Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Stability analysis and tuning strategies for a novel next generation regulatory controller

Kapil Mukati
Babatunde Ogunnaike
Department of Chemical Engineering
University of Delaware

Abstract

The popular PID controller, even though versatile, possesses well-known characteristics that limits its achievable performance, and an intrinsic structure that makes its tuning not as transparent. We recently developed a novel 4-mode control scheme that takes full advantage of the modern electronic hardaware components, and whose tuning parametersare related *directly* to controller performance attributes (robustness, set-point tracking, and disturbance rejection); its achievable performance is better, and it can be designed and implemented much more directly and transparently. . In this paper we present rigorous robust stability results for the proposed controller for any given plant/model mismatch. These results are then used to generate procedures for selecting values for the controller tuning parameters. The controller design and tuning is illustrated via simulation on a nonlinear polymerization process.

1. Introduction

The PID controller is the most commonly used controller in industrial practice with perhaps only 5-10% of loops on which it is not routinely applied [1]. However, the controller parameters are not related directly to the three critical controller performance attributes of robustness, set-point tracking and disturbance rejection; therefore designing the controller to achieve desired performance in each of these attributes is not straightforward. In fact, it is well-known that one cannot tune a PID controller to achieve good set-point tracking and disturbance rejection simultaneously.

A study of industrial control loops by Ender [2] found that more than 30% of installed controllers operate in manual mode while, of the loops operating in automatic mode, 65% are poorly tuned. It is also well-established that PID controllers perform poorly for dead-time dominant, inverse response or highly nonlinear processes. Under these conditions, it is typical to enhance the performance of PID controllers with the Smith predictor for dead-time dominant process, and with adaptive [3] or auto-tuning techniques [4] for nonlinear process. Even then, there are well-known technical issues associated with implementing these strategies in practice.

Alternatives to the PID controller (fuzzy control, general linear control, state feedback and observers etc. [5]) can achieve better performance but usually at the expense of sacrificing simplicity. There is therefore still a need for an alternative controller that is sufficiently simple to implement, that can achieve better performance, and whose structure allows for transparent tuning related directly to controller performance attributes. We recently developed such a controller that combines the simplicity of classical PID controller with the efficiency of model predictive controller, while avoiding the tuning problem associated with both. The details of the development, design and implementation of this regulatory controller (the RTD-A controller) have been discussed elsewhere [6]. The key characteristic is that its three primary tuning parameters, θ_R, θ_T, and θ_D, are not only related directly to the attributes of robustness, set-point tracking, and disturbance rejection, they are also normalized to lie between 0 and 1. These normalized tuning parameters arise naturally from the formulation in a manner that makes it possible to tune the controller directly for each performance attributes independently. An auxiliary fourth parameter, θ_A, related to the overall controller aggressiveness, arises naturally from the model predictive formulation independently of the three main tuning parameters; it is also normalized between 0 and 1.

Controller Features and Performance: The salient features of the controller are as follows [6]:

1. *Process characterization*: A discrete-time version of the same model employed for tuning classical PID controllers, the first-order-plus-

dead-time (FOPDT) model obtainable from a "process reaction curve":

$$\hat{y}(k+1) = a\hat{y}(k) + bu(k-m) \quad (1)$$

2. *Control Strategy*: A single control move, $u(k)$, to be held over an N-step horizon beyond the delay period, m, is computed and implemented at each time instant, k, in standard receding horizon fashion. Under these conditions the "uncorrected" model prediction is obtained as:

$$\hat{y}(k+m+i) = a^{m+i}\hat{y}(k) + a^{i-1}b\mu(k,m) + b\eta_i u(k) \quad (2)$$

