TRACKING OF MULTIPLE MANEUVERING TARGETS USING MULTISCAN JPDA AND IMM FILTERING

Sumedh Puranik *Jitendra K. Tugnait*

Department of Electrical & Computer Engineering
Auburn University, Auburn, Alabama 36849, USA
Email: puransp, tugnajk@eng.auburn.edu

ABSTRACT

We consider the problem of tracking multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach and joint probabilistic data association (JPDA) technique. But unlike the standard single scan JPDA approach, we exploit a multiscan joint probabilistic data association (Mscan-JPDA) approach to solve the data association problem. The algorithm is illustrated via a simulation example involving tracking of three maneuvering targets and a multiscan data window of length two.

1. INTRODUCTION

We consider the problem of tracking multiple maneuvering targets in presence of clutter using switching multiple target motion models. The switching multiple model approach has been found to be very effective in modeling maneuvering targets [1]-[4],[9]. In this approach various modes of target motion are represented by distinct kinematic models, and in a Bayesian framework, the target maneuvers are modeled by switching among these models controlled by a Markov chain. While tracking multiple targets in the presence of clutter, one has to solve the problem of measurement origin uncertainty, i.e. how to associate the data available at the sensor(s) with various targets or clutter (false measurements). In the Bayesian framework the standard JPDA algorithm uses only a single (latest) scan data available at the sensors. To use more information to solve the data association problem, the idea of using multiple scans of data (current and past scans) seems to have been initially proposed by Drummond [8]. Drummond describes some practical issues involved but does not discuss detailed problem formulation and technical issues for multiscan JPDA. Roecker [6] has extended Drummond's ideas where he has discussed problem formulation and solution in some detail. In a simulation example presented in [6] it has been shown that performance improvement in multiple target tracking can be achieved via multiscan JPDA as compared to single scan JPDA with most of the improvement gains achieved via a window size of 2 or 3 scans.

In [6] only non-maneuvering targets (i.e. one model per target) have been considered. In this paper, we extend Roecker's approach to highly maneuvering targets where we allow multiple kinematic motion models per target. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach and multiple scan joint probabilistic data association (Mscan-JPDA) technique. The algorithm is illustrated via a simulation example involving tracking of three maneuvering targets and multiscan data window of length two.

The paper is organized as follows. The basic multiscan JPDA problem is explained in Sec. 2 followed by the presentation of the problem formulation in Sec. 3. The proposed IMM-based multiscan JPDA algorithm is described in Sec. 4 for the case of a sliding multiscan window of size 2. A computer simulation example is presented in Sec. 5.

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2. MULTISCAN JOINT PROBABILISTIC DATA ASSOCIATION

For multiple target tracking in presence of clutter, the JPDA algorithm has been developed [1],[3] which computes the probabilistic weight for measurement-to-track association jointly across the set of all targets and clutter. Basically it defines all the feasible joint events for the known number of targets and clutter. Each feasible joint event is a unique event that represents the association of measurements to targets and clutter. A disadvantage of JPDA is that it uses only the data present in the current scan; in multiscan JPDA, we use multiple scans. In a single scan JPDA we define single scan joint events. Similarly in the multiscan scenario we define multiple-scan joint events as follows. A marginal association event $\theta_{ir}(k)$ is said to be effective at time scan k when the validated measurement $y_k^{(i)}$ is associated with (i.e. originates from) target r ($r = 0, 1, \cdots, N$ where $r = 0$ means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event Θ_k is said to be effective when a set of marginal events $\{\theta_{ir}(k)\}$ holds true simultaneously. That is, $\Theta_k = \bigcap_{i=1}^{m} \theta_{ir_i}(k)$ where r_i is the index of the target to which measurement $y_k^{(i)}$ is associated in the event under consideration, ($i = 1, 2, \cdots, m$). In the multiscan case with a scan window size L (L-scan-back) and $k_s = k - L + s$, we define multiscan joint events

$$\Theta_{kL} = \bigcap_{s=1}^{L} \bigcap_{i=1}^{m} \theta_{ir_{is}}(k_s)$$

where $\theta_{ir_{is}}(k_s)$ is the marginal association event that at time scan k_s, the ith validated measurement $y_{k_s}^{(i)}$ is associated with target r_{is}. Let $|\{\Theta_k\}|$ denote the total number of feasible joint events in the single scan case. In the multiscan case we get total number of multiscan feasible joint events as $|\{\Theta_k\}| \times |\{\Theta_{k-1}\}| \times \cdots \times |\{\Theta_{k-L+1}\}|$, derived from a Cartesian product of joint events present in all scans considered in the scan window. As one can see, the number of feasible multiscan joint events grow exponentially with multiscan window length, and even for a multiscan window of length two or three, this number can be large.

3. PROBLEM FORMULATION

Assume that there are total N targets with the target set denoted as $\mathcal{T}_N := \{1, 2, \cdots, N\}$. Assume that the dynamics of each target can be modeled by one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \cdots, n\}$. For target r ($r \in \mathcal{T}_N$), the event that model i is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_k^i(r)$. For the j-th hypothesized model (mode), the state dynamics and measurements of target r ($r \in \mathcal{T}_N$) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r), \qquad (1)$$

$$z_k(r) = h^j(x_k(r)) + w_k^j(r) \qquad (2)$$

where $x_k(r)$ is the system state of target r at t_k and of dimension n_x (assuming all targets share a common state space), $z_k(r)$ is the (true) measurement vector (i.e. due to target r) at t_k and of dimension n_z, $F^j_{k-1}(r)$ and $G^j_{k-1}(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r and h^j is the nonlinear transformation of $x_k(r)$ to $z_k(r)$ for model j. A first-order linearized version of (2) is given by

