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Sagittal Gait Synthesis for a Five-Link Biped Robot

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Abstract – This paper presents a method for synthesising the gait of a planar five-link biped walking on level ground. Both the single support phase (SSP) and the double support phase (DSP) are considered. The compatible trajectories of the hip and the swing limb are first designed, which has the advantage of decoupling the biped into three subsystems, namely a trunk and two lower limbs and thus, substantially simplifies the problem. The hip and the swing limb trajectories are approximated with time polynomial functions and their coefficients are determined through the constraint equations cast in terms of coherent physical characteristics of gait. Special constraints are developed to eliminate the impact effect in spite of physical impact at the heel strike, which avoids the sudden jump of angular velocities and thus reduces the control difficulty. Other constraints considered in this work include the system stability during the DSP and repeatability of the gait. The effectiveness of the proposed method is confirmed by computer simulations.

I. INTRODUCTION

A biped robot is a class of walking robots that imitates human locomotion. The design of reference trajectories for gait cycle is a crucial step for biped motion control. However, there is a lack of systematic methods for synthesizing the gait and most of the previous work has been based on trial and error [1]. Vukobratovic *et al*. [2] have studied locomotion by using human walking data to prescribe the motion of the lower limbs. Two immediate problems arise when using human walking data directly. Firstly, a complex dynamic model is required. Secondly, the designer has no freedom to synthesize the joint angle profiles based on tangible gait characteristics, such as the walking speed, step length or step elevation. None of the aforementioned approaches gives a well-defined method for the generation of biped gait pattern. Hurmuzlu [3] developed a parametric formulation that ties together the objective functions and the resulting gait patterns. The objective functions are cast in terms of coherent physical characteristics of gait, which has been used to generate the walking pattern of a 5-link biped during the single support phase (SSP). In light of prevailing literature, Humuzlu's method fills the gap in the design of walking machines regarding the specification of objective functions. However, to have continuous and repeatable gait, the postures at the beginning and the end of each step have to be identical. This requires the selection of specific initial conditions, objective functions and associated gait parameters. This selection, however, can be extremely challenging, if not impossible. This issue has not been addressed in Humuzlu's original work [3].

The above problem of selecting proper initial conditions to generate repeatable gait can be remedied by using numerical methods by approximating the reference trajectories through time polynomial functions [4] or periodic spline interpolation [5,6], etc. One advantage of this technique is that extra constraints, such as repeatability of gait, can be easily included by adding the coefficients to the polynomials. Disadvantage includes that the computing load is high for large biped systems.

Chow and Jacobson [7] studied the optimal biped locomotion and first drew attention to the hip motion and suggested that the hip trajectory be synthesized prior to joint angle profiles. The advantage is to decouple the biped model into three subsystems — a torso and two lower limbs. Huang *et al.,* [6] adopted this idea and synthesized walking patterns for a 7-link biped robot.

Since a biped robot tips over easily, it is important to consider the stability during gait synthesis. Methods [6,8] have been proposed for generating walking patterns based on the concept of zero moment point (ZMP) [2]. In the previous work [8,9], the ZMP trajectory was first designed and the hip motion and joint angle profiles are then derived. The advantage of this method is that the stability can be guaranteed. However, not all desired ZMP can be achieved due to limited hip motion and, even if it can be achieved, large hip acceleration is often resulted from the desired ZMP, which makes the control task difficult.

From prevailing locomotion literature on biped walking pattern design, it has been noticed that several important issues need to be further investigated. Firstly, most studies have focused on motion generation during the SSP, and the DSP has received less attention. The DSP plays an important role in keeping a biped walking stably with a wide range of speeds, and thus cannot be neglected. Secondly, impact, occurring at the transition between the SSP and the DSP, makes control task difficult due to the discontinuity of the angular velocities and may have destablising effects on biped motion. Thus the impact effect should be taken into account in the gait synthesis. Other important issues, which need to be considered in the gait design, include requirements of stability and repeatability of the gait.