$i = 1,2,...,N$, with $\eta_i = (1-a^i)/(1-a)$, and

$$\mu(k,m) = \sum_{i=1}^{m} a^i u(k-i) \quad (3)$$

3. *Error Decomposition and Model Update*: The model error, $e(k) = y(k) - \hat{y}(k)$, is explicitly decomposed into separate and distinct *conditional* estimates of the effect of plant/model mismatch, $e_m(k)$, and unmeasured disturbances, $e_D(k)$, as:

$$e(k) = e_m(k) + e_D(k) \quad (4)$$

using Bayesian principles [6] to determine $e_D(k)$ estimates as

$$e_D(k) = \theta_R e_D(k-1) + (1-\theta_R)e(k) \quad (5)$$

where θ_R, $(0<\theta_R<1)$, emerges from the formulation as the robustness parameter. From here, the future disturbance effect is predicted according to:

$$\hat{e}_D(k+j) = e_D(k) + \frac{(1-\theta_D)}{\theta_D}\left[1-(1-\theta_D)^j\right]\nabla e_D(k) \quad (6)$$

for $m+1 \le j \le m+N$, where

$$\nabla e_D(k) = e_D(k) - e_D(k-1) \quad (7)$$

θ_D, $(0<\theta_D<1)$, is the disturbance rejection tuning parameter.

The model prediction in (2) is updated with (6) to obtain:

$$\tilde{y}(k+m+i) = \hat{y}(k+m+i) + \hat{e}_D(k+m+i) \quad (8)$$

4. *Control action computation*: At each discrete point in time, the *single* control move, $u(k)$, is determined to minimize the predicted deviation of the plant output from the desired set-point trajectory, y^*, over the next N discrete steps in the future. For a set point y_d, the desired set-point trajectory given by:

$$y^*(k+j) = \theta_T{}^j y^*(k) + (1-\theta_T{}^j)y_d(k);\ 1 \le j \le N$$

with θ_T, $(0<\theta_T<1)$, as the set-point tracking tuning parameter. The value chosen for N, or

$$\theta_A = 1 - e^{-\left(\frac{(N-1)\Delta t}{\tau}\right)} \quad (9)$$

determines the overall controller aggressiveness. The closed form solution of the least squares optimization problem is easily shown to yield:

$$u(k) = \frac{\sum_{i=1}^{N}\eta_i\psi_i(k)}{b\sum_{i=1}^{N}\eta_i^2} \quad (10)$$

where:

$$\psi_i(k) = y^*(k+i) - a^{(m+i)}\hat{y}(k) - a^{(i-1)}b\mu(k,m) - \hat{e}_D(k+m+i)$$

Effect of Tuning Parameters: Figure 1 shows simulations of nominal controller performance on a plant model, $P(s) = \frac{e^{-s}}{10s+1}$, to demonstrate how the tuning parameters, θ_T, θ_D, and θ_R directly influence the controller performance attributes of set-point tracking, disturbance rejection and robustness respectively; it also shows the effect of the overall aggressiveness parameter θ_A and how, in each case, the controller becomes more conservative as each tuning parameter value is increased from 0 to 1. The other tuning parameters are kept at a fixed value of 0.1 and sampling time is 0.1. These results are meant to illustrate the ease and transparency of tuning this controller.

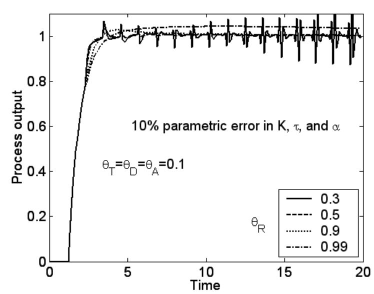

Figure 1(a): Effect of θ_R on robustness.

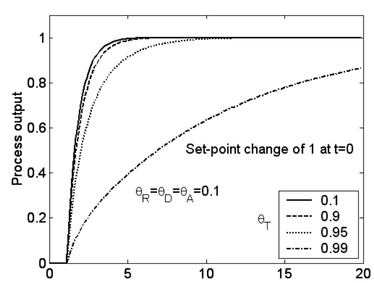

Figure 1(b): Effect of θ_T on set-point tracking.

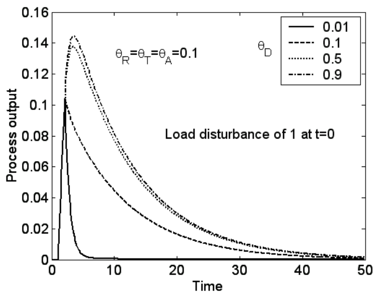

Figure 1(c): Effect of θ_D on disturbance rejection.