$$z_k(r) = H^j_k(r)x_k(r) + w^j_k(r) \tag{3}$$

where $H^j_k(r)$ is the Jacobian matrix of h^j evaluated at some value of the estimate of state $x_k(r)$ (see Sec. 4). The process noise $v^j_{k-1}(r)$ and the measurement noise $w^j_k(r)$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q^j_{k-1} (same for all targets) and R^j_k (same for all targets), respectively. At the initial time t_0, the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}^j_0(r)$ and the known covariance $P^j_0(r)$. The probability of target r in model j at t_0, $\mu^j_0(r) = P\{M^j_0(r)\}$, is also assumed to be known. The switching from model $M^i_{k-1}(r)$ to model $M^j_k(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P\{M^j_k(r)|M^i_{k-1}(r)\}$. Henceforth, t_k will be denoted by k.

Note that, in general, at any time k, some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time k. The measurement set (not yet validated) generated at time k is denoted as $Z_k := \{z^{(1)}_k, z^{(2)}_k, \cdots, z^{(m)}_k\}$ where m is the number of measurements generated at time k. Variable $z^{(i)}_k$ $(i = 1, \cdots, m)$ is the ith measurement within the set. The validated set of measurements at time k will be denoted by Y_k, containing $\overline{m}$ $(\leq m)$ measurement vectors. The cumulative set of validated measurements up to time k is denoted as $\mathcal{Z}^k = \{Y_1, Y_2, \cdots, Y_k\}$.

We make the following (standard) assumptions: It is assumed that the number of targets (N) is known and that for each target track has been initiated, and our objective is to maintain the tracks. Assuming there are no unresolved measurements (i.e. measurements associated with two or more targets simultaneously), any measurement therefore is either associated with a single target or caused by clutter. Clutter is modeled as independently and identically distributed (i.i.d.) with uniform spatial distribution over the entire validation region (across all targets). State estimate of individual targets conditioned on the modes, joint events and set of measurements are mutually independent and Gaussian distributed i.e. states of the targets are not coupled and estimation is carried out independently. Multiscan window of length two will be used to compute multiscan joint probabilities. Extension to higher lengths is straightforward but tedious.

The goal is to find the filtered state estimate for target r $(r \in \mathcal{T}_N)$

$$\widehat{x}_{k|k}(r) := E\{x_k(r)|\mathcal{Z}^k\} \tag{4}$$

and the associated error covariance matrix

$$P_{k|k}(r) := E\{[x_k(r) - \widehat{x}_{k|k}(r)][x_k(r) - \widehat{x}_{k|k}(r)]'|\mathcal{Z}^k\} \tag{5}$$

where $x_k(r)'$ denotes the transpose of $x_k(r)$.

4. IMM/MSCAN-JPDA FILTERING ALGORITHM

We now extend the single scan IMM/JPDA filtering algorithm of [5] to apply to the multiscan case. As in [6] we will follow a sliding window multiscan approach. The approach of [5], in turn, is based on the approaches of [1], [3]. As the IMM/PDAF algorithm is well-explained in [3, Sec. 4.5], the JPDAF algorithm is well-explained in [1, Sec. 9.3] and [3, Sec. 6.2], and the IMM/JPDA filter is given in detail in [5], we will only briefly outline the basic steps in "one cycle" (i.e. processing needed to update for a new set of measurements and a new multiscan window) of the IMM/JPDA multiscan filter. We assume that the scan window size is two. Given state estimate at time $k-1$ based on data up to time $k-1$, in Sec. 4.1 we provide first scan steps (using data up to time k) and in Sec. 4.2 we provide the second scan steps (using data up to time $k+1$).

Assumed available: Given the state estimate $\widehat{x}^j_{k-1|k-1}(r) := E\{x_{k-1}(r)|M^j_{k-1}(r), \mathcal{Z}^{k-1}\}$, the associated covariance $P^j_{k-1|k-1}(r)$ and the conditional mode probability $\mu^j_{k-1}(r) = P[M^j_{k-1}(r)|\mathcal{Z}^{k-1}]$ at time $k-1$ for each mode $j \in \mathcal{M}_n$ and each target $r \in \mathcal{T}_N$.

4.1. First Scan Steps

Step 1.1. Interaction – mixing of the estimate from the previous time $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$: The expressions for the predicted mode probability $\mu^{j-}_k(r) := P\{M^j_k(r)|\mathcal{Z}^{k-1}\}$ and the mixing probability $\mu^{i|j}(r) := P\{M^i_{k-1}(r)|M^j_k(r), \mathcal{Z}^{k-1}\}$ are as in [5, Sec. 4.2]. Similarly, the expressions for the mixed estimate $\widehat{x}^{0j}_{k-1|k-1}(r) := E\{x_{k-1}(r)|M^j_k(r), \mathcal{Z}^{k-1}\}$ and the associated covariance $P^{0j}_{k-1|k-1}(r) := E\{[x_{k-1}(r) - \widehat{x}^{0j}_{k-1|k-1}(r)][x_{k-1}(r) - \widehat{x}^{0j}_{k-1|k-1}(r)]'|M^j_k(r), \mathcal{Z}^{k-1}\}$ are as in [5, Sec. 4.2].

