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Swing-up Control of a Serial Double Inverted Pendulum

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***Abstract*— In this paper, a swing-up control scheme for a serial double inverted pendulum is proposed. The control scheme is to swing up the pendulum in three steps, Step 1: to swing up the first pendulum, Step 2: to swing up the second pendulum while stabilizing the first pendulum at the upright position, and Step 3: to stabilize the two pendulums around the unstable equilibrium state.**

In each step a control scheme and a switching rule from a step to next step are given. The rule bases on the states of the system. For the controller of Step 1, the energy control method is applied to swing up the first pendulum. For the controller of Step 2, a new method is given. The method combines a stabilization control method of the first pendulum using sliding mode control method and a swing-up control method of the second pendulum using energy control method. And for a controller in Step 3, the sliding mode controller stabilizing both of the pendulums is used. A numerical simulation is given to show the effectiveness of the proposed scheme.

I. INTORODUCTION

Inverted pendulums are typical examples of nonlinear and underactuated mechanical systems and well known in control engineering for verification and practice of various kinds of control theories. Inverted pendulum systems have several types, e.g., a single pendulum, a parallel double pendulum, a serial double pendulum and a two dimensional pendulum, etc. And many control methods have been proposed to control the inverted pendulum systems, such as feedback stabilization[1][4], energy based control[1][4], bang-bang control[3], sliding mode control[6], robust control[2], hybrid control[4][5], partial linearization[7][8] (see [1] for more details). In spite of these existing methods, to control the inverted pendulums is still an open research topic. In particular, the serial double inverted pendulum is strongly nonlinear and highly underactuated than a single inverted pendulum and the control of the serial double pendulum is a difficult problem. And a solution of this problem is applicable to other nonlinear and underactuated control problems. Hence, this paper considers the swing-up control problem for the serial double inverted pendulum.

For swing-up control of the serial double inverted pendulum, various methods are already proposed, e.g., the method in [3] is a combination of feedforward controller that swings up pendulums and feedback controller that stabilizes the pendulum at the upright position. In [2], a control scheme which transfers the state of pendulum from an arbitrary equilibrium point to another arbitrary equilibrium point among four equilibrium points; (first pendulum) Down- (second pendulum) Down position, Up-Down position, Down-Up position, and Up-Up position was proposed, and this method was applied to the swing-up control, i.e., to transfer the state from Down-Down position to Up-Up position. However, those methods were the controllers for a rotation type pendulum and they do not consider the restriction to the movement of a carriage. The controller proposed in this paper swings up and stabilizes the cart type serial double inverted pendulum and includes control scheme to restrict traveling position of the cart.

The proposed controller consists of three steps, Step 1: to swing up the first pendulum, Step 2: to swing up the second pendulum while stabilizing the first pendulum at the upright position, and Step 3: to stabilize the two pendulums around the unstable equilibrium state.

In each step, a control scheme and a switching rule from a step to the next one are given. Although the control law in Step1 corresponds to the control problem transferring the state from Down-Down position to Up-Down position in [2], and Step2 corresponds to transferring the state from Up-Down position to Up-Up position, the control laws in these steps are different from those in [2] in including a control law to restrict traveling position of the cart. Another difference is, in designing the control law to swing up the second pendulum that the controller of [2] does not consider the stabilization of the first pendulum, while the controller given in this paper swings up the second pendulum and also stabilizes the first pendulum. That is, the proposed controller is given by a stabilization control law for the first pendulum added by a swing-up control law for a second pendulum. For the stabilizing controller of the first pendulum at the unstable equilibrium point, a state feedback stabilization controller is derived by using a sliding mode control method[6]. And for swinging up the second pendulum, a modified energy control method given by [1] is used. Since the sliding mode controller has

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strong robustness to disturbance and the swing-up input is considered as disturbance, the added swing-up control input does not affect the stabilization in the stabilizing the first pendulum.

In Section II, the dynamics equation is derived for the serial double inverted pendulum system as depicted in Fig.1.

Section III is for a controller of Step 1. A control scheme which swings up the first pendulum is given. For swinging up the second pendulum, an energy control method described in [1] is applied. To prevent the cart from exceeding the rail, in swinging up the pendulum, a control law which suppresses the movement of the cart is considered.

In Section IV, a control law to swing up the second pendulum while stabilizing the first pendulum is given.

In Section V, for a controller in Step 3, a sliding mode controller is used in stabilizing both of the pendulums by linearizing the system around the unstable equilibrium state.

In Section VI, a numerical simulation is given to show the effectiveness of the proposed scheme in this paper.