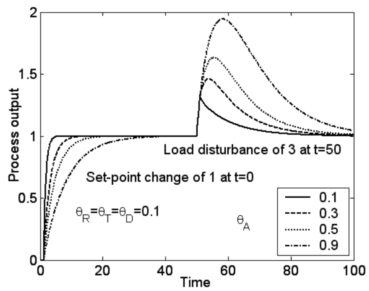

Figure 1(d): Effect of θ_A on overall aggressiveness.

The focus of this paper from here on is the development of a procedure for determining acceptable tuning parameters for the controller. The procedure, based on robust stability considerations, is presented in section 2 and illustrated in section 3 using a polymerization reactor simulation.

2. Theoretical Stability Analysis

State Variable Formulation: When the controller in (10) is implemented on a plant *with explicit parametric plant/model mismatch,* the closed-loop behavior may be represented in the following state variable form (see Appendix):

$$x(k+1) = Rx(k) + Qu(k) + \begin{bmatrix}1-\theta_T & 0 & 0 & 0 & 0\end{bmatrix}^T y_d + \begin{bmatrix}0 & b^0 z^{-m^0} & 0 & 0 & 0\end{bmatrix}^T d$$

$$y(k) = \begin{bmatrix}0 & 1 & 0 & 0 & 0\end{bmatrix} \cdot x(k) \qquad (11)$$

In state feedback form, the controller is given by

$$u(k) = Sx(k) + \lambda B y_d \qquad (12)$$

y_d and d are set-point and disturbance variables respectively, and the indicated matrices are shown fully in the Appendix. The system characteristic equation is now obtained as:

$$zI - R - QS = 0\,. \qquad (13)$$

Robust Stability Analysis: For any given plant/model mismatch, Eq. (13) may now be used to determine the set of tuning parameter values such that the closed-loop system is stable (i.e. all roots of the characteristic equation are within the unit circle). It can be shown that one of these roots is the set-point tracking parameter, θ_T. Hence, the only condition θ_T must satisfy is $0<\theta_T<1$; it does not affect closed-loop stability in any other way. Eq. (13) is thus used to determine acceptable values for only 3 parameters, θ_A, θ_D, and θ_R.

For representing plant/model mismatch, we employ the multiplicative uncertainty description where, for each model parameter, M, its relationship to the "true" but unknown parameter M^0 and the model error ΔM % is:

$$M^0 = M\left(1 + \frac{\Delta M}{100}\right) \qquad (14)$$

3. Illustrative Example

We now apply this technique to design and implement the RTD-A controller on a simulation of an isothermal polymerization reactor discussed in [7], where initiator flow rate is used to control number average molecular weight (NAMW). The nonlinear, four-state, state-space model is used to represent the plant; an approximate FOPDT model used for controller design is obtained from a step response (see Figure 2). Note that this model is an approximation of the real process and has inherent parametric and structural uncertainty. To design the RTD-A controller for "worst case" 10% parametric uncertainty, (i.e. the process steady state gain, dead time, and time constant are assumed to be underestimated by 10%), we employ (13) to generate Figure 3, a plot of closed-loop stability regions as a function of θ_R and θ_D, and, for simplicity, for only 3 values of θ_A (0.1, 0.5 and 0.9). The region above each curve is the stability region.

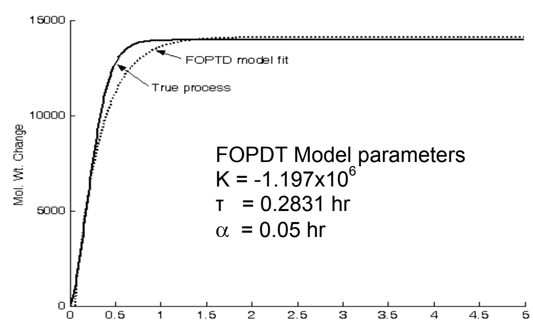

Figure 2: First-Order-Plus-Time-Delay model fit to the true process response.