Step 1.2. Predicted state $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$:

State prediction:

$$\widehat{x}^j_{k|k-1}(r) := E\{x_k(r)|M^j_k(r), \mathcal{Z}^{k-1}\} = F^j_{k-1}\widehat{x}^{0j}_{k-1|k-1}(r). \tag{6}$$

State prediction error covariance:

$$P^j_{k|k-1}(r) = \widetilde{F}^j_{k-1}P^{0j}_{k-1|k-1}(r)\widetilde{F}^{j\prime}_{k-1} + \widetilde{G}^j_{k-1}Q^j_{k-1}\widetilde{G}^{j\prime}_{k-1}. \tag{7}$$

The mode-conditioned predicted measurement of target r, $\widehat{z}^j_k(r)$, and the covariance $S^j_k(r)$ of the mode-conditioned residual $\nu^{j(i)}_k(r) := z^{(i)}_k - \widehat{z}^j_k(r)$ are as in [5].

Step 1.3. Measurement validation: This is exactly as in Step 3.3 of [5]. Denote the volume of validation region for the whole target set by V_k where $V_k = \sum_{r=1}^N V_k(r)$, and $V_k(r)$ is the validation region for target r.

Step 1.4. State estimation with validated measurements $(\forall j \in \mathcal{M}_n, \forall r \in \mathcal{T}_N)$: From among all the raw measurements at time k, i.e., $Z_k := \{z^{(1)}_k, z^{(2)}_k, \cdots, z^{(m(k))}_k\}$, define the set of validated measurement for sensor 1 at time k as $Y_k := \{y^{(1)}_k, y^{(2)}_k, \cdots, y^{(\overline{m}(k))}_k\}$ where $\overline{m}(k)$ is the total number of validated measurement at time k and $y^{(i)}_k := z^{(l_i)}_k$ where $1 \leq l_1 < l_2 < \cdots < l_{\overline{m}(k)} \leq m(k)$ when $\overline{m}(k) \neq 0$. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \cdots, \overline{m}(k), \quad r = 0, \cdots, N \tag{8}$$

where ω_{ir} =1 if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ_k is represented by the event matrix

$$\widehat{\Omega}(\Theta_k) = [\widehat{\omega}_{ir}(\Theta_k)] \quad i = 1, \cdots, \overline{m}(k), \quad r = 0, \cdots, N \tag{9}$$

where $\widehat{\omega}_{ir}(\Theta_k) = 1$ if $\theta_{ir}(k) \subset \Theta_k$, else it is 0. A feasible association event is one where a measurement can have only one source, i.e. $\sum_{r=0}^N \widehat{\omega}_{ir}(\Theta_k) = 1 \;\; \forall i$, and where at most one measurement can originate from a target, i.e. $\delta_r(\Theta_k) := \sum_{i=0}^{\overline{m}(k)} \widehat{\omega}_{ir}(\Theta_k) \leq 1$ for $r = 1, \cdots, N$.

The above joint events Θ_k are mutually exclusive and exhaustive. Define the binary measurement association indicator $\tau_i(\Theta_k) := \sum_{r=1}^{N} \widehat{\omega}_{ir}(\Theta_k), \quad i = 1, \cdots, \overline{m}(k)$, to indicate whether the validated measurement $y_k^{(i)}$ is associated with a target in event Θ_k. Furthermore, the number of false (unassociated) measurements in event Θ_k is $\phi(\Theta_k) = \sum_{i=1}^{\overline{m}(k)} [1 - \tau_i(\Theta_k)]$. We will limit our discussion to nonparametric JPDA [2],[5]. One can evaluate the likelihood that the target r is in model j_r as [5] $\Lambda_k^{j_r}(r) :=$

$$p[Y_k|M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{\Theta_k} p[Y_k|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] P\{\Theta_k\}. \tag{10}$$

The first term in the last line of (10) can be written as

$$p[Y_k|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}] = \sum_{j_1=1}^{n} \cdots \sum_{j_{r-1}=1}^{n} \sum_{j_{r+1}=1}^{n} \cdots \sum_{j_N=1}^{n}$$
$$p[Y_k|\Theta_k, M_k^{j_1}(1), \cdots, M_k^{j_N}(N), \mathcal{Z}^{k-1}] \times P\{M_k^{j_1}(1), \cdots,$$
$$M_k^{j_{r-1}}(r-1), M_k^{j_{r+1}}(r+1), \cdots, M_k^{j_N}(N)|\Theta_k, M_k^{j_r}(r), \mathcal{Z}^{k-1}\}. \tag{11}$$

The second term in the last line of (10) turns out to be [5]

$$P\{\Theta_k\} = \frac{\phi(\Theta_k)!\, \epsilon}{\overline{m}(k)!} \prod_{s=1}^{N} (P_D)^{\delta_s(\Theta_k)} (1 - P_D)^{1-\delta_s(\Theta_k)} \tag{12}$$

where P_D is the detection probability (assumed to be the same for all targets) and $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant. We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term in (11) is

$$p[Y_k|\Theta_k, M_k^{j_1}(1), \cdots, M_k^{j_N}(N), \mathcal{Z}^{k-1}]$$
$$\approx \prod_{i=1}^{\overline{m}(k)} p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}], \quad \theta_{ir_i}(k) \subset \Theta_k \tag{13}$$

where the conditional pdf of the validated measurement $y_k^{(i)}$ given its origin and target mode, is given by $p[y_k^{(i)}|\theta_{ir_i}(k), M_k^{j_{r_i}}(r_i), \mathcal{Z}^{k-1}] = \mathcal{N}(y_k^{(i)}; \widehat{z}_k^{j_{r_i}}(r_i), S_k^{j_{r_i}}(r_i))$ if $\tau_i(\Theta_k) = 1$, else it equals $1/V_k$ where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp[-\frac{1}{2}(x-y)' P^{-1}(x-y)]. \tag{14}$$