II. Dynamics

The dynamics of the serial double inverted pendulum system as depicted in Fig.1 is given

$$I_\theta\ddot{\theta} + M_1\cos(\theta-\phi)\ddot{\phi} + M_1\dot{\phi}^2\sin(\theta-\phi) + (c_1+c_2)\dot{\theta} - c_2\dot{\phi} - M_2 g\sin\theta + M_2\cos\theta\ddot{z} = 0 \quad (1)$$

$$M_1\cos(\theta-\phi)\ddot{\theta} + I_\phi\ddot{\phi} - M_1\dot{\theta}^2\sin(\theta-\phi) - c_2(\dot{\theta}-\dot{\phi}) - M_3 g\sin\phi + M_3\cos\phi\ddot{z} = 0 \quad (2)$$

where I_θ,I_ϕ,M_1,M_2,M_3 are expressed as

$$\begin{aligned} I_\theta &= I_1 + m_2L^2 + n_2L^2 \\ I_\phi &= I_2 + J_{n_1} + J_{n_2} \\ M_1 &= m_2 l_2 L \\ M_2 &= m_1 l_1 + m_2 L + n_2 L \\ M_3 &= m_2 l_2 \end{aligned}$$

and the parameters of the pendulums are defined in Table I.

The control input u as

$$u = \ddot{z} \quad (3)$$

The control objective in this paper is to swing up both pendulums from the downward position to the upward position and to stabilize the pendulums at the upward position.

III. Swing up the first pendulum (Step1)

In this section, a control method to swing up the first pendulum is proposed as a controller of Step 1. The energy control method using Lyapunov method developed in [1] is applied.

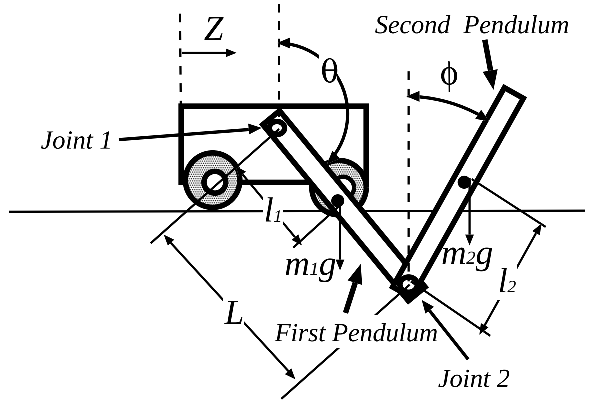

Fig. 1. The serial double inverted pendulum

TABLE I
DEFINITIONS OF PARAMETERS

z	position of the cart
θ	angular position of the first pendulum from the vertical line
ϕ	angular position of the second pendulum from the vertical line
$m_i\ (i=1,2)$	mass of the *i* th pendulum
$n_i\ (i=1,2)$	mass of the *i* th joint of pendulums
$l_i\ (i=1,2)$	length from the *i* th joint to the center of mass of the *i* th pendulum
$J_{n_i}\ (i=1,2)$	inertia of mass of the *i* th joint around the center of gravity
$I_i\ (i=1,2)$	inertia of the *i* th pendulum around the joint
$c_i\ (i=1,2)$	viscosity of each joint
L	length of the first pendulum
g	gravity acceleration

From (1) and (3), by neglecting second pendulum and viscosity friction, the dynamics of θ is

$$\begin{aligned} \ddot{\theta} &= \frac{M_2}{I_\theta}(g\sin\theta - u\cos\theta) \\ &= \frac{m_1 l_1}{I_1}(g\sin\theta - u_\theta\cos\theta) \end{aligned} \quad (4)$$

where $u_\theta := u$ is defined as a control input to swing up the first pendulum. This equation is equivalent to the dynamics of a single inverted pendulum. Therefore, as a control method which swings up the first pendulum, the control method presented in [1] and [4] which swings up the single pendulum is applied.

For the swing-up control, a control strategy is presented in [1] and [4] by using Lyapunov method.

Define a Lyapunov function as

$$V_1 = \frac{1}{2}E_1^2 \tag{5}$$

where E_1 is the energy of the first pendulum given as

$$E_1 = \frac{1}{2}I_1\dot{\theta}^2 + m_1 g l_1(\cos\theta - 1) \tag{6}$$

When the pendulum is at the upright position, i.e., $\phi = 0$ and $\dot{\phi} = 0$, then $E_1 = 0$ and when the pendulum is at the pendant position, i.e., $\phi = \pi$ and $\dot{\phi} = 0$, then $E_1 = -2m_1 l_1 g$.

Calculating the derivative of V_1 along the trajectory of (4) yields

$$\dot{V}_1 = -m_1 l_1(E_1\dot{\theta}\cos\theta)u_\theta \tag{7}$$

From (7), to make $\dot{V} < 0$, the control input is given as

$$u_\theta = u_{a1}\text{sign}(E_1\dot{\theta}\cos\theta) \qquad (u_{a1} > 0) \tag{8}$$

From (7) and (8), the derivative of V_1 is

$$\dot{V}_1 = -m_1 l_1(E_1\dot{\theta}\cos\theta)(u_{a1}\text{sign}(E_1\dot{\theta}\cos\theta)) \leq 0 \tag{9}$$

It follows that $V_1(t)$ is a non-increasing function and $\dot{V}_1 \to 0$ as $t \to \infty$. When $\dot{V}_1 = 0$, there are three cases, a) $\cos\theta = 0$, b) $\dot{\theta} = 0)$ or c) $E_1 = 0$. Since the horizontal position of the pendulum is not an equilibrium point, case a) can not be maintained. Also, if case b) can be kept without control input, then $\theta = 0$ must hold simultaneously, which follows $E_1 = 0$. Otherwise, the pendulum will fall again. Therefore, the necessary and sufficient condition for $\dot{V}_1 \equiv 0$ is $E_1 = 0$. Thus,

$$\lim_{t\to\infty} E_1(t) = 0, \quad \lim_{t\to\infty} V_1(t) = 0 \tag{10}$$

are obtained. Therefore, using the input of (8), the energy of the first pendulum increases to zero, and the first pendulum is swung up.

