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Attitude Determination and Orbital Estimation Using Earth Position and Magnetic Field Vector Measurements

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Abstract - An attitude determination routine based on geometric relations coupled with an orbital position estimator is designed. The proposed determination algorithm utilizes Earth position and magnetic field vector measurements. Orbital position data is provided by an Extended Kalman Filter (EKF) estimation of the Keplerian orbital elements. This estimator uses only measurements of the magnitude of the Earth's magnetic field. Coupling the orbital position estimator and attitude determination routine results in a fully autonomous satellite navigation system. The proposed attitude determination routine significantly reduces the computational efforts necessary to accurately estimate the attitude. It also eliminates the need of error-prone gyros while only requiring a nadir vector measurement. Simulation of the proposed system on the CATSAT (Cooperative Astrophysics and Technology SATellite) model results in accurate orbital position and attitude estimates for secondary fault detection and isolation implementation.

I. Introduction

For three-axis stabilized satellites, an accurate measure of inertial attitude of all three axes is needed for accurate control. As a satellite rotates about the Earth, sensors must measure the satellite attitude with respect to a fixed reference point (such as the Earth, sun and stars). Stars are the only group of objects that can be considered inertially fixed with respect to the satellite. However, star sensors need extensive star catalogs for determination and may be a computational burden for small satellite missions. Since stars are the only inertially fixed points available, attitude determination from objects such as the sun and the Earth require knowledge of satellite position with respect to the Earth. Such orbital information must either be up-linked from a ground station or be calculated autonomously by the satellite itself. Global positioning systems have also recently been used for semi-autonomous orbital estimates [1].

This paper deals with the use of only magnetometer data for orbital position estimation. One of the earliest papers by Psiaki *et al.* [2] used a square root information filter implementation of the Extended Kalman Filter (EKF) to estimate the Keplerian Orbital Elements (KOE) [3]. Shorshi *et al.* [4] applied an EKF to estimate the KOE by using only the dynamic equations of a mass under a central force while neglecting the drag terms when an estimate is unnecessary. The estimation of KOE utilizing an EKF was later combined with an attitude estimation routine to yield an algorithm that used magnetometer and gyro data filtered through an EKF to obtain both the orbital and the attitude estimates [5], [6]. Another recent study by Deutschmann *et al.* [7] excludes gyro measurements and also estimates the angular rates by additional sun sensor measurements.

Inertial attitude determination involves the determination of both the orbital position and the orientation of a body fixed coordinate system with respect to an orbital reference coordinate system. One possible way to determine the latter is by utilizing an EKF to estimate the attitude quaternion and the axis rates [6], [8], [9]. This requires the estimation of seven states. Since both an update and propagation stage are used for both states and the error covariance matrix, the computational burden of such a process is enormous. For many satellites with slow CPU and limited RAM, such a computational load is undesirable.

An alternative is to determine the attitude directly from vector observations. The first standard method used on many missions is the TRIAD algorithm [10]. Using two vector sets, the attitude information is found in a deterministic manner. Matrix inversion of a three dimensional system is utilized, directly yielding the directional cosine (attitude) matrix. Unfortunately, some of the attitude information must be discarded to prevent the system from being over-determined, due to the existence of multiple solutions for a given attitude matrix. Another method known as the QUEST (QUaternion ESTimate) [10] overcomes some of the shortcomings by combining all vector observations in an optimal manner by minimizing a specified loss function of sensor data. This method assumes accurate knowledge of the measurement and process noise, a priori attitude information and gyro data.

This paper proposes a simple attitude determination routine that directly processes the magnetic field and Earth position vector measurements. Proposed method uses a similar approach with the TRIAD algorithm, requiring a minimal amount of sensor measurements and relatively inexpensive measurement hardware. The computational requirements are also significantly reduced. The algorithm is based on a geometric development, where six independent equations can be formed from the six independent variables that create the directional cosine matrix. Using a numerical solution method, a minimum error approximate solution to these equations can be found. It is important to note that the attitude determination routine is reliant on the accuracy of the orbital estimator for both set of vectors.

The resulting completely autonomous navigation system would be adequate as a primary routine if a coarse heading is sufficient. Otherwise, the proposed attitude determination and orbital estimation routines can be used as a part of the contingency system.