The second term on the right-side of (11) is given by $\prod_{s=1, s\neq r}^{N} \mu_k^{j_s -}(s)$. Moreover, $P\{\Theta_k|M_k^j(r), \mathcal{Z}^{k-1}, Y_k\}$

$$= \frac{1}{c} p[Y_k|\Theta_k, M_k^j(r), \mathcal{Z}^{k-1}] P\{\Theta_k\} =: \beta_k^j(r, \Theta_k)$$

where c is such that $\sum_{\Theta_k} \beta_k^j(r, \Theta_k) = 1$. The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_k^{j_r}(r)$ (needed in Step 1.5 later) via (10)-(14). Define the target and mode-conditioned innovations $\nu_k^j(r, \Theta_k) := y_k^{(i)} - \widehat{z}_k^j(r)$ if $\theta_{ir}(k) \subset \Theta_k$, else 0. Using $\widehat{x}_{k|k-1}^j(r)$ and its covariance $P_{k|k-1}^j(r)$, one computes the state update $\widehat{x}_{k|k}^j(r)$ and its covariance $\widetilde{P}_{k|k}^j(r)$ according to the standard PDAF [5].

Step 1.5. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): $\mu_k^j(r) := P[M_k^j(r)|\mathcal{Z}^k]$

$$= P[M_k^j(r)|\mathcal{Z}^{k-1}] p[Y_k|M_k^j(r), \mathcal{Z}^{k-1}] = \frac{1}{c} \mu_k^{j-}(r) \Lambda_k^j(r)$$

where c is such that $\sum_{j=1}^{n} \mu_k^j(r) = 1$.

Step 1.6. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final state estimate update at time k is given by $\widehat{x}_{k|k}(r) = \sum_{j=1}^{n} \widehat{x}_{k|k}^j(r) \mu_k^j(r)$ and its covariance $P_{k|k}(r)$ is given by

$$\sum_{j=1}^{n} \left\{ P_{k|k}^j(r) + [\widehat{x}_{k|k}^j(r) - \widehat{x}_{k|k}(r)][\widehat{x}_{k|k}^j(r) - \widehat{x}_{k|k}(r)]' \right\} \mu_k^j(r).$$

4.2. Second Scan Steps

Here we update to scan $k+1$, given data up to time $k+1$, with a sliding scan window of size two, scans $\{k, k+1\}$.

Step 2.1. Interaction − mixing of the estimate from the previous time ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

$$\mu_k^j(r, \Theta_k) := P\{M_k^j(r)|\mathcal{Z}^k, \Theta_k\}$$
$$= c' p[Y_k|M_k^j(r), \mathcal{Z}^{k-1}, \Theta_k] P\{M_k^j(r)|\mathcal{Z}^{k-1}, \Theta_k\}$$
$$= c \beta_k^j(r, \Theta_k) \mu_k^{j-}(r). \tag{15}$$

predicted mode probability: $\mu_{k+1}^{j-}(r, \Theta_k) :=$

$$P\{M_{k+1}^j(r)|\mathcal{Z}^k, \Theta_k\} = \sum_{i=1}^{n} p_{ij} \mu_k^i(r, \Theta_k). \tag{16}$$

mixing probability: $\mu^{i|j}(r, \Theta_k) :=$

$$P\{M_k^i(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = p_{ij} \mu_k^i(r, \Theta_k) / \mu_{k+1}^{j-}(r, \Theta_k). \tag{17}$$

mixed estimate: $\widehat{x}_{k|k}^{0j}(r, \Theta_k) := E\{x_k(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$

$$= \sum_{i=1}^{n} \widehat{x}_{k|k}^i(r, \Theta_k) \mu^{i|j}(r, \Theta_k). \tag{18}$$

covariance of the mixed estimate: $P_{k|k}^{0j}(r, \Theta_k) :=$

$$E\{[x_k(r) - \widehat{x}_{k|k}^{0j}(r, \Theta_k)][x_k(r) - \widehat{x}_{k|k}^{0j}(r, \Theta_k)]'|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$$
$$= \sum_{i=1}^{n} \{P_{k|k}^i(r, \Theta_k) + [\widehat{x}_{k|k}^i(r, \Theta_k) - \widehat{x}_{k|k}^{0j}(r, \Theta_k)]$$
$$\times [\widehat{x}_{k|k}^i(r, \Theta_k) - \widehat{x}_{k|k}^{0j}(r, \Theta_k)]'\} \mu^{i|j}(r, \Theta_k). \tag{19}$$

Step 2.2. Predicted state ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

State prediction: $\widehat{x}_{k+1|k}^j(r, \Theta_k) :=$

$$E\{x_{k+1}(r)|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = F_k^j \widehat{x}_{k|k}^{0j}(r, \Theta_k). \tag{20}$$

State prediction error covariance: $P_{k+1|k}^j(r, \Theta_k)$

$$:= E\{[x_{k+1}(r) - \widehat{x}_{k+1|k}^j(r, \Theta_k)][x_{k+1}(r) - \widehat{x}_{k+1|k}^j(r, \Theta_k)]'|$$
$$M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\} = F_k^j P_{k|k}^{0j}(r, \Theta_k) F_k^{j'} + G_k^j Q_k^j G_k^{j'}. \tag{21}$$

