

LMS-based Structural Health Monitoring Methods for the ASCE Benchmark Problem

J. Geoffrey Chase, Luciana R. Barroso, and Kyu-Suk Hwang

Abstract— A structure's health or level of damage can be monitored by identifying changes in structural or modal parameters. This research directly identifies changes in structural stiffness due to modelling error or damage using a structural health monitoring method based on Adaptive Least Mean Square (LMS) filtering theory. The focus in developing these methods is on simplicity to enable real-time implementation with minimal computation. An LMS filtering based approach is used to analyze the data from the IASC-ASCE Structural Health Monitoring Task Group Benchmark problem. The proposed methods accurately identify damage to within 1%, with convergence times of 0.4 – 13.0 seconds for the twelve different 4 and 12 degree of freedom Benchmark Problems and modal parameters match to within 1%. Finally, the method presented is computationally simple, requiring no more than 1.4Mcycles of computation.

I. INTRODUCTION

Structural Health Monitoring (SHM) is the process of examining the current state of a structure's condition and determining the existence, location, and degree of damage that may exist, particularly after a damaging input, such as an earthquake or other large environmental load. Current SHM methods are based on the idea of vibration-based damage detection where changes in modal parameters, such as frequencies, mode shapes and modal damping, are a result of changes in the physical mass, damping and stiffness properties of the structure (Doebbling et al, 1996). SHM can simplify typical procedures of visual or localized experimental methods, such as acoustic or ultrasonic methods, magnetic field methods, radiography, eddy-current methods or thermal field methods (Doherty, 1997), as it does not require visual inspection of the structure and its connections or components. Doebbling et al (1996a) has an excellent review of the numerous different approaches for vibration-based damage detection methods. However, the various studies apply different methods to different

structures, rendering side-by-side comparison difficult.

In 1999, under the auspices of the International Association for Structural Control (IASC) and the Dynamics committee of the American Society of Civil Engineers (ASCE) Engineering Mechanics Division, the SHM Task Group was formed and charged with studying the efficacy of various SHM methods. The IASC-ASCE SHM Task Group developed a series of Benchmark SHM problems and established a set of specific Benchmark results for a specially designed test structure in the Earthquake Engineering Research Laboratory at the University of British Columbia (Johnson et al, 2000). After the Benchmark problem was established, SHM research for civil structure was concentrated on applying different techniques to the Benchmark problem to examine the relative and absolute effectiveness of different algorithms.

SHM in Civil structures is useful for determining the damage state of a structure. In particular, the ability to assess damage in real-time or immediately after an earthquake would allow Civil Defence authorities to determine which structures were safe. Current methods are more applicable to steel frame or bridge structures where vibration response may be more linear. These problems typically have known, or reasonably estimated, input loads. However, the insensitivity of modal parameters to (localised) damage in some cases can be a major limitation for the larger number of methods that rely on identifying these parameters to assess and locate damage. This research uses adaptive filtering to assess the damage directly without using modal parameters.

The most common method for identification of civil structural model parameters is the Eigensystem Realization Algorithm (ERA). The ERA method is based on knowledge of the time domain free response data. In ERA, a discrete Hankel matrix is formed, and the state and output matrices for the resulting discrete matrix are determined. These matrices are transformed to the corresponding continuous time system. The natural frequencies are found by determining the eigenvalues of the continuous time system. Dyke et al (2000) use cross correlation functions in conjunction with the ERA method for identification of the modal parameters, which are used to identify frequency and damping parameters. Caicedo et al (2000) introduces SHM

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methods based on changes in the component transfer functions of the structure, or transfer functions between the floors of a structure, and use the ERA to identify the natural frequencies of each component transfer function. Lus and Betti (2000) also proposed a damage identification method based on ERA with a Data Correlation and Observer/Kalman Identification algorithm. Bernal and Gunes (2000) also used the ERA with Observer/Kalman Identification for identifying modal characteristics when the input is known, and used a Subspace Identification algorithm when the input cannot be measured.