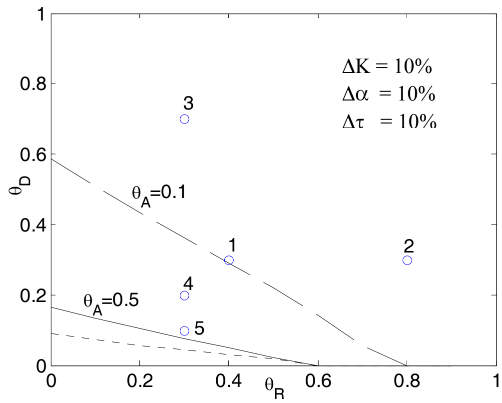

Figure 3: Stability regions for polymerization reactor control: dashed, θ_A=0.1; solid, θ_A=0.5; dotted, θ_A=0.9. The stability region is above each curve.

Controller Tuning and Implementation: The first set of simulations are performed under the following conditions: "Start-up," in which a set-point change of 15,000 in NAMW is made at t=0 hr, followed by a disturbance in the form of a one-third reduction in monomer concentration introduced at t=2.5 hr. If an aggressive controller performance is desired, the overall aggressiveness parameter, θ_A should be small. For a choice of θ_A = 0.1, closed loop system stability requires combinations of θ_D and θ_R values in the area above the dashed curve in Figure 3; two possible choices are labeled (1) and (2) both with θ_D = 0.3. For point 1 (θ_R=0.4), aggressive performance (close to the edge of instability) is expected; increasing θ_R to 0.8 (point 2 in Figure 3) should enhance robustness. Figure 4(a) shows that this, in fact, is the case. Note especially the control action plot.

To illustrate tuning for disturbance rejection, a disturbance in the form of a one-third reduction in monomer concentration is introduced at t=0h. The responses are shown in Figure 4(b). For a relatively less aggressive controller, selecting θ_A = 0.5 and θ_D = 0.7 with θ_R=0.3 (point 3 in Figure 3, considerably far from the stability boundary) results in the response shown in the dashed lines. A choice of θ_D = 0.2 (point 4 in Figure 3) should provide better disturbance rejection; and this is seen in the dotted line responses. Decreasing θ_D further to 0.1 (point 5), should result in even better disturbance rejection; however, this parameter combination is close to the stability boundary. The responses shown in the solid lines reflect this: faster disturbance rejection is achieved with control action showing the onset of oscillations. The set-point tracking parameter θ_T is fixed at 0.9 for all the simulations.

Comparison with PID control: The performance of the RTD-A controller may also be compared with that of an IMC tuned PID controller (IMC-PID) and a minimum Integral-Time-Absolute-Error tuned PID controller (ITAE-PID). For aggressive control, a choice of θ_A = 0.1 restricts acceptable θ_D and θ_R values to those lying in the region above the dashed line in Fig 3. Choosing θ_D=0.1 for aggressive disturbance rejection requires a companion θ_R > 0.7. We consider a final choice as follows: θ_D=0.1, θ_A=0.1, θ_R=0.8 along with θ_T=0.9. The tuning parameters for the IMC-PID and ITAE-PID are determined using the tuning rules in [8]. (For the ITAE-PID controller, we employed the tuning parameters for set-point response because the closed-loop system was unstable with parameters tuned for disturbance rejection.) The ITAE-PID and IMC-PID tuning parameters are, respectively: K_C=-3.55.10^{-6}, τ_I=0.3677, τ_D=0.0174 and K_C=-4.29.10^{-6}, τ_I=0.3081, τ_D=0.0230. The resulting simulation results are shown in Figures 5(a) and 5(b) where the RTD-A controller is seen to achieve better set-point tracking and disturbance rejection.

The main point here is not so much the improved performance; it is that the RTD-A controller tuning parameters are transparently related to controller attributes and are easier to choose. If we wish to modify any aspect of the observed response, the required changes are clear. For example, if the overshoot in set-point tracking response is undesirable, we simply increase the θ_T value to say, 0.95, to obtain the more gradual transition to set-point shown in the dash-dotted curve in the top half of Figure 5a.