Using (2) and (20), the mode-conditioned predicted measurement of target r is

$$\widehat{z}_{k+1}^j(r, \Theta_k) := h^j(\widehat{x}_{k+1|k}^j(r, \Theta_k)). \tag{22}$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu_{k+1}^{j(i)}(r, \Theta_k) := z_{k+1}^{(i)} - \widehat{z}_{k+1}^j(r, \Theta_k)$ is given by

$$S_{k+1}^j(r, \Theta_k) := E\{\nu_{k+1}^{j(i)}(r, \Theta_k) \nu_{k+1}^{j(i)}(r, \Theta_k)'|M_{k+1}^j(r), \mathcal{Z}^k, \Theta_k\}$$

$$= H^j_{k+1}(r,\Theta_k)P^j_{k+1|k}(r,\Theta_k)H^{j'}_{k+1}(r,\Theta_k) + R^j_{k+1} \quad (23)$$

where $H^j_{k+1}(r,\Theta_k)$ is the first order derivative (Jacobian matrix) of $h^j(.)$ at $\widehat{x}^{j(0)}_{k+1|k}(r,\Theta_k)$.

Step 2.3. Measurement validation: For target r, the validation region is taken to be the same for all models and Θ_k's, i.e., as the largest of them. Let

$$(j_r, \overline{\Theta}_k) := \arg\left\{\max_{j\in\mathcal{M}_n,\,\Theta_k} |S^j_{k+1}(r,\Theta_k)|\right\}. \quad (24)$$

Then measurement $z^{(i)}_{k+1}$ ($i = 1, 2, \cdots, m(k+1)$) is validated if and only if

$$[z^{(i)}_{k+1} - \widehat{z}^{j_r}_{k+1}(r,\overline{\Theta}_k)]'[S^{j_r}_{k+1}(r,\overline{\Theta}_k)]^{-1}[z^{(i)}_{k+1} - \widehat{z}^{j_r}_{k+1}(r,\overline{\Theta}_k)] < \gamma \quad (25)$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is

$$V_{k+1}(r) := c_{n_z}\gamma^{n_z/2}|S^{j_r}_{k+1}(r,\overline{\Theta}_k)|^{1/2} \quad (26)$$

where n_z is the dimension of the measurement and c_{n_z} is the volume of the unit hypersphere of this dimension. The volume of validation region for the whole target set is approximated by $V_{k+1} = \sum_{r=1}^N V_{k+1}(r)$.

Step 2.4. State estimation with validated measurements ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): From among all the raw measurements at time $k+1$, i.e., $Z_{k+1} := \{z^{(1)}_{k+1}, z^{(2)}_{k+1}, \cdots, z^{(m(k+1))}_{k+1}\}$, define the set of validated measurement for sensor 1 at time $k+1$ as

$$Y_{k+1} := \{y^{(1)}_{k+1}, y^{(2)}_{k+1}, \cdots, y^{(\overline{m}(k+1))}_{k+1}\} \quad (27)$$

where $\overline{m}(k+1)$ is the total number of validated measurement at time $k+1$. and

$$y^{(i)}_{k+1} := z^{(l_i)}_{k+1} \quad (28)$$

where $1 \le l_1 < l_2 < \cdots < l_{\overline{m}(k+1)} \le m(k+1)$ when $\overline{m}(k+1) \neq 0$. Note that all targets share a common validated measurement set Y_{k+1}. We now consider joint probabilistic data association across targets as in Sec. 4.1. Define the validation matrix

$$\Omega = [\omega_{ir}] \quad i = 1, \cdots, \overline{m}(k+1), \quad r = 0, \cdots, N \quad (29)$$

where ω_{ir} =1 if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ_{k+1} is represented by the event matrix

$$\widehat{\Omega}(\Theta_{k+1}) = [\widehat{\omega}_{ir}(\Theta_{k+1})] \quad i = 1, \cdots, \overline{m}(k+1), \quad r = 0, \cdots, N \quad (30)$$

where

$$\widehat{\omega}_{ir}(\Theta_{k+1}) = \begin{cases} 1 & \text{if } \theta_{ir}(k+1) \subset \Theta_{k+1} \\ 0 & \text{otherwise.} \end{cases} \quad (31)$$

A feasible association event is one where a measurement can have only one source $\sum_{r=0}^N \widehat{\omega}_{ir}(\Theta_{k+1}) = 1 \quad \forall i$, and where at most one measurement can originate from a target $\delta_r(\Theta_{k+1}) := \sum_{i=0}^{\overline{m}(k+1)} \widehat{\omega}_{ir}(\Theta_{k+1}) \le 1$ for $r = 1, \cdots, N$. The above joint events Θ_{k+1} are mutually exclusive and exhaustive.

Define the binary measurement association indicator $\tau_i(\Theta_{k+1}) := \sum_{r=1}^N \widehat{\omega}_{ir}(\Theta_{k+1})$, $i = 1, \cdots, \overline{m}(k+1)$, to indicate whether the validated measurement $y^{(i)}_{k+1}$ is associated with a target in event Θ_{k+1}. Furthermore, the number of false (unassociated) measurements in event Θ_{k+1} is $\phi(\Theta_{k+1}) = \sum_{i=1}^{\overline{m}(k+1)}[1 - \tau_i(\Theta_{k+1})]$. We will limit our discussion to nonparametric JPDA [1],[3]. One can evaluate the likelihood that the target r is in model j_r as

$$\Lambda^{j_r}_{k+1}(r) := p[Y_{k+1}|M^{j_r}_{k+1}(r), \mathcal{Z}^k]$$
$$= \sum_{\Theta_k}\sum_{\Theta_{k+1}} p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M^{j_r}_{k+1}(r), \mathcal{Z}^k]$$
$$\times P\{\Theta_{k+1}\}P\{\Theta_k|M^{j_r}_{k+1}(r), \mathcal{Z}^k\}. \quad (32)$$