The objective of this paper is to propose a method for synthesizing cyclic gaits for biped walking. The important issues, which have not been investigated properly in previous literature, will be considered. Compatible hip and swing limb trajectories will be designed first, which has the advantage of simplifying the problem by decoupling the biped into three subsystems. The joint angle profiles for a full gait cycle including both the SSP and the DSP will then be obtained. The constraint functions and gait parameters are to be chosen such that repeatable gait will be generated. Smooth joint angle profiles will be generated at all time, including the transition between the SSP and the DSP, which removes the velocity

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jump caused by impact. Certain gait parameters will be chosen such that a large stability region during the DSP will be obtained. The paper is organized as follows. In Section 2, we formulate the constraints for the hip and swing limb trajectories for both the SSP and the DSP, and solve these trajectories by time polynomials. Simulations are carried out in Section 3. The conclusions are given in Section 4.

II. GENERATION OF WALKING PATTERN

2.1. Walking cycle

In this paper, we select the 5-link biped model taken from Hurmuzlu's work [3] for gait synthesis. The planar biped model has five rigid links connected by pin joints as shown in Fig. 1. One link represents the upper body and two links are connected as each lower limb, representing a thigh and a shank. $\theta_i(t)$ ($i=1,2,\ldots,5$) is the absolute angle between the i^{th} link and the vertical direction. There is an actuator located at each joint. We assume massless feet to simplify the work. Although we neglect the dynamics of the feet, we assume that the biped can apply torques at the ankles. As the biped requires steady motion, the joint angle profiles need to be continuous and repeatable.

A complete step can be divided into a SSP and a DSP. The SSP is characterized by one limb (the swing limb) moving in the forward direction while another limb (the stance limb) is pivoted on the ground. This phase begins with the swing limb tip leaving the ground and terminates with the swing limb touching the ground. Its time period is denoted as T_S. In the DSP, both lower limbs are in contact with the ground while the upper body can move forward slightly. The time period of this phase is denoted as T_D. In the following step, the roles of the swing limb and the stance limb are exchanged.

It has been noticed that the joint angle profiles can be determined if compatible trajectories for the hip and the tip of the swing limb can be prescribed. This approach has the advantage in that it decouples the biped into three separate subsystems: two lower limbs and a torso, which significantly simplifies the problem. The compatible hip and swing limb trajectories should satisfy the geometric condition that the distance between the hip and each tip of the lower limb is less than the length of the whole limb and greater than the difference between the lengths of the shank and the thigh, which does not allow the singular configurations at any time. This condition guarantees the existence of the joint angle profiles during the whole walking period. Furthermore we prescribe that both knees only bend in one direction, thus, the joint angle profiles will be uniquely determined by the trajectories of the hip and the swing limb. From the viewpoint of natural human walking, it is desirable that the torso is kept at the upright position. Giving the trajectory $\theta_3(t)=0$ in both the SSP and the DSP, our main task here is to generate the motion of the lower limbs. To satisfy the biped walking under various ground conditions, it is natural to design the trajectory for the swing limb tip first, followed by the design of the compatible hip trajectory.