Wavelet analysis approaches for SHM and damage detection may found in Corbin et al (2000) and Hou et al (2000). Damage, and the moment when the damage occurs, can be detected by a spike or an impulse in the plots of higher resolution details from wavelet decomposition of the acceleration response data. Wavelets offer the advantage of determining not only the extent of the damage but also the time of its occurrence, which can be correlated to the input record for greater understanding of what occurred.

The major drawback of all of these approaches is their inability to be implemented in real-time, as the event occurs. More specifically, the wavelet and ERA methods require the entire measured response to process and identify damage. Further, their reliance on modal properties, which can be subject to noise, has potential problems. In addition, modal properties have been shown in some cases, to be non-robust in the presence of strong noise and insensitive to small amounts of damage (Hou et al, 2000).

Adaptive identification methods were employed to identify modal parameters by Sato and Qi (1998) and Loh et al (2000). Loh et al (2000) used the adaptive fading Kalman filter technique, and Sato and Qi (1998) an Adaptive H_∞ Filter, to achieve real-time capable or near real-time capable results. What these approaches provide in real-time identification of modal parameters comes with significant computational cost and complexity.

This paper presents the development of a much simpler and efficient algorithm than existing methods for continuously monitoring the status of a steel frame structure. This task is accomplished by taking advantage of an LMS filter's ability to adaptively model noisy signals to identify changes in structural parameters in comparison to a base structural model..

II. PROBLEM DEFINITION

A seismically excited structure is can be modeled using standard linear equations of motion with an additional time varying stiffness term to account for damage:

$$\mathbf{M} \cdot \{\ddot{\mathbf{v}}\} + \mathbf{C} \cdot \{\dot{\mathbf{v}}\} + (\mathbf{K} + \Delta\mathbf{K}) \cdot \{\mathbf{v}\} = -\mathbf{M} \cdot \ddot{\mathbf{x}}_g \quad (1)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping and stiffness matrices of the model, respectively, $\ddot{\mathbf{v}}$, $\dot{\mathbf{v}}$ and \mathbf{v} are the responses of the damaged structure, and $\Delta\mathbf{K}$ contains changes in the stiffness of the structure and can be a function of time. By tracking the changes in the stiffness matrix via the $\Delta\mathbf{K}$ term, the structure's condition can be directly monitored without having to identify modal parameters or mode shapes first. These changes can be time varying or result without an input from simple modelling error.

Damping changes, $\Delta\mathbf{C}$, could also be identified and can occur due to hysteresis. However, hysteretic damping could also be seen as oscillations in $\Delta\mathbf{K}$, rather than absolute changes, and identified that way. Change in the mass matrix, $\Delta\mathbf{M}$, is not likely to be significant, hence it is ignored. Finally, the approach can be generalized to more detailed or complex models of the system and variations, as required. Finally, for steel framed structures, as in the Benchmark Problem, it is the stiffness that is most likely to change substantially.

To determine $\Delta\mathbf{K}$ using adaptive LMS a new form of is defined with time varying scalar parameters, α_i , to be determined. For a three story example, the $\Delta\mathbf{K}$ matrix is sub-divided into three matrices with entries of 1, -1 and 0 to allow independent identification of changes in k_1 , k_2 and k_3 , the story stiffnesses, from the α_i coefficients.

$$\Delta\mathbf{K} = \alpha_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha_1 + \alpha_2 & -\alpha_2 & 0 \\ -\alpha_2 & \alpha_2 + \alpha_3 & -\alpha_3 \\ 0 & -\alpha_3 & \alpha_3 \end{bmatrix} \quad (2)$$

where:

$$\alpha_1 = \Delta k_1, \quad \alpha_2 = \Delta k_2, \quad \alpha_3 = \Delta k_3 \quad (3)$$

hence,

$$\Delta\mathbf{K} = \begin{bmatrix} \Delta k_1 + \Delta k_2 & -\Delta k_2 & 0 \\ -\Delta k_2 & \Delta k_2 + \Delta k_3 & -\Delta k_3 \\ 0 & -\Delta k_3 & \Delta k_3 \end{bmatrix} = \sum_{i=1}^3 \alpha_i \Delta K_i \quad (4)$$