The first term in the last line of (32) can be written as

$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M^{j_r}_{k+1}(r), \mathcal{Z}^k]$$
$$= \sum_{j_1=1}^n \cdots \sum_{j_{r-1}=1}^n \sum_{j_{r+1}=1}^n \cdots \sum_{j_N=1}^n p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M^{j_1}_{k+1}(1),$$
$$\cdots, M^{j_{r-1}}_{k+1}(r-1), M^{j_r}_{k+1}(r), M^{j_{r+1}}_{k+1}(r+1), \cdots, M^{j_N}_{k+1}(N), \mathcal{Z}^k]$$
$$\times P\{M^{j_1}_{k+1}(1), \cdots, M^{j_{r-1}}_{k+1}(r-1), M^{j_{r+1}}_{k+1}(r+1),$$
$$\cdots, M^{j_N}_{k+1}(N)|\Theta_k, \Theta_{k+1}, M^{j_r}_{k+1}(r), \mathcal{Z}^k\}. \quad (33)$$

The second term (apriori joint association probabilities) in the last line of (32) turns out to be ([2, Sec. 6.2])

$$P\{\Theta_{k+1}\} = \frac{\phi(\Theta_{k+1})!\,\epsilon}{\overline{m}(k+1)!}\prod_{s=1}^N (P_D)^{\delta_s(\Theta_{k+1})}(1-P_D)^{1-\delta_s(\Theta_{k+1})} \quad (34)$$

where P_D is the detection probability (assumed to be the same for all targets) and $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant. The third term in the last line of (32) is given by

$$P\{\Theta_k|M^{j_r}_{k+1}(r), \mathcal{Z}^k\} = c'P\{M^{j_r}_{k+1}(r)|\Theta_k, \mathcal{Z}^k\}P\{\Theta_k|\mathcal{Z}^k\}$$
$$= c\mu^{j_r-}_{k+1}(r,\Theta_k)P\{\Theta_k\}p[Y_k|\Theta_k, \mathcal{Z}^{k-1}] \quad (35)$$

where

$$p[Y_k|\Theta_k, \mathcal{Z}^{k-1}] = \sum_j p[Y_k|\Theta_k, M^j_k(r), \mathcal{Z}^{k-1}]\mu^{j-}_k(r). \quad (36)$$

We assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term on the right-side of (33) can be written as

$$p[Y_{k+1}|\Theta_k, \Theta_{k+1}, M^{j_1}_{k+1}(1), \cdots, M^{j_{r-1}}_{k+1}(r-1), M^{j_r}_{k+1}(r),$$
$$M^{j_{r+1}}_{k+1}(r+1), \cdots, M^{j_N}_{k+1}(N), \mathcal{Z}^k] \approx$$
$$\prod_{i=1}^{\overline{m}(k+1)} p[y^{(i)}_{k+1}|\theta_{ir_i}(k+1), \Theta_k, M^{j_{r_i}}_{k+1}(r_i), \mathcal{Z}^k], \quad \theta_{ir_i}(k+1) \subset \Theta_{k+1} \quad (37)$$

where the conditional pdf of the validated measurement $y^{(i)}_{k+1}$ given its origin and target mode, is given by

$$p[y^{(i)}_{k+1}|\theta_{ir_i}(k+1), \Theta_k, M^{j_{r_i}}_{k+1}(r_i), \mathcal{Z}^k] =$$
$$\begin{cases} \mathcal{N}(y^{(i)}_{k+1}; \widehat{z}^{j_{r_i}}_{k+1}(r_i,\Theta_k), S^{j_{r_i}}_{k+1}(r_i,\Theta_k)) & \text{if } \tau_i(\Theta_{k+1}) = 1, \\ 1/V_{k+1} & \text{if } \tau_i(\Theta_{k+1}) = 0. \end{cases} \quad (38)$$

The second term on the right-side of (33) is given by

$$P\{M^{j_1}_{k+1}(1), \cdots, M^{j_{r-1}}_{k+1}(r-1), M^{j_{r+1}}_{k+1}(r+1),$$

$$\cdots, M_{k+1}^{j_N}(N)|\Theta_k, \Theta_{k+1}, M_{k+1}^{j_r}(r), \mathcal{Z}^k\}$$

$$= \prod_{s=1, s\neq r}^{N} P\{M_{k+1}^{j_s}(s)|\Theta_{k+1}, \Theta_k, M_{k+1}^{j_r}(r), \mathcal{Z}^k\}$$

$$= \prod_{s=1, s\neq r}^{N} P\{M_{k+1}^{j_s}(s)|\mathcal{Z}^k, \Theta_k\} = \prod_{s=1, s\neq r}^{N} \mu_{k+1}^{j_s -}(s, \Theta_k). \quad (39)$$

The probability of the joint association events Θ_{k+1} and Θ_k given that model j is effective for target r from time k through $k+1$ is

$$P\{\Theta_{k+1}, \Theta_k | M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}\}$$

$$= \frac{1}{c} p[Y_{k+1}|\Theta_{k+1}, \Theta_k, M_{k+1}^j(r), \mathcal{Z}^k] P\{\Theta_{k+1}\} P\{\Theta_k | M_{k+1}^j(r), \mathcal{Z}^k\}$$

$$=: \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \quad (40)$$

where the first term can be calculated from (33) and (37) - (39), the second term from (34), the third term from (35), and c is a normalization constant such that $\sum_{\Theta_{k+1}} \sum_{\Theta_k} \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) = 1$.