2.2. Trajectories of the swing limb

The trajectory of the tip of the swing limb during the SSP is

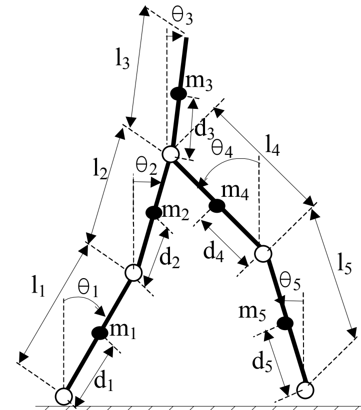

Fig. 1 Five-link biped model

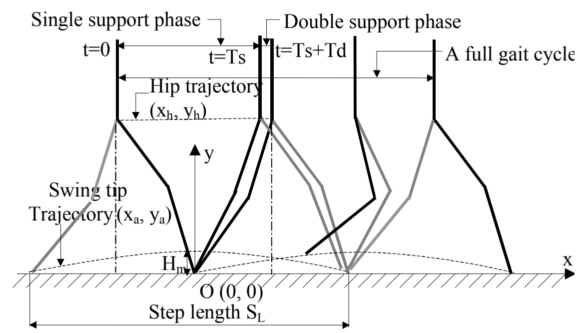

Fig. 2 Full gait cycle of a planar five-link biped

an important factor in biped walking. In this section, we develop constraint equations that can be used for solving the swing limb trajectory. The trajectory of the tip of the swing limb is denoted by the vector $X_a : (x_a(t), y_a(t))$, where $(x_a(t), y_a(t))$ is the coordinate of the swing limb tip position with the origin of the coordinate system located at the tip of the supporting limb (see Fig. 2). We use a third order polynomial and a fifth order polynomial functions for the x_a and y_a, separately. They are shown below:

$$X_a : \begin{cases} x_a(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ y_a(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{cases} \quad 0 \le t \le T_S \quad (1)$$

Next we develop constraint equations that can be used for solving the coefficients, a_i and b_j ($i=0,\ldots 3$ and $j=0,\ldots 5$). We cast the gait patterns in terms of four basic quantities: step length S_L, step period for the SSP T_S, maximum clearance of the swing limb H_m and its location S_m. Other constraints used for designing the swing limb motion are repeatable gait and minimizing the effect of impact. The constraint relations are described as follows:

(1) ***Geometrical constraints***: The swing limb has to be lifted off the ground at the beginning of the step and has to be landed back at the end of it. We will enforce this condition by the following equation:

$$y_a(0) = 0 \quad (2)$$

$$y_a(T_S) = 0 \quad (3)$$

(2) ***Maximum clearance of the swing limb:*** During the swing phase, the tip of the swing limb has to stay clear off the ground to avoid accidental contact. In some previous work [3,10], the parabolic relation between $x_a(t)$ and $y_a(t)$ has been assumed. Although this strategy has the advantage of being the simplest form that allows the prescription of the desired step length and the tip maximum clearance independently, it is unlikely satisfies the requirement of repeatable gait. In this work, we synthesize the swing limb trajectory by setting the following relation:

$$x_a(T_m) = S_m \tag{4}$$

$$y_a(T_m) = H_m \tag{5}$$

$$\dot{y}_a(T_m) = 0 \tag{6}$$

Where H_m is the maximum clearance of the swing limb, S_m is the x-coordinate of the swing limb tip corresponding to the maximum clearance, and T_m is the time instant when the tip of the swing limb reaches to the maximum clearance. Note that T_m is not prescribed.

(3) ***Repeatability of the gait:*** The requirement for repeatable gait imposes the initial posture and angular velocities to be identical to those at the end of the step. Furthermore, since during the DSP both tips are in contact with the ground and remain stationary, the initial velocities in both horizontal and vertical direction must remain zero. Subsequently, the following relations must hold:

$$x_a(0) = -\frac{S_L}{2} \tag{7}$$

$$x_a(T_S) = \frac{S_L}{2} \tag{8}$$

$$\dot{x}_a(0) = 0 \tag{9}$$

$$\dot{y}_a(0) = 0 \tag{10}$$

(4) ***Minimizing the effect of impact:*** During locomotion, when the swing limb contacts the ground at heel strike, impact occurs, which causes sudden changes in the joint angular velocities. The effect of impact on the sudden jumps in the joint angular velocities is shown by (A3) in the Appendix for the case that the impact at the contact point is perfectly plastic (*i.e.,* the tip of the swing limb sticks to the ground after impact). Thus, by keeping the velocities of the swing tip zero before impact, the sudden jump in the joint angular velocities can be eliminated (Refer to (A3)). The above conditions lead to:

$$\dot{x}_a(T_S) = 0 \tag{11}$$

$$\dot{y}_a(T_S) = 0 \tag{12}$$

Equations (2)-(12) can be used for solving ten polynomial coefficients a_i, b_j ($i=0,\ldots 3$ and $j=0,\ldots 5$) and T_m. The trajectory of the swing limb with the above coefficients satisfies the requirements, such as repeatable gait with no destabilizing effect from impact and with prescribed gait parameters.