Rewriting Equation (1) using Equations (2) – (4) yields:

$$\mathbf{M} \cdot \{\ddot{\mathbf{v}}\} + \mathbf{C} \cdot \{\dot{\mathbf{v}}\} + \mathbf{K} \cdot \{\mathbf{v}\} + \sum_{i=1}^n \alpha_i \Delta K_i \mathbf{v} = \mathbf{F} \quad (5)$$

where n is the number of degrees-of-freedom (DOF) of the model and \mathbf{F} is the known, or estimated, input load vector. Note that n is the maximum number of coefficients to identify changes in each story stiffness. A lesser number can be used if some storys are assumed not to suffer

damage. Similarly, a greater number could be used for a more complex structural model with more DOF per story.

The varying stiffness term is simply the error between the, in this case, linear model and real measurements (\bar{v} , $\dot{\bar{v}}$ and $\ddot{\bar{v}}$). Hence, $\Delta K \bar{v}$ is the linear model error.

$$\sum_{i=1}^n \alpha_i \Delta K_i \bar{v} = \mathbf{F} - \mathbf{M} \ddot{\bar{v}} - \mathbf{C} \dot{\bar{v}} - \mathbf{K} \bar{v} \quad (6)$$

where \bar{v} , $\dot{\bar{v}}$ and $\ddot{\bar{v}}$ are measured values obtained either directly and/or from a dynamic state estimator. Equation (6) is only valid at any point in time if the α_i have the correct values. Therefore, at any discrete time, k , the difference between the linear model and actual measurements can be defined:

$$y_k = \mathbf{F}_k - \mathbf{M} \ddot{\bar{v}}_k - \mathbf{C} \dot{\bar{v}}_k - \mathbf{K} \bar{v}_k \quad (7)$$

The elements of the vector signal y_k can each be readily modelled in real-time using an adaptive LMS filter so that the coefficients α_i can be readily determined from the reduced noise modelled signal.

$$y_k = \sum_{i=1}^n \alpha_i \Delta K_i \bar{v}_k \quad (8)$$

More specifically, if each element of the vector signal y_k is modelled using an adaptive filter then the α_i are then determined using the linear system of equations defined in Equation (8) at each time step.

III. ADAPTIVE LMS APPROACHES TO SHM

In adaptive LMS filtering, the coefficients are adjusted from sample-to-sample to minimize the Mean Square Error (MSE), between a measured noisy scalar signal and its modelled value from the filter.

$$e_k = \hat{y}_k - W_k^T X_k = \hat{y}_k - \sum_{i=0}^{m-1} w_k(i) x_{k-i} = \hat{y}_k - \hat{n}_k \quad (9)$$

where W_k is the adjustable filter coefficient vector or weight vector at time k , \hat{y}_k is the noisy measured signal to be modelled or approximated, X_k is vector the input to the filter model of current and previous filter outputs, x_{k-i} , so $W_k^T X_k$ is the vector dot product output from the filter to model a scalar signal \hat{y}_k , and m is the number of taps or prior time steps considered. The Widrow-Hopf LMS algorithm for updating the weights to minimise the error, e_k is defined (Ifeachor and Jervis, 1993):

$$W_{k+1} = W_k + 2\mu \cdot e_k \cdot X_k \quad (10)$$

where μ is a positive scalar that controls the stability and rate of convergence. The general computational procedure for the basic adaptive LMS algorithm is summarized in Ifeachor and Jervis (1993).