The following updates are done for each target r ($r \in \mathcal{T}_N$). Calculate $\Lambda_{k+1}^{j_r}(r)$ (needed in Step 2.5 later) via (32)-(39). Define the target and mode-conditioned innovations

$$\nu_{k+1}^j(r, \Theta_{k+1}, \Theta_k) :=$$

$$\begin{cases} y_{k+1}^{(i)} - \widehat{z}_{k+1}^j(r, \Theta_k) & \text{for } i = 1, \cdots, \overline{m}(k+1) \\ & \text{ if } \theta_{ir}(k+1) \subset \Theta_{k+1} \\ 0 & \text{otherwise.} \end{cases} \quad (41)$$

Using $\widehat{x}_{k+1|k}^j(r, \Theta_k)$ (from (20)) and its covariance $P_{k+1|k}^j(r, \Theta_k)$ (from (21)), one computes the state update $\widehat{x}_{k+1|k+1}^j(r)$ and its covariance $P_{k+1|k+1}^j(r)$ as follows.

$$\text{Kalman gain: } W_{k+1}^j(r, \Theta_k)$$

$$= P_{k+1|k}^j(r, \Theta_k) H_{k+1}^{j'}(r, \Theta_k)[S_{k+1}^j(r, \Theta_k)]^{-1}. \quad (42)$$

$$\text{State estimate update: } \widehat{x}_{k+1|k+1}^j(r)$$

$$:= E\{x_{k+1}(r)|M_{k+1}^j(r), \mathcal{Z}^k, Y_{k+1}\}$$

$$= \sum_{\Theta_{k+1}} \sum_{\Theta_k} \widehat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \quad (43)$$

$$\widehat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) = \widehat{x}_{k+1|k}^j(r, \Theta_k) +$$

$$W_{k+1}^j(r, \Theta_k) \nu_{k+1}^j(r, \Theta_{k+1}, \Theta_k). \quad (44)$$

$$\text{Covariance of } \widehat{x}_{k+1|k+1}^j(r):$$

$$P_{k+1|k+1}^j(r) = \sum_{\Theta_{k+1}} \sum_{\Theta_k} \sum_{i=1}^{4} A_i(\Theta_{k+1}, \Theta_k) \beta_{k+1}^j(r, \Theta_{k+1}, \Theta_k) \quad (45)$$

where $A_1(\Theta_{k+1}, \Theta_k) =$

$$E\left\{x_{k+1}(r) x_{k+1}'(r) | M_{k+1}^j(r), \Theta_{k+1}, \Theta_k, \mathcal{Z}^k, Y_{k+1}\right\}$$

$$= \widehat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) + P_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k), \quad (46)$$

$$P_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k)$$

$$= P_{k+1|k}^j(r, \Theta_k) - W_{k+1}^j(r, \Theta_k) S_{k+1}^j(r, \Theta_k) W_{k+1}^{j'}(r, \Theta_k) \quad (47)$$

if $\theta_{ir}(k+1) \subset \Theta_{k+1}$, $1 \le i \le \overline{m}(k+1)$, and

$$P_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) = P_{k+1|k}^j(r, \Theta_k) \quad (48)$$

otherwise,

$$A_2(\Theta_{k+1}, \Theta_k) = -\widehat{x}_{k+1|k+1}^j(r, \Theta_{k+1}, \Theta_k) \widehat{x}_{k+1|k+1}^{j'}(r), \quad (49)$$

$$A_3(\Theta_{k+1}, \Theta_k) = -\widehat{x}_{k+1|k+1}^j(r) \widehat{x}_{k+1|k+1}^{j'}(r, \Theta_{k+1}, \Theta_k), \quad (50)$$

$$A_4(\Theta_{k+1}, \Theta_k) = \widehat{x}_{k+1|k+1}^j(r) \widehat{x}_{k+1|k+1}^{j'}(r). \quad (51)$$

Step 2.5. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$): $\mu_{k+1}^j(r) :=$

$$P[M_{k+1}^j(r)|\mathcal{Z}^{k+1}] = P[M_{k+1}^j(r)|\mathcal{Z}^k] p[Y_{k+1}|M_{k+1}^j(r), \mathcal{Z}^k]$$

$$= \frac{1}{c} \mu_{k+1}^{j-}(r) \Lambda_{k+1}^j(r) \quad (52)$$

where c is such that $\sum_{j=1}^n \mu_{k+1}^j(r) = 1$ and

$$\mu_{k+1}^{j-}(r) = \sum_{\Theta_k} \mu_{k+1}^{j-}(r, \Theta_k) P\{\Theta_k\} p[Y_k|\Theta_k, \mathcal{Z}^{k-1}]. \quad (53)$$

Step 2.6. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final state estimate update at time $k+1$ is given by

$$\widehat{x}_{k+1|k+1}(r) = \sum_{j=1}^{n} \widehat{x}_{k+1|k+1}^j(r) \mu_{k+1}^j(r) \quad (54)$$

and its covariance is given by $P_{k+1|k+1}(r) =$

$$\sum_{j=1}^{n} \Big\{ P_{k+1|k+1}^j(r) + [\widehat{x}_{k+1|k+1}^j(r) - \widehat{x}_{k+1|k+1}(r)]$$

$$\times [\widehat{x}_{k+1|k+1}^j(r) - \widehat{x}_{k+1|k+1}(r)]' \Big\} \mu_{k+1}^j(r). \quad (55)$$

5. SIMULATION EXAMPLE

We now consider tracking three maneuvering targets in clutter. We carry out state estimation for each target using IMM multiscan JPDA with a scan window size of two and compare our results with single scan IMM/JPDA algorithm of [5].