2.3. Trajectories of hip

Hip motion has significant effect on the stability of the biped system. Here, the trajectory of the hip is designed for the SSP and the DSP, separately, which are denoted by the coordinate of the hip position as $X_{hS} : (x_{hS}(t), y_{hS}(t))$ in the SSP and $X_{hD} : (x_{hD}(t), y_{hD}(t))$ in the DSP. A third order polynomial function is used to describe x_{hS} and x_{hD}, respectively. With a general function of vertical hip motion, they are shown below:

$$x_{hS}(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3;\ 0 \le t \le T_S \tag{13a}$$

$$x_{hD} = d_0 + d_1 t + d_2 t^2 + d_3 t^3;\ \ 0 \le t \le T_D \tag{13b}$$

$$y_{hS}(t) = y_h(t)\ ;\qquad 0 \le t \le T_S \tag{13c}$$

$$y_{hD}(t) = y_h(t);\qquad 0 \le t \le T_D \tag{13d}$$

Next, we develop constraint equations which include the additional quantities: positions of the hip at the beginning of the SSP and the DSP: S_{S0} and S_{D0}, step period for the DSP: T_D, and the height of the hip: H_h. Besides the constraints of repeatable gait and minimizing the effect of impact, the stability of the biped walking during the DSP is also considered. The constraint relations are described as follows:

(1) ***Vertical hip motion:*** One desired feature of biped gait is to keep the minimum vertical motion of the gravity center, which requires minimum vertical motion of the hip. For the sake of simplicity, we assume y_{hS} and y_{hD} a constant at any time during the whole gait cycle, *i.e.*,

$$y_{hS}(t) = H_h \tag{14}$$

$$y_{hD}(t) = H_h \tag{15}$$

H_h should be given such that the robot does not go through the singular configurations.

(2) ***Repeatability of the gait:*** To keep the gait repeatable, the posture and angular velocity at the beginning of the SSP must be identical to that at the end of the DSP. Thus, the following relations must hold:

$$x_{hS}(0) = -S_{S0} \tag{16}$$

$$\mathrm{x_{hD}}(T_D) = \frac{1}{2} S_L - S_{S0} \tag{17}$$

$$\dot{x}_{hS}(0) = V_{h1} \tag{18}$$

$$\dot{x}_{hD}(T_D) = V_{h1} \tag{19}$$

where V_{h1} is the hip velocity at the beginning of each step, which will be determined later.

(3) ***Continuity of the gait:*** The hip trajectory must be continuous during the whole gait cycle, *i.e.*, the horizontal displacements and velocities of the hip at the end of the SSP and the beginning of the DSP must be identical respectively, which leads to:

$$\mathrm{x_{hD}}(0) = S_{D0} \tag{20}$$

$$x_{hS}(T_S) = S_{D0} \tag{21}$$

$$\dot{x}_{hS}(T_S) = V_{h2} \tag{22}$$

$$\dot{x}_{hD}(0) = V_{h2} \tag{23}$$

V_{h2} will be determined later.

(4) ***Stability of the gait during the DSP:*** The horizontal velocity of the hip is the main factor that affects the stability of biped locomotion. As discussed in Section 1, previous research focused on deriving the hip trajectories to execute a desired ZMP. The disadvantages are that not all desired ZMP trajectories can be attained and the hip acceleration may be

very large. In this work, we propose the following procedure to select the initial and final hip velocity V_{h1} and V_{h2} :

(i) generate a series of smooth $x_{hS}(t)$ and $x_{hD}(t)$ trajectories by selecting various V_{h1} and V_{h2} ;

(ii) obtain a set of solutions of $x_{hS}(t)$ and $x_{hD}(t)$ with a large stability margin.