The organization of this paper is given as follows. First,

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the dynamic equations that govern the orbital dynamics are given. Then an Extended Kalman Filter is formulated for the orbital estimation problem. Next, the aforementioned attitude determination method is explained. Finally, the results of the coupled orbital estimator and the attitude determination routine as simulated on the CATSAT[1] (Cooperative Astrophysics and Technology SATellite) model are demonstrated for various orbital inclinations.

II. Orbital Dynamics

The state vector for the EKF is comprised of the six classical Keplerian orbital elements in addition to a term representing drag friction:

$$X^T(t) = [a, e, i, \Omega, \omega, \theta, C_d] \tag{1}$$

where *a* is the semi-major axis, *e* is the eccentricity, *i* is the inclination, Ω is the right ascension of the ascending node, ω is the argument of the perigee, θ is the true anomaly and C_d is the drag coefficient. Analysis of the dynamics of the orbital parameters when the satellite is exposed to perturbing forces is performed using the variation of parameters method [3], which considers the direct effect of the perturbing forces on orbital parameters. Analytical descriptions of the rates of change of each parameter is obtained instead of a numerical integration routine. Results of the analysis [3] are given as:

$$\begin{aligned}
\dot{a} &= \frac{2a}{r}(2a - r)\frac{f_t}{v} \\
\dot{e} &= \frac{2f_t}{v}(cos\,\theta + e) - \frac{f_n}{v}\frac{r}{a}sin\,\theta \\
\dot{i} &= \frac{f_l}{v_\theta}\,cos\,\theta^* \\
\dot{\Omega} &= \frac{f_l}{v_\theta}\frac{sin\,\theta^*}{\sin\,i} \\
\dot{\omega} &= \frac{2f_t}{v \cdot e}\,sin\,\theta + \frac{f_n}{v}\left(2 + \frac{r}{a \cdot e}\,cos\,\theta\right) \\
\dot{\theta} &= \frac{h}{r^2} - \dot{\Omega} \cdot cos\,i
\end{aligned} \tag{2}$$

where f_t, f_n, f_l are the tangential, inward normal and lateral orbital perturbing forces, respectively. The first two forces are "in-plane" perturbations, whereas the last acts outside the orbital plane. These forces are assumed small compared to the central gravitational force. Additionally,

$$\begin{aligned}
h &= \sqrt{\mu \cdot a(1 - e^2)} \\
r &= \frac{a(1 - e^2)}{1 + e \cdot cos\,\theta} \\
v &= \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \\
\theta^* &= \theta + \omega
\end{aligned} \tag{3}$$

where μ is the Earth gravitation constant. As the drag coefficient is constant, $\dot{C}_d = 0$, which is included in the state dynamics, so as to allow for the estimation of its nominal value.

[1]CATSAT is a small satellite mission sponsored by NASA and developed by the Space Science Center of the University of New Hampshire to detect gamma ray bursts in deep space.

III. EKF Orbital Estimator

The relation between the magnitude of Earth's magnetic field and the orbital elements is highly nonlinear. Therefore, an EKF [11] is used for the estimation of orbital elements instead of a linear KF, to incorporate the nonlinear effects present both in the measurement model and in the dynamic equations. The system and the measurement models are given as:

$$\dot{X}(t) = f[X(t), t] + w(t) \tag{4a}$$

$$y_k = h_k[X(t_k)] + v_k \tag{4b}$$

where $f[X(t), t]$ is the nonlinear system equations, $w(t)$ is the white, zero-mean process noise, $h_k[X(t_k)]$ is the nonlinear measurement model, and v_k is the white, zero-mean measurement error. Development of the measurement update and the propagation stages of the filter equations [11] are reviewed in the following section.

A. Measurement Update Stage

The error between the measurement and the estimated magnetic field vector is used to update the state estimates to force the convergence of magnetic field estimates. The measurement model is defined as

$$y_k = |\vec{B}(X_k, t_k)| + v_k \tag{5}$$

where $\vec{B}(X_k, t_k)$ represents the measured magnetic field vector. The update equation is given as

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k\left[y_k - |\tilde{B}(\hat{X}_k(-), t_k)|\right] \tag{6}$$

where $\hat{X}_k()$ represents the estimated states. Pre-update and post-update variables are also represented by - and +, respectively. $\tilde{B}(\hat{X}_k(-), t_k)$ is the estimated magnetic field vector using the International Geomagnetic Reference Field (IGRF) model. The IGRF is a complex 10th-order spherical harmonic model of the Earth's magnetic field and models secular variations to the 8th order. The IGRF model used in this work uses the spherical harmonic coefficients calculated for the epoch 2000. The secular variation terms vary slightly each year but can be accurately calculated by modifying the coefficients of the same epoch. Details of the IGRF model can be found in [12].