The True Trajectories: Target 1 starts at location [10500 1740 40] in Cartesian coordinates in meters. The initial velocity is [-140 299.9 0] in m/s. Target stays at constant altitude with a constant speed of 331 m/s. Its trajectory is a straight line with constant velocity between 0 and 15 sec., a coordinated turn of -0.32 rad/s with a constant acceleration of 109 m/s^2 between 15 and 25 s, and a straight line with a constant velocity between 25 and 35 s. Target 2 starts at location [9800 1960 40] in Cartesian coordinates in meters. The initial velocity is [0 299 0] in m/s. The target stays at a constant altitude with a constant speed of 299 m/s. Its trajectory is a straight line with constant velocity between 0 and 15 sec., a coordinated turn of 0.32 rad/s with a constant acceleration of 94 m/s^2 between 15 and 25 s, and a straight line with a constant velocity between 25 and 35 s. Target 3 starts at location [9200 1740 40] in Cartesian coordinates in meters. The initial velocity is [0 299 0] in m/s. The target stays at a constant altitude with a constant speed of 299 m/s. It's trajectory is a straight line with a constant velocity between 0 and 35 sec.

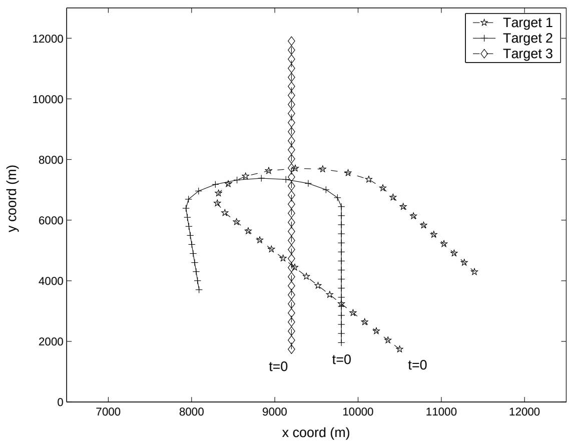

Figure 1. Trajectories (xy positions) of the three targets.

The Target Motion Models: The motion models are identical for all three targets. In each mode target dynamics are modeled in cartesian coordinates as $x_k = Fx_{k-1}+Gv_{k-1}$ where state of the target is position, velocity and acceleration in each of the three Cartesian coordinates (x, y and z). Thus x_k is of dimension 9 (n_x=9). Three models are considered in the following discussion and they are exactly as in [5, Sec. 5]. The initial model probabilities for three targets are identical: $\mu_0^1 = 0.8$, $\mu_0^2 = 0.1$ and $\mu_0^3 = 0.1$. The mode switching probability matrix for three targets is also identical and as in [5, Sec. 5].

The Sensor: A single sensor (radar) is used to obtain three measurements: range, and azimuth and elevation angles. The measurement noise w_k^j is assumed to be zero-mean white Gaussian with known covariance matrix $R = \text{diag}[400\text{m}^2,\ 49\text{mrad}^2,\ 4\text{mrad}^2]$. The sensor is assumed to be located at the origin of the coordinate system. The sampling interval was $T = 1$s and it was assumed that the probability of detection $P_D = 0.997$.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda = 0.1/\text{m} - \text{rad}^2$. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for the sensor were based on the threshold $\gamma = 16$. With the measurement vector of dimension 3, this leads to a gate probability $P_G = 0.9989$ (see p. 96 of [3]).

Simulation Results: The results were obtained from 100 Monte Carlo runs. Fig. 1 shows the true trajectories of the three targets. Fig. 2 shows the RMSE (root mean-square error) for the filtered position estimates for the three targets as a function of time. It is seen from Fig. 2 that the multiscan approach does provide a significant improvement over the single scan approach when the multiple maneuvering targets are in close proximity.

6. CONCLUSIONS

We investigated the problem of tracking multiple maneuvering targets in the presence of clutter using switching multiple target motion models. A novel suboptimal filtering algorithm was developed by applying the basic interacting multiple model (IMM) approach and multiscan joint probabilistic data association (JPDA) technique. Past work (see [6]) on this problem is restricted to non-maneuvering targets. The algorithm was illustrated via a simulation example involving tracking of three maneuvering targets and a multiscan data window of length two. The simulation example shows a significant improvement in target position estimate by the proposed IMM multiscan JPDA (with a scan window size of two) compared to the results of the single scan IMM/JPDA algorithm of [5].

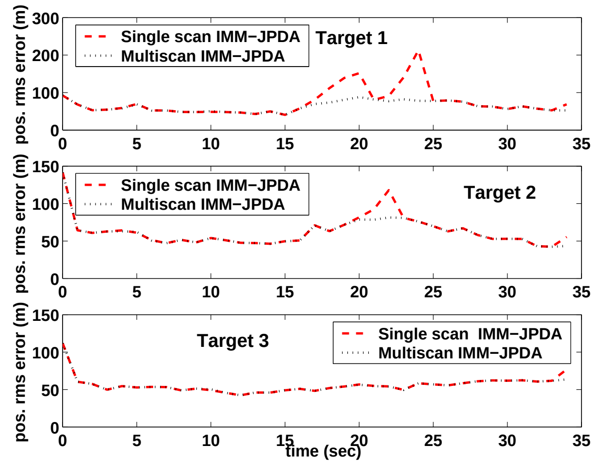

Figure 2. Root mean-square error (RMSE) in position using single scan IMM/JPDAF [5] and the proposed multiscan (window size 2 scans) IMM/Mscan-JPDAF algorithms.

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