Stability margin is defined here as the minimum distance between the ZMP and the boundary of the stable region, which is the line between the two tips of the supporting limbs. Equations (16)-(23) and the stability constraints can be used to solve all eight coefficients, c_i *and* d_i ($i=0,\ldots3$). For each set of the selected values of V_{h1} and V_{h2} , $x_{hS}(t)$ and $x_{hD}(t)$ can be solved uniquely and they are used to find out the ZMP trajectory. By using computer iterative calculation, we can easily obtain a set of V_{h1} and V_{h2} to solve for the trajectories of $x_{hS}(t)$ and $x_{hD}(t)$ with the largest stability margin.

The ZMP criterion is not studied in the SSP here by assuming applying a pair of enough long massless feet to simplify the modeling and trajectory design.

2.4. Biped joint angle profile

With the designed hip and swing leg tip trajectories and the biped kinematic model, the joint angle profiles can be expressed by the following equations

$$\begin{cases} \theta_1(t) = \arcsin(\dfrac{A_1C_1 + B_1\sqrt{A_1^2+B_1^2-C_1^2}}{A_1^2+B_1^2}) \\ \theta_2(t) = \theta_1(t) + \arcsin(\dfrac{A_1\cos(\theta_1(t)) - B_1\sin(\theta_1(t))}{l_2}) \\ \theta_3(t) = 0 \\ \theta_4(t) = \arcsin(\dfrac{A_4C_4 + B_4\sqrt{A_4^2+B_4^2-C_4^2}}{A_4^2+B_4^2}) \\ \theta_5(t) = \theta_4(t) + \arcsin(\dfrac{A_4\cos(\theta_4(t)) - B_4\sin(\theta_4(t))}{l_5}) \end{cases} \quad (24)$$

where for the SSP,

$$A_1 = x_{hS}(t),\ B_1 = y_{hS}(t),\ C_1 = \frac{A_1^2+B_1^2+l_1^2-l_2^2}{2l_1},$$

$$A_4 = x_{hS}(t) - x_{aS}(t),\ B_4 = y_{hS}(t) - y_{aS}(t),$$

$$C_4 = \frac{A_4^2+B_4^2+l_4^2-l_5^2}{2l_4}$$

and for the DSP,

$$A_1 = x_{hD}(t),\ B_1 = y_{hD}(t),\ C_1 = \frac{A_1^2+B_1^2+l_1^2-l_2^2}{2l_1},$$

$$A_4 = \frac{1}{2}S_L - x_{hD}(t),\ B_4 = y_{hD}(t),\ C_4 = \frac{A_4^2+B_4^2+l_4^2-l_5^2}{2l_4}$$

Equation (24) can be used as reference inputs for control purpose.

III. SIMULATIONS

In this section, the joint profiles for a five-link biped walking on level ground with both the SSP and the DSP are simulated based on the method discussed in Section 2. The values of the parameters m_i, I_i, l_i and d_i of the five-link biped robot are listed in Table 1, and the walking speed is chosen approximately 1m/s with $S_L=0.72m$, $T_S=0.6s$, $T_D=0.1s$, $H_m=0.05m$ and $S_m=0m$.

Fig. 3a shows the horizontal displacements of the hip and the tips of both the swing limb and the stance limb. Fig. 3b shows the trajectories of the tip of the swing limb. All the trajectories are smooth, *i.e.,* all the velocities are continuous. Fig. 4 shows the motion of lower limb joints for two steps. Fig. 4a is the profiles of the joint angles and Fig. 4b is the angular velocities during the SSP and the DSP, respectively. It is seen that both joint angles and their angular velocities are repeatable, and the velocities are continuous at the instant of impact showing that the discontinuity of the angular velocities has been removed.