For SHM, a noisy signal vector, y_k , for the linear model error is obtained from a simulation of the non-linear

Benchmark model, and can be modelled using adaptive LMS filters.

$$y_k = \begin{bmatrix} (\hat{y}_k)^1 \\ \vdots \\ (\hat{y}_k)^n \end{bmatrix} = \begin{bmatrix} (W_k^T X_k)^1 \\ \vdots \\ (W_k^T X_k)^n \end{bmatrix} \quad (11)$$

where each W_k^T is updated individually for n different input signals and $(W_k^T X_k)^i$ is the output for the i^{th} individual adaptive LMS filter. In this two step method, adaptive LMS filters approximate the noisy signal, $\bar{y}_k \approx y_k$ for each step, where \bar{y}_k is the estimate of y_k with dimension $n \times 1$. Hence, from Equation (8), the filter approximation, \bar{y}_k is defined:

$$\mathbf{S} \bar{y}_k = \sum_{i=1}^n \alpha_i \Delta K_i \bar{v}_k = [\Delta K_1 \bar{v}_k \quad \cdots \quad \Delta K_n \bar{v}_k] \alpha_k = A \alpha_k \quad (12)$$

where dimensions of matrix A are $n \times n$ and α_k is a $n \times 1$ vector of coefficients α_i at time k . Therefore, the α_i values can be determined analytically by solving Equation (12) as long as the matrix is full rank. This two step method is a fast and simple approach, and robust to noise because of its use of LMS filters.

A second approach is to couple these filters and solve in a single step as a One-Step method. The linear model error, estimated between the measured noisy signal and its modelled value from the filter, defined in Equation (11) can be expressed:

$$e_k = y_k - \sum_{j=0}^{m-1} \sum_{i=1}^n \alpha_{ij} \Delta K_i \bar{v}_k = y_k - Q_k \quad (13)$$

where \bar{v}_k and y_k are noisy signals, Q_k is a $n \times 1$ vector, α_{ij} are weights where $i = 1, \dots, n$ and $j = 1, \dots, m$. Hence, the change in k_i will be the sum over j of α_{ij} . This averaged approach essentially low pass filters the signal \bar{v}_k and reduces the impact of noise. An exact unfiltered solution would simply use $m = 1$. Note that there are no prior time steps involved when estimating error at time k , because y_k is not stationary, and the error, e_k , in Equation (13) is the error at this time step is really a function of the response at time k only.

Hence, the mean square error (MSE) can be defined:

$$e_k^T e_k = y_k^T y_k + Q_k^T Q_k - 2y_k^T Q_k \quad (14)$$

Adaptive LMS minimises the MSE with respect to the weights α_{ij} , and the optimum solution occurs when the gradient is zero.

$$\nabla MSE_{\alpha_{ij}} = -2e_k^T \frac{\partial Q_k}{\partial \alpha_{ij}} \quad (15)$$

where $\nabla MSE_{\alpha_{ij}}$ is one element of an $n \times m$ matrix ∇MSE and the gradient term for Q is an $n \times 1$ vector defined:

$$\frac{\partial Q_k}{\partial \alpha_{ij}} = \frac{\partial}{\partial \alpha_{ij}} \left(\sum_{j=0}^{m-1} \sum_{i=1}^n \alpha_{ij} \Delta K_i \bar{v}_k \right) = \Delta K_i \bar{v}_k \quad (16)$$

Therefore,

$$\nabla MSE = -2 \left[e_k^T \Delta K_i \bar{v}_k \right] \quad (17)$$

where ∇MSE is a $n \times m$ matrix and $[e_k^T \Delta K_i \bar{v}_k]$ is the same across an entire row for all $i = 1, \dots, n$ rows. The weight matrix of dimension $n \times m$ can then be updated.

$$w_{k+1} = w_k - \mu \nabla MSE \quad (18)$$

$$= w_k + 2\mu \left[e_k^T \Delta K_i \bar{v}_k \right]$$

where the term $[e_k^T \Delta K_i \bar{v}_k]$ is the (i,j) th element and is the same for all m elements in the i^{th} row.