The Kalman gain matrix K_k in Eq.(6) is defined as

$$K_k = P_k(-)\,{H_k}^T\left[H_k P_k(-) {H_k}^T + R_k\right]^{-1} \tag{7}$$

where H_k is the measurement matrix, $P_k(-)$ is the pre-update estimation error covariance matrix, and R_k is the covariance matrix of v_k. Once the states have been updated, the estimation error covariance matrix is updated as follows:

$$P_k(+) = [I - K_k H_k]P_k(-)[I - K_k H_k]^T + K_k R_k {K_k}^T \tag{8}$$

The measurement matrix H_k, relates the differential of the norm of the magnetic field vector to the differential of the orbital elements. As the relations for the magnetic field and the nonlinear measurement model are highly involved, the corresponding derivations and calculations are simplified and briefly mentioned.

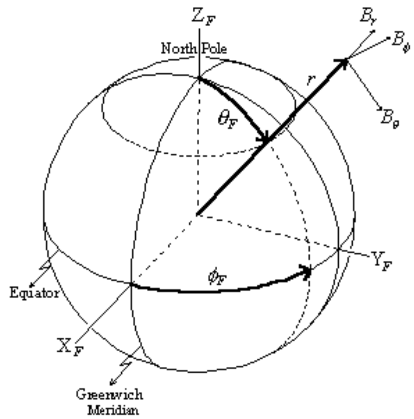

Fig. 1. Magnetic Spherical Coordinates

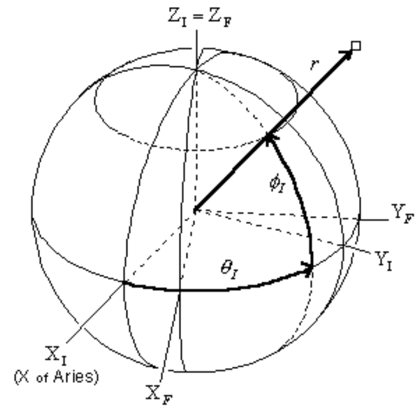

Fig. 2. Earth-Centered Inertial Spherical Coordinates

Measurement Model: The measurement matrix H_k is defined as

$$H_k = \left.\frac{\partial h}{\partial X}\right|_{\hat{X}_k(-),t_k} = \left.\frac{\partial |\hat{B}(R_F)|}{\partial X}\right|_{\hat{X}_k(-),t_k} \tag{9}$$

where $|\hat{B}(R_F)|$ refers to the norm of the estimated magnetic field vector resolved in the Earth-fixed magnetic spherical coordinate system. Figure 1 shows the components of the magnetic field vector in the Earth-fixed spherical coordinate system. The measurement model (i.e. the magnitude of the estimated magnetic field vector) has no direct functional relation with the orbital states, therefore Eq.(9) can be expanded by chain rule to yield

$$H_k = \left.\frac{\partial |\hat{B}|}{\partial \hat{B}} \cdot \frac{\partial \hat{B}}{\partial R_F} \cdot \frac{\partial R_F}{\partial R_I} \cdot \frac{\partial R_I}{\partial X}\right|_{\hat{X}_k(-),t_k} \tag{10}$$

where R_F and R_I represent the spacecraft position vector in the Earth centered fixed and the Earth centered inertial coordinate systems, respectively. Definition of the latter can be seen in Figure 2 (X of Aries is an inertially fixed vector that points from the Earth's center to the center of the sun on the vernal equinox).

The partial derivatives of R_I with respect to the filter states are calculated analytically according to the spherical geometry. The partial derivatives of the magnetic field with respect to the position vector are calculated using the magnetic field model equations. Details of the matrix development can be found in [13]. Measurement matrix calculations impose a significant computational burden on the flight processors. For this reason, calculations can be made analytically and offline. Therefore, during the execution only the states are needed for the propagation of the algorithm.