Fig. 5 shows the horizontal displacements of the gravity centre of the biped, hip and ZMP during the DSP. The grey area is the stability region with the top and bottom lines representing the locations of the two feet. It can be seen that the center of gravity (CG) and the hip, especially the ZMP, remain approximately at the centre of the stability region, which ensures a large stability margin. Fig. 6 shows the reaction forces exerted on the lower limbs carried out by the forward dynamic simulation. For the above gait parameters, the vertical ground reaction forces are always upward showing that a firm contact without lifting is guaranteed. In addition, the no-slip condition can be guaranteed with a friction coefficient of 0.4. Fig. 7 is the stick diagram of the five-link biped walking on level ground. From this diagram, one can observe the overall motion of the biped during both the SSP and the DSP. The solid lines represent the SSP while the dashed lines represent the DSP. The asterisk shows the CG of the biped during walking. The posture of the biped at the end of each step is close to that at the beginning of each step, and the displacement of CG trajectory is almost horizontal. Overall, Fig. 6-7 show that the gait pattern designed based on the method proposed in this paper is quite natural and the contact constraints between the lower limbs and the ground can be satisfied with a moderate friction coefficient.

Table 1. Parameters of the biped robot

Link	m_i (kg)	I_i (kgm^2)	l_i (m)	d_i (m)
Torso	14.79	3.30×10^{-2}	0.486	0.282
Thigh	5.28	3.30×10^{-2}	0.302	0.236
Leg	2.23	3.30×10^{-2}	0.332	0.189

IV. CONCLUSIONS

In this paper, a systematic approach is presented for gait synthesis for five-link biped walking in the sagittal plane. Unlike most of the previous work focusing on the SSP, our model includes both the SSP and the DSP, which gives the