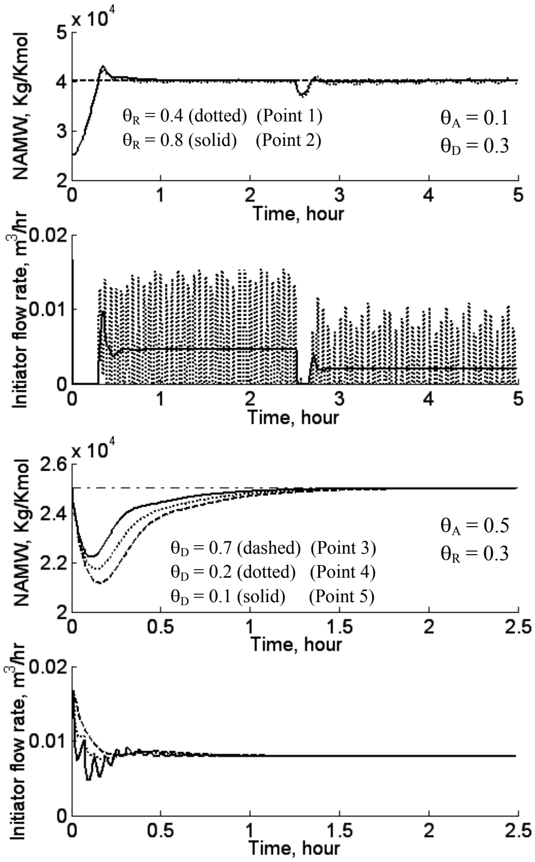

Figure 4: Controller performance for various tuning parameters: worst case 10% plant/model mismatch. (a) Top: Robustness (start-up and disturbance rejection); (b) Bottom: Disturbance rejection characteristics.

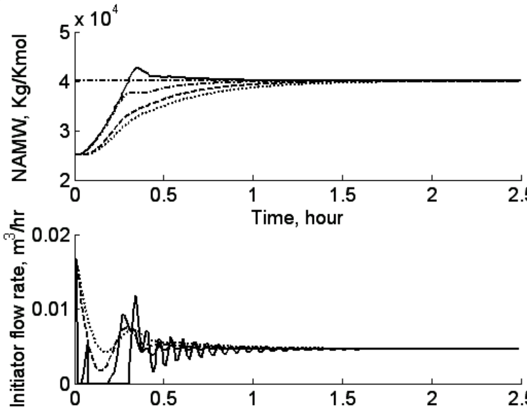

Figure 5(a): Polymerization reactor control simulation for set-point change. (solid) RTD-A with θ_T=0.9, (dash-dot) RTD-A with θ_T=0.95, (dashed) IMC-PID, (dotted) ITAE-PID.

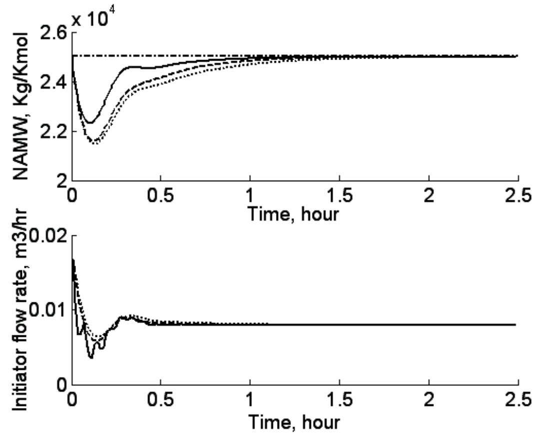

Figure 5(b): Polymerization reactor control simulation for disturbance rejection (solid) RTD-A, (dashed) IMC-PID, (dotted) ITAE-PID.

4. Summary and Conclusions

In this paper, stability analysis results and their use for controller tuning have been presented for a novel regulatory controller. We have demonstrated how tuning this controller is easy and transparent because (i) the tuning parameters are directly related to the controller performance attributes; and (ii) they are naturally normalized to lie between 0 and 1, and formulated such that decreasing their value from 1 to 0 makes the controller more aggressive in each of the corresponding attribute. How to apply the methodology in practice was illustrated using a simulation of an isothermal polymerization reactor where it was shown that the controller performed well even in the presence of parametric and structural uncertainties. Future work will focus on reducing the stability plots to generalized formulas that can be used to choose the RTD-A controller parameters more conveniently.