B. Propagation Stage

Orbital dynamics of Eq.(2) constitute the system model previously given in Eq.(4a)

$$\dot{X}(t) = f(X(t), t)$$

The derivatives of the states are approximated by the forward Euler approximation. Hence, the estimated state at the next time step is

$$\hat{X}_{k+1}(-) = \hat{X}_k(+) + f(\hat{X}_k(+)) \cdot \Delta T \tag{11}$$

where ΔT is the time step between measurements. The propagation of the covariance matrix is carried out as follows

$$P_{k+1}(-) = A_k(\hat{X}_k(+))\, P_k(+)\, {A_k}^T(\hat{X}_k(+)) + Q_k \tag{12}$$

where Q_k is the process noise covariance matrix (used for tuning purposes) and A_k is the transition matrix approximation, which is computed using a first order Taylor series expansion:

$$A_k(\hat{X}_k(+)) = I + F_k(\hat{X}_k(+)) \cdot \Delta T \tag{13}$$

Here $F_k(\hat{X}_k(+))$ denotes the Jacobian matrix based on the partial derivative of the nonlinear system dynamics with respect to the orbital states:

$$F_k(\hat{X}_k(+)) = \left.\frac{\partial f(X(t), t)}{\partial X}\right|_{X=\hat{X}_k(+)} \tag{14}$$

After predicting the filter states and the covariance matrix at the next step, the algorithm proceeds with the updates from the sensor measurements of the following time step.

IV. Attitude Determination

The attitude determination routine requires two sets of vectors. These vector sets are essentially comprised of two nonparallel vectors expressed in two different coordinate systems: the satellite centered body (SCB) and the satellite centered inertial (SCI) coordinate frames. (The SCB frame is defined by the satellite principal inertial axes, whereas the SCI frame is in the same orientation with the Earth centered inertial frame except the origin is translated to the satellite's center of mass). The magnetic field vector is one of these vectors, and the *nadir* vector, which points from the satellite to the center of the Earth, will be used as the second vector.

As the orbital position estimator converges, the inertial position of the satellite with respect to the center of the Earth is assumed known. Consequently, both the magnetic field and the Earth position vectors are known in the SCI coordinate frame. Assuming that both vectors are measured in the SCB frame, the necessary sets of vectors are complete and differ only by the satellite attitude. The transform relating each vector set is merely the directional cosine matrix. This relation can be shown as:

$$\vec{B}_{SCB} = A_{IB} \cdot \vec{B}_{SCI}$$
$$\vec{C}_{SCB} = A_{IB} \cdot \vec{C}_{SCI} \quad (15)$$

where $\vec{B}_{SCB}$, $\vec{B}_{SCI}$, $\vec{C}_{SCB}$ and $\vec{C}_{SCI}$ are the magnetic field and nadir vectors resolved in the SCB and the SCI coordinates, respectively. A_{IB} is the attitude (directional cosine) matrix which transforms a given vector from the SCI to the SCB coordinate system and is defined as

$$A_{IB} = [\vec{n}\ \vec{o}\ \vec{a}]^T = \begin{bmatrix} n_x & n_y & n_z \\ o_x & o_y & o_z \\ a_x & a_y & a_z \end{bmatrix} \quad (16)$$

$\vec{n}, \vec{o}, \vec{a}$ represent the respective principal inertial axes of the satellite in the SCI coordinate frame. Indeterminacy of Eq.(15) can be solved by using the knowledge that $\vec{n}, \vec{o}, \vec{a}$ form an orthogonal triad, such that $\vec{a}$ can be replaced by

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \times \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} = \begin{bmatrix} n_y \cdot o_z - n_z \cdot o_y \\ n_z \cdot o_x - n_x \cdot o_z \\ n_x \cdot o_y - n_y \cdot o_x \end{bmatrix} \quad (17)$$

Equations (15, 17) yield the nonlinear system

$$F_1(n_x, n_y, n_z, o_x, o_y, o_z) = F_1(X_a) = 0 \quad (18)$$

to be solved in the attitude determination routine. The expansion of Eq.(18) can be found in the Appendix. Using Taylor series expansion and neglecting higher order terms,

$$F_1(X_a) = F_1(X_{a0}) + (X_a - X_{a0}) \cdot J(X_{a0}) = 0 \quad (19)$$

where X_{a0} denotes the current iterated value of the state vector X_a, and $J(X_{a0})$ represents the Jacobian of $F_1(X_a)$. The closed form solution of Eq.(19) requires the inverse of the Jacobian to be computed at every iteration. Since this is not trivial for high-dimensional systems, an appropriate numerical method (e.g. Gaussian elimination/back substitution) can be used to find an approximate solution for X_a.