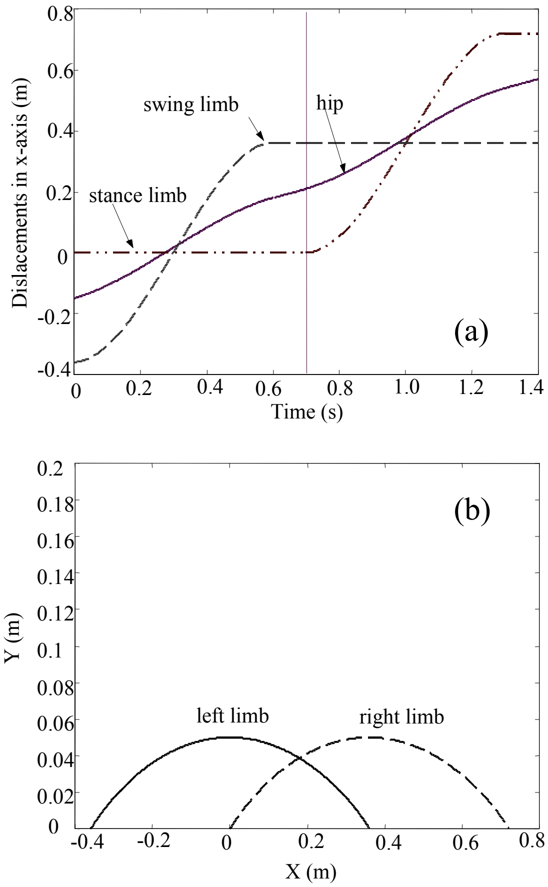

Fig. 3 Trajectories of the hip and lower limb
(a) *x-t* and (b) *y-x*

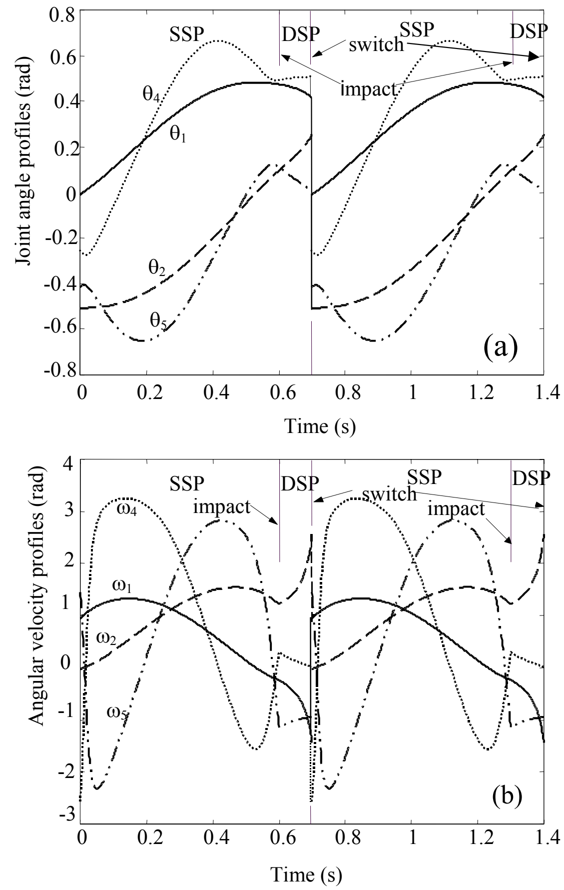

Fig.4 Profiles of joint angle and angular velocity
(a) joint angles and (b) angular velocities

biped a wider range of walking speeds and more stable locomotion. A method of formulating the compatible trajectories for the hip and the swing limb is employed. This method has the advantage of decoupling the biped into three subsystems, namely a trunk and two lower limbs, which significantly simplified the problem. The trajectories of the hip and the swing limb are approximated with time polynomial functions and their coefficients are solved using the constraint equations cast in terms of the step length, step period and maximum step clearance etc. Special constraints are developed to eliminate the impact effects during the biped motion in spite of physical impact at heel strike occurring in the system. Other important constraints used in this paper include the stability and repeatability of the gait. Computer simulations were carried out to demonstrate the effectiveness of the proposed method. The results show that with all the above designed criteria satisfied, the gait pattern appears natural and only moderate requirement on the friction coefficient of the ground is required to keep the contact constraints between the limbs and the ground.

We believe this research can provide a valuable tool for generating motion patterns of biped gait, which is crucial for biped motion control. The advantage of decoupling the biped into subsystems significantly simplifies the problem, which makes it feasible to generate other optimal gait patterns, such as energy efficient walking. It is also a stepping stone for using more complicated objective functions. The avoidance of impact occurring at heel strike to eliminate the sudden jump of the angular velocity and large impulsive forces to each joint reduces the control difficulty and enhances the system stability, which is the main concern in the development of bipedal walking machines.

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APPENDIX: Dynamic model of a planar five-link biped

We consider a planer five-link biped model with both the SSP and the DSP. By applying Lagrange formulation, the equation of motion during the SSP can be written in the following general form:

$$D(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = T \qquad \text{(A1)}$$

where $D(\theta) \in R^{5\times5}$ is the positive definite and symmetric inertia matrix, $H(\theta,\dot{\theta}) \in R^{5\times5}$ and $G(\theta) \in R^{5}$ are matrices of centrifugal and Coriolis terms and gravity terms, and θ, $\dot{\theta}$, $\ddot{\theta}$, $T \in R^{5}$ represent the generalized coordinates, velocities, accelerations and torques, respectively.

The motion of the DSP can be derived based on the Lagrange dynamic equations with constraint conditions in the following form:

$$\begin{cases} D(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = J^T(\theta)\lambda + T \\ \Phi(\theta) = 0 \end{cases} \qquad \text{(A2)}$$

where $\Phi(\theta)$ represents the constraint functions: $\Phi(\theta) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x_e - x_b - L \\ y_e - y_b \end{bmatrix} = 0$, J is the Jocobian matrix: $J = \frac{\partial f_i}{\partial \theta_j}$, λ is the Lagrangian multiplier.