5. References

1. Koivo, H.N., & Tanttu, J.T. (**1991**). *Proc. IFAC Intelligent Tuning and Adaptive Control Symposium*, Singapore, 75.

2. Ender, D.B. (1993*). Control Engineering*, September, 180.

3. Astrom, K.J., & Hagglund, T. (**1984**). Automatic tuning of simple regulators with specifications on phase and amplitude margins. *Automatica*, 20(5), 645-651.

4. Kraus, T.W., & Mayron, T.J. (**1984**). Self tuning PID controllers based on a pattern recognition approach. *Control Engineering Practice*, 106-111.

5. Astrom, K.J., & Hagglund, T. (**2001**). The future of PID control. *Control Engineering Practice*, 1163-1175.

6. Ogunnaike, B.A. and K. Mukati, (2003) "Development, Design and Implementation of an alternative structure for Next Generation regulatory controllers" Preprints of the Annual AIChE meeting, San Francisco, CA.
7. Maner B.R., Doyle F.J., Ogunnaike B.A., and R.K. Pearson, (1996) "Nonlinear model predictive control of a simulated multivariable polymerization reactor using second order Volterra models," *Automatica*, Vol. 32, No. 9, 1285-1301.
8. Ogunnaike, B.A., & Ray, W.H. (**1994**). Process Dynamics, Modeling and Control. *Oxford University Press,* New York.

APPENDIX: State variable formulation:

$$\begin{bmatrix} y^*(k+1) \\ y(k+1) \\ \hat{y}(k+1) \\ e_D(k+1) \\ \mu(k+1) \end{bmatrix} = \begin{bmatrix} \theta_T & 0 & 0 & 0 & 0 \\ 0 & a^0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 1-\theta_R & -(1-\theta_R) & \theta_R & 0 \\ 0 & 0 & 0 & 0 & a \end{bmatrix} \cdot \begin{bmatrix} y^*(k) \\ y(k) \\ \hat{y}(k) \\ e_D(k) \\ \mu(k) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ b^0 z^{-m^0} \\ bz^{-m} \\ 0 \\ a - a^{m+1} z^{-m} \end{bmatrix} u(k) + \begin{bmatrix} 1-\theta_T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} y_d + \begin{bmatrix} 0 \\ b^0 z^{-m^0} \\ 0 \\ 0 \\ 0 \end{bmatrix} d$$

R is the matrix in the first term and ***Q*** is the vector in the second term on RHS.

$$y(k) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} y^*(k) \\ y(k) \\ \hat{y}(k) \\ e_D(k) \\ \mu(k) \end{bmatrix}.$$

The state feedback is given by:

$$u(k) = \begin{bmatrix} \lambda A & 0 & \lambda C & \lambda(D + z^{-1}E) & \lambda F \end{bmatrix} \cdot \begin{bmatrix} y^*(k) \\ y(k) \\ \hat{y}(k) \\ e_D(k) \\ \mu(k) \end{bmatrix} + \lambda B y_d$$

$$S = \begin{bmatrix} \lambda A & 0 & \lambda C & \lambda(D + z^{-1}E) & \lambda F \end{bmatrix}$$

where, a^0, b^0, and m^0 are true discretized FOPDT process parameters given by:

$$a^0 = \exp\left(\frac{-\Delta t}{\tau^0}\right); b^0 = K^0\left[1 - \exp\left(\frac{-\Delta t}{\tau^0}\right)\right]; m^0 = round\left(\frac{\alpha^0}{\Delta t}\right)$$

Δt is the sampling time, and

$$A = \sum_{i=1}^{N} \eta_i \theta_T{}^i,$$

$$B = \sum_{i=1}^{N} \eta_i (1 - \theta_T{}^i),$$

$$C = -\sum_{i=1}^{N} \eta_i a^i,$$

$$D = -\sum_{i=1}^{N} \eta_i (1 + \alpha(i)),$$

$$E = \sum_{i=1}^{N} \eta_i \alpha(i),$$

$$F = -\sum_{i=1}^{N} \eta_i a^{i-1} b,$$

$$\alpha(i) = \frac{\theta_D}{1-\theta_D}\left(1 - (1-\theta_D)^{m+i}\right)$$