Detailed simulation results [13] show that the Newton-Raphson iteration technique requires only two iterations to converge. Unfortunately, the convergence is not assured due to the inherent nature of the numerical method employed. To increase the number of successful determinations, a second set of equations is also used:

$$\begin{bmatrix} n_x \cdot B^x_{SCI} + n_y \cdot B^y_{SCI} + n_z \cdot B^z_{SCI} - B^x_{SCB} \\ o_x \cdot B^x_{SCI} + o_y \cdot B^y_{SCI} + o_z \cdot B^z_{SCI} - B^y_{SCB} \\ n_x \cdot o_x + n_y \cdot o_y + n_z \cdot o_z \\ n_x \cdot C^x_{SCI} + n_y \cdot C^y_{SCI} + n_z \cdot B^z_{SCI} - C^x_{SCB} \\ o_x \cdot C^x_{SCI} + o_y \cdot C^y_{SCI} + o_z \cdot C^z_{SCI} - C^y_{SCB} \\ n_x^2 + n_y^2 + n_z^2 - 1 \end{bmatrix} = 0 \quad (20)$$

This set uses the unity constraint of vectors $\vec{n}$ and $\vec{o}$

$$n_x^2 + n_y^2 + n_z^2 = 1$$
$$o_x^2 + o_y^2 + o_z^2 = 1 \quad (21)$$

and

$$\vec{n} \cdot \vec{o} = 0 \quad (22)$$

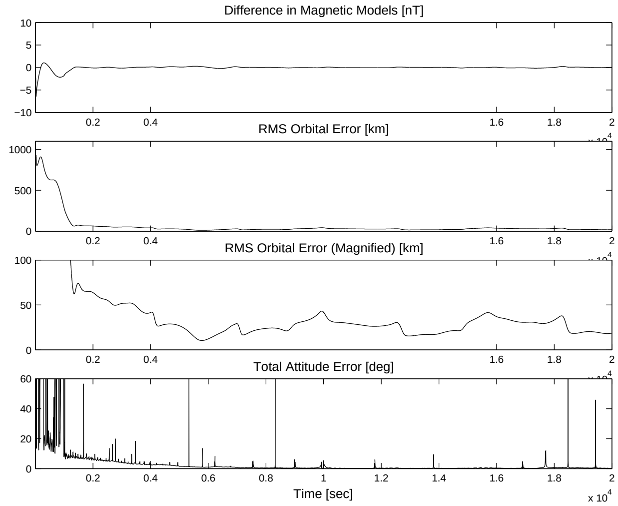

Fig. 3. Orbital and Attitude Estimator Results for *i*=45^o

In cases where the first set (F_1) of equations does not meet the performance requirements or converge, the Newton-Raphson solution method (or applied numerical method) is repeated for Eq.(20).

V. Simulation Results

The orbital estimation and attitude determination routines are tested on the University of New Hampshire CATSAT simulation model. The orbital estimator results in acceptable RMS orbital position errors between 29 and 42 km upon convergence, with magnitude errors ranging from 5 km to 60 km. The attitude determination routine produces accurate results within 0.481^o to 1.080^o of total angular error for all axes, over the range mentioned for the orbital position estimator. Efficiency of the estimation routine is demonstrated in Figure 3 for an orbital inclination of 45^o. The results are investigated separately for the orbital estimator and the attitude determination routine in the following sections.

A. Orbital Estimator Results

Obtaining success in convergence and in rates of convergence in a global sense is difficult due to the complex topography of the magnetosphere and varying initial conditions of the estimator and the orbit. Therefore, the simulations are performed only under varying orbital inclinations, since the inclination is the most significant parameter affecting the observability of the orbit. All other initial conditions remain constant. Initial conditions used for the simulated orbit and the orbital estimator are given in Table 1.

Table 1. Initial conditions for the simulated orbit and the orbital estimator

	a (km)	e	Ω (deg)	ω (deg)	θ (deg)
Orbital IC	6921.2	0.001	45	90	0
Estimator IC	6926.2	0.00101	48	86	-4

The orbital estimator is initialized with an inclination of 2^o less than the actual inclination and tested for values ranging between 15^o and 75^o. The orbital position error results are listed in Table 2.