At the end of the SSP, the swing limb contacts the ground surface. The generalized velocities will be subject to a sudden change resulting from the impact event. During impact, there will be impulsive forces between the contact points of both lower limbs and the ground. The velocities of the two contact points immediately after impact are zero under the assumption of perfectly plastic impact. Using Lagrange impact model, the corresponding equations describing the new angular velocity after impact can be written as:

$$\dot{\theta}^{+} = \dot{\theta}^{-} + D^{-1}J^T\left[JD^{-1}J^T\right]^{-1}\left[-\dot{x}_a^{-} \quad -\dot{y}_a^{-}\right]^T \qquad \text{(A3)}$$

where $\dot{\theta}^{+}, \dot{\theta}^{-} \in R^5$ represent the velocity immediately after and before impact, respectively. Equation (A3) shows that if the linear velocity of the swing limb tip is zero immediately before it touches down on the ground, the discontinuity of the biped angular velocities due to an impact event will be avoided.

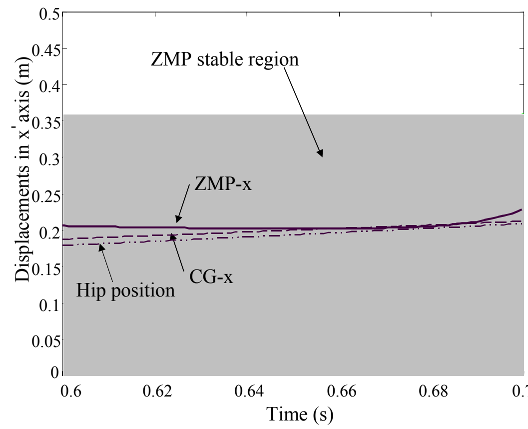

Fig. 5 CG, hip and ZMP trajectories during DSP

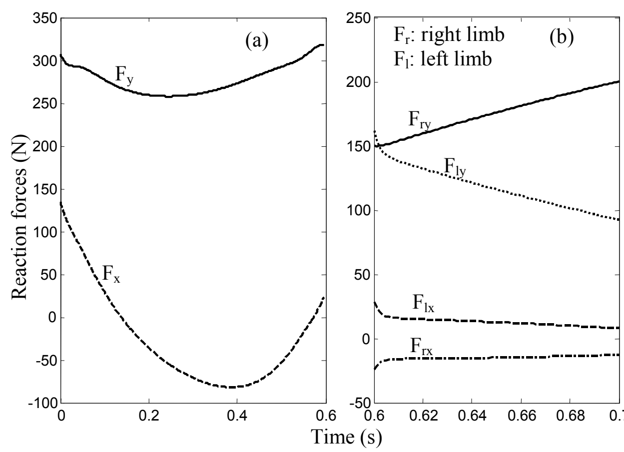

Fig. 6 Reaction forces (a)SSP and (b)DSP

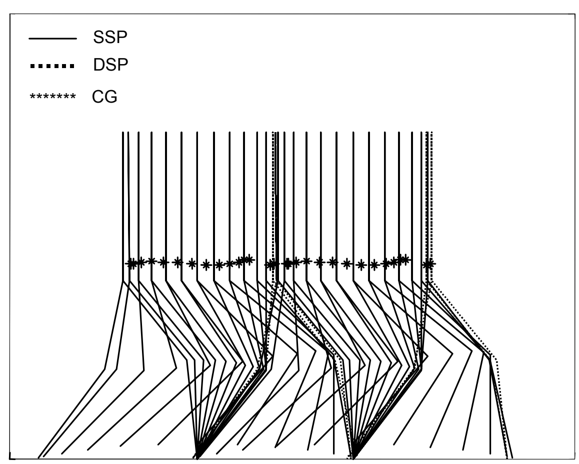

Fig. 7 Stick diagram of a biped walking