Finally, decoupling the gradient estimation by approximating ΔK_i as a zero matrix with a 1.0 for the (i,i) element results in the following weight update formula:

$$w_{k+1} = w_k + 2\mu \left[e_k^T(i) \bar{v}_k(i) \right]_{n \times m} \quad (19)$$

Without these coupling terms the gradient is calculated based only on changes in diagonal elements of the stiffness matrix reducing the likelihood of coupling terms producing a near zero gradient when the error is small, near final convergence. Note that the error calculation Equation (13) still uses the ΔK_i as originally defined, and Equation (19) only changes the means by which weights are updated. This One-Step approach also leads to a computationally simpler approach by eliminating additional multiplication and addition operations.

IV. APPLICATION TO THE BENCHMARK PROBLEM

The IASC-ASCE task group on SHM was established in 1999 and the group developed a series of benchmark SHM problems (Johnson et al, 2000). In the 12 DOF model, the structure is assumed to act as a shear building with three DOF per floor: translation in x- and y-direction and rotation. In this paper, only the 12 DOF model and the simpler, one direction, 4 DOF model are considered, and the displacements, velocities and accelerations of each story are assumed to be either measured and/or estimated.

The following set of parameters are used in all simulations, unless otherwise stated:

- Input load(s) = $1 \times 10^6 \sim 10^7 \sin(30t) N$
- Sample rate = 100 Hz
- $\mu = 0.3$
- Number of taps, $m = 5$

The convergence rate of the weights in the algorithm depends on the LMS parameter μ and the number of taps used. Even though faster convergence for each different case of the Benchmark problem can be achieved by varying those parameters, they would typically be fixed in a practical application. The values used here were developed

by trial and error to illustrate the methods developed and may not be completely optimal.

Table 1 shows the convergence times for the One Step method and the Two Step method for all the 4 and 12 DOF sudden failure cases examined. These times are the time taken for α_l (change in stiffness of the first floor in y-direction) to reach 90 and 95 percent of the actual change from the time when the damage occurred. The first story is examined due to its dominance in seismic structural response and rotational DOF can take longer to converge in the 12 DOF model. The times are very quick, indicating the potential for both methods in real-time or adaptive algorithms.

Table 1: Convergence times (seconds) for α_l

Case	Damage Pattern	One Step method		Two Step method	
		90 %	95 %	90 %	95 %
1	1	0.33	0.41	0.20	0.21
	2	0.31	0.33	0.21	0.22
	3	0.11	0.12	0.20	0.21
	4	0.31	0.34	0.21	0.22
3	1	0.33	0.41	0.20	0.21
	2	0.31	0.33	0.21	0.22
	3	0.21	0.32	0.08	0.08
	4	0.21	0.32	0.08	0.08
4	1	0.21	0.22	0.09	0.01
	2	0.13	0.15	0.21	0.23
	3	0.29	0.32	0.22	0.28
	4	0.29	0.32	0.21	0.28

Figures 1 and 2 shows the One-Step method results for case 1 and damage patterns 1 for a sudden failure occurring at 5 seconds, as well as for gradual failure starting at 5 seconds and taking 5 seconds to finish. The figure shows that only the first story coefficient changes as expected. Additionally, the fast convergence times are illustrated in the rapid tracking of the actual damage function, particularly for the sudden failure. The gradual failure convergence is not quite as fast due to the fact that the filter does not track as fast while the error is changing every step.

For more complex cases the plots are not necessarily clear, however two results emerge. First, rotational DOF in the 12 DOF model take longer to converge, up to 10 seconds, which may not be suitable for adaptive control applications. Second, some ai parameters that end up zero valued, may not initially be zero while convergence is not complete, also inhibiting the applicability for adaptive control applications. These results indicate that this approach will work best for shear building model based

approaches to SHM, but is still effective for identifying damage when rotational DOF are included.