Table 2. Orbital position errors for varying inclinations

Inclination i (deg)	Initial Error (km)	Orbital Position RMS Error After Convergence (km)	Initial Magnetic Field Magnitude Error (nT)
15	1018	108	-321.359
20	894	194	-160.694
25	929	209	-65.095
30	710	52	-34.953
45	929	27	-223.009
60	995	39	-566.325
75	1068	30	-651.412

The orbital position error in Table 2 refers to the difference between the actual and the estimated rectangular coordinates of the center of mass of the satellite. The simulation duration is $2 \cdot 10^4$ s, corresponding to approximately 3.5 orbits according to the average orbital period [3] given by

$$T_{orbit} = 2\pi \cdot \sqrt{\frac{a^3}{\mu_{earth}}} \tag{23}$$

where μ_{earth} is the gravitational constant equal to $3.986 \cdot 10^5 \frac{km^3}{s^2}$. A sampling time of 2 s between measurements is used for the simulations. Convergence of the orbital estimator often occurs within one-half of an orbit.

It can be clearly seen that the steady-state estimation error falls drastically for inclinations higher than 30^o. This is due to the poor magnetic field magnitude variation at low inclinations. The magnitude varies a great deal more at higher inclinations because of the dipole shape of the magnetic field.

B. Attitude Determination Results

To quantify the accuracy of the attitude determination routine, an approximate error expression based on the difference between the actual and the estimated directional cosine matrices is established:

$$A_{err} = \begin{bmatrix} \Delta\vec{n} \\ \Delta\vec{o} \\ \Delta\vec{a} \end{bmatrix} = \begin{bmatrix} \vec{n}_{act} - \vec{n}_{est} \\ \vec{o}_{act} - \vec{o}_{est} \\ \vec{a}_{act} - \vec{a}_{est} \end{bmatrix} \tag{24}$$

Taking the RMS of each error vector indicates the accuracy of the estimated attitude information

$$E_k = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\Delta\vec{k}_i)^2} \qquad (k = n, o, a) \tag{25}$$

where m is the number of data points after convergence. To obtain the equivalent angular difference, the error vector is assumed to be approximately perpendicular to the actual vector and the error angle is obtained as follows:

$$\alpha_k \simeq tan^{-1}\left(\frac{E_k}{1}\right) \tag{26}$$

A single indicator of the total attitudinal error is chosen to be

$$\beta = \sqrt{{\alpha_n}^2 + {\alpha_o}^2 + {\alpha_a}^2} \tag{27}$$

where β is the total angular error. The attitudinal error simulation results are given in Table 3 for varying inclinations.

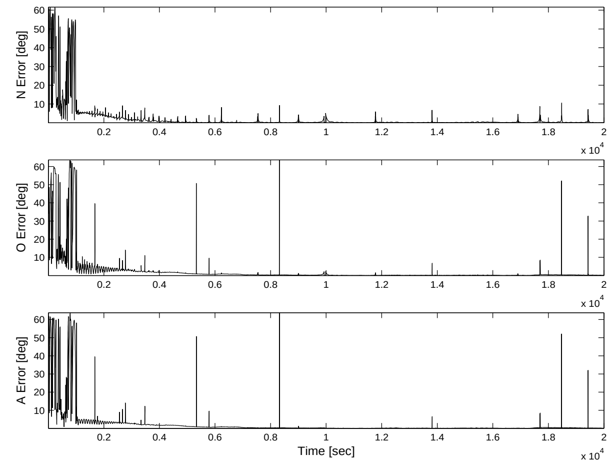

Fig. 4. Attitude Error Vectors for i=45^o

Table 3. Attitudinal error results for varying inclinations

i (deg)	Orbital Position RMS Error After Convergence (km)	α_n (deg)	α_o (deg)	α_a (deg)	β (deg)
15	108	0.761	2.324	2.319	3.370
20	194	1.609	3.680	3.691	5.455
25	209	1.739	4.021	4.032	5.954
30	52	0.489	0.985	0.965	1.463
45	27	0.294	0.281	0.256	0.481
60	39	0.417	0.382	0.333	0.656
75	30	0.640	0.753	0.436	1.080

