S. Freudenberg, R. H. Middleton, and J. H. Braslavsky, "Robustness of zero shifting via generalized sampled-data hold functions," *IEEE Transactions on Automatic Control*, vol. AC-42, no. 12, pp. 1681–1692, 1997.

[11] M. Rossi and D. Miller, "Model reference control using static generalized sampled-data hold functions," in *Proceedings of the 38'th IEEE Conference on Decision and Control*, pp. 4583–4588, 1999.

[12] T. Chen and B. Francis, *Optimal Sampled-Data Control Systems*. Springer-Verlag, 1995.

[13] Y. C. Juan and P. Kabamba, "Optimal hold functions for sampled-data regulation," *Automatica*, vol. 27, no. 1, pp. 177–181, 1991.

[14] E. J. Davison and T. N. Chang, "Decentralized stabilization and pole assignment for general proper systems," *IEEE Transactions on Automatic Control*, vol. AC-35, no. 6, pp. 652–664, 1990.

[15] W. S. Levine and M. Athans, "On the determination of the optimal constant output feedback gains for linear multivariable systems," *IEEE Transactions on Automatic Control*, vol. AC-15, pp. 44–48, 1970.

[16] A. F. Vaz and E. J. Davison, "The decentralized control expanding system problem," *Systems and Control Letters*, vol. 10, pp. 301–308, 1988.

[17] J. A. Nelder and R. Mead, "A simplex method for function optimization," *Computer Journal*, vol. 7, pp. 308–313, 1965.