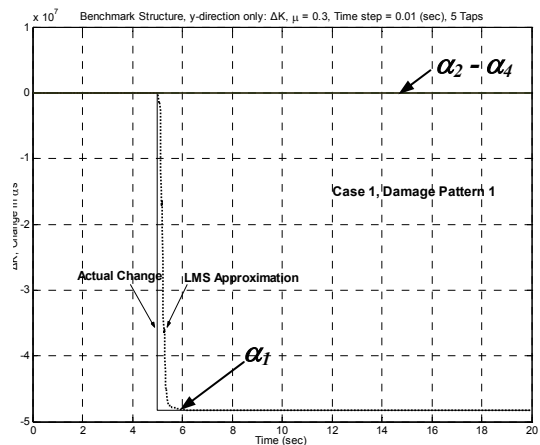


Fig. 1: Case 1 Damage Pattern 1 sudden failure.

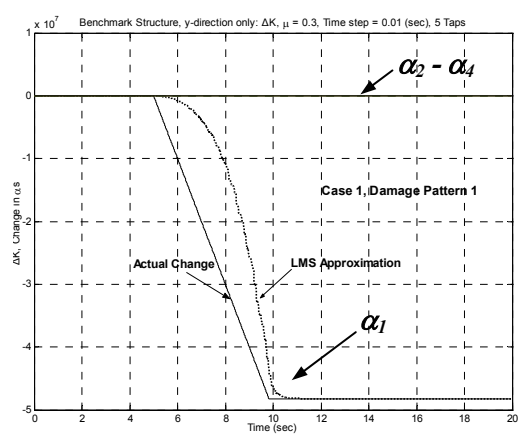


Fig. 2: Case 1 Damage Pattern 1 gradual failure

Modal parameters for each of the cases studied can be determined by reconstructing the damaged stiffness matrix and finding the eigenvalues of the system. In each case the final natural frequencies are all within 1% for all 4, or 12, modes of the structure. This performance matches those in the literature for the cases that have been previously reported.

For real-time applications however it is the convergence times that are most important along with the computational intensity. From an examination of the One Step method, there are, conservatively, 1400 single cycle operations per time step, including memory storage and retrieval for a 4 DOF model. If a sampling rate of 100 Hz is used, then 0.14 MHz (or mega-cycles) of computation is required and 1.4 MHz are required for a sampling rate of 1000 Hz. A 12 DOF model would require approximately 3 times more computational effort. A current Digital Signal Processing (DSP) chip operates at 300 – 1000 MHz. At a single operation per chip clock cycle, and many such chips have up to four operations per cycle, computation of the

One Step method is well within this range. The Two Step method would involve approximately ten times more computation due to the matrix solutions required. Therefore, SHM for civil structures using the adaptive LMS filtering based methods as presented could be readily implemented in real-time, even without any significant computational simplifications or parallelization.

V. CONCLUSIONS

This paper presents SHM methods for civil structures using adaptive Least Mean Square filtering theory. Damage that occurs in the structure can be identified by changes in the stiffness matrix. One Step and Two Step adaptive LMS based methods were developed and tested. All of the 4 and 12 DOF cases of the SHM Task group’s Benchmark problems were tested using the proposed methods, and the results show that the adaptive LMS filtering is very effective for identifying damage in real-time.

The different variations are compared and the method without coupling terms in the gradient calculation is seen to converge the fastest. However, the final results for all methods converge to the desired final values. In each case the changes in stiffness are determined directly and then the modal parameters presented are calculated for comparison. The resulting modal parameters are well within 1% of the IASC-ASCE Benchmark problem results.

The methods presented require only 0.14 – 1.4 Mega-cycles of computation and can operate on a sample to sample basis without requiring the entire record. Hence, they are all suitable for real-time implementation, and the One Step method without coupling in the gradient calculation has convergence times for the Benchmark problem under 0.41 seconds making it suitable for adaptive control applications. Convergence times for the Two Step method presented are faster, however the computational costs are significantly higher. Finally, the convergence times of the adaptive LMS methods presented improve as sampling rate increases from the 100 Hz of the Benchmark problem to a still practicable value of 1000 Hz. Overall, these methods provide accurate, robust identification of damage with stability, little computational cost, and fast convergence.

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