The correlation between the orbital estimator and the attitudinal steady-state error is obvious from the data. The attitude estimate increases in accuracy with increasing orbital estimate accuracy. The ascending trend in attitudinal error for inclinations about 75^o is due to the fact that the magnetic poles and the Earth's rotation axis do not align. The radial component of the Earth's magnetic field shows a peak for locations corresponding to latitudes between 70^o and 75^o [13]. This leads to a loss of independence of the equations used to obtain the attitude estimates as a result of the near co-alignment of the magnetic and the nadir-pointing vectors. This phenomenon occurs even for smaller orbital inclinations at certain instances where the satellite travels near the magnetic poles. This can be seen in Figure 4, which shows the attitude determination results at an inclination of 45^o. The spikes correspond to brief intervals where the determination routine fails. The autonomous navigation routine implemented here is completely decoupled from the specific satellite model used and works outside the control loop. The actual model uses the *ORBINT* code developed by the NASA Jet Propulsion Lab (JPL) for orbit propagation. It should be noted that the IGRF model is used for both the true satellite model and the orbital estimator magnetic field information; although both have different initial conditions. As the IGRF model is a highly complex nonlinear function of the orbital position, these varying initial conditions suffice to effect the true and orbital estimator model outputs to differ significantly.

VI. CONCLUSIONS

This paper presents a satellite attitude determination algorithm coupled with an orbital position estimator utilizing Earth position and magnetic field measurements. The satellite position estimates are provided by Extended Kalman Filtering of Keplerian Orbital Elements. The proposed attitude determination routine is a computationally efficient and simple algorithm, which directly processes two vector measurements to obtain attitude information without requiring error-prone gyros.

The resulting autonomous navigation algorithm is implemented on the University of New Hampshire - CATSAT simulation model. The results show that RMS orbital position errors vary between 27 km and 42 km, depending on the orbital inclination. The orbital estimator succeeds to overcome initial position errors exceeding 1000 km. This algorithm works well for near-circular orbits. The EKF is designed to incorporate an atmospheric drag estimate as well as the Keplerian orbital element estimates.

Over the same range of convergence, the attitude determination routine accuracy varies in total angular error from 0.481^o to 1.080^o. The large angular deviation at higher inclinations occur because of the position of the magnetic poles. The simulations show that the band of optimal results for orbital estimation and attitude determination is obtained between 40^o and 70^o.

Both the orbital estimator and the attitude determination routine are observed to converge to acceptable estimates within an orbital period. The orbital estimator uses only magnetometer data, while the attitude determination routine requires only an additional Earth position measurement to complete the autonomous navigation algorithm. The proposed method is a simple and inexpensive solution for smaller satellites, and reduces the extra computational requirements imposed by adding the attitude states to the EKF. The need for any priori knowledge of the attitude is also eliminated, since the attitude is determined by a direct processing of vector measurements.

The results are satisfactory for smaller satellites similar to CATSAT, for which the mission requirements are less stringent. However, if a more accurate navigation system is deserved, the proposed algorithm may preferably be used as a secondary system.

Future work involves developing a computationally more efficient orbital estimation algorithm in place of the EKF.

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VIII. APPENDIX: EQUATION (18)

$$\begin{bmatrix} n_x \cdot B^x_{SCI} + n_y \cdot B^y_{SCI} + n_z \cdot B^z_{SCI} - B^x_{SCB} \\ o_x \cdot B^x_{SCI} + o_y \cdot B^y_{SCI} + o_z \cdot B^z_{SCI} - B^y_{SCB} \\ (n_y \cdot o_z - n_z \cdot o_y) \cdot B^x_{SCI} + (n_z \cdot o_x - n_x \cdot o_z) \cdot B^y_{SCI} + (n_x \cdot o_y - n_y \cdot o_x) \cdot B^z_{SCI} - B^z_{SCB} \\ n_x \cdot C^x_{SCI} + n_y \cdot C^y_{SCI} + n_z \cdot B^z_{SCI} - C^x_{SCB} \\ o_x \cdot C^x_{SCI} + o_y \cdot C^y_{SCI} + o_z \cdot C^z_{SCI} - C^y_{SCB} \\ (n_y \cdot o_z - n_z \cdot o_y) \cdot C^x_{SCI} + (n_z \cdot o_x - n_x \cdot o_z) \cdot C^y_{SCI} + (n_x \cdot o_y - n_y \cdot o_x) \cdot C^z_{SCI} - C^z_{SCB} \end{bmatrix} = 0$$