So far, the swing-up of the first pendulum by using the control input u_θ of (8) is shown. But moving distance of the cart is not considered in the control input u_θ of (8) and it may exceed the width of the rail. Hence, a new control input to suppress the moving distance of a cart is introduced.

Now, the feedback control input u_z to control the cart into the center of a rail,

$$u_z = -K_1 \cdot x_1 \qquad (K_1 > 0) \tag{11}$$

is considered, where, $x_1 = [z, \dot{z}]^T$ and K_1 is feedback gain matrix which control x_1 to 0.

To ensure $\dot{V} < 0$, u_z has the same sign as that of (8). Therefore, a control input is proposed by combining (8) and u_z as

$$u = \begin{cases} u_{a1}\text{sign}(E_1\dot{\theta}\cos\theta) + u_z & (\text{sign}(u_\theta) = \text{sign}(u_z) \\ & \text{or} \quad |u_\theta| > |u_z|) \\ u_{a1}\text{sign}(E_1\dot{\theta}\cos\theta) & (\text{sign}(u_\theta) \neq \text{sign}(u_z) \\ & \text{and} \quad |u_\theta| \leq |u_z|) \end{cases} \tag{12}$$

(12) is the control input which swings up the first pendulum and controls the moving distance of a cart.

In this section, in designing the control law which swings up the first pendulum, influences of the second pendulum and viscosity friction are ignored. However, these influences exist in fact, and it is predicted that the energy of the first pendulum decreases by these influences. In case of single inverted pendulum, previous method presented in [1],[4] is also designed ignoring the influence of viscosity friction. This influence is avoided by setting up a large input gain and giving energy to a system larger than the energy loss of the system by the influence of viscosity friction.

Considering the case of the serial double inverted pendulum, influence of viscosity friction makes always loss of the energy of the first pendulum. However, there are two cases that the influence of the second pendulum may lose the energy of the first pendulum, and it may give energy to the first pendulum. Therefore, we assume that we can also ignore the influence of second pendulum by making the input gain u_{a1} large.

When the first pendulum swung up in the neighborhood $\theta = 0$ by controller (12), the controller will be switched to the controller of Step 2.

IV. Swing up the second pendulum while stabilizing the first pendulum (Step 2)

In this section, a control method which swings up the second pendulum while stabilizing the first pendulum is proposed. The proposed method consists of two control laws, to stabilize the first pendulum at the unstable equilibrium point, and to swing up the second pendulum. For the stabilizing controller of the first pendulum at the unstable equilibrium point, a state feedback stabilization controller is derived by using sliding mode controller[6]. And for swinging up the second pendulum, a modified energy control method described in [1] is used.

First, the control law which stabilizes the first pendulum is considered. By neglecting ϕ in (1), the dynamics of θ becomes

$$I_\theta\ddot{\theta} + (c_1 + c_2)\dot{\theta} - M_2 g\sin\theta + M_2\cos\theta\ddot{z} = 0 \tag{13}$$

The control input u_1 is defined as

$$u_1 = \ddot{z} \tag{14}$$

The following state variables are chosen as

$$\begin{aligned} x_2 &= [x_{21},\ x_{22},\ x_{23},\ x_{24}]^T \\ &= [\theta,\ \dot{\theta},\ z,\ \dot{z}]^T \end{aligned} \tag{15}$$

With (14) and (15), the linearized state-space equation of (13) around the unstable equilibrium point of the first pendulum $((\theta,\dot{\theta})=(0,0))$ is

$$\begin{aligned}\dot{x}_2 &= A_2x_2+B_2u_1\\ &= \begin{bmatrix} 0 & 1 & 0 & 0\\ \frac{M_2 g}{I_\theta} & -\frac{c_1+c_2}{I_\theta} & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix} x_2 + \begin{bmatrix} 0\\ -\frac{M_2}{I_\theta}\\ 0\\ 1\end{bmatrix} u_1\end{aligned} \tag{16}$$

A sliding mode controller is used to keep robustness for stabilization of the first pendulum. The control input u_1 which stabilizes (16) is

$$u_1 = -(S_2B_2)^{-1}(S_2A_2x_2+R_2\mathrm{sign}(\sigma_2)+K_2\sigma_2) \tag{17}$$
$$\sigma_2 = S_2x_2 \tag{18}$$

where $R_2>0, K_2>0$ and S_2 is the solution of the following Riccati equation with $\epsilon_2>0$

$$\begin{aligned}&P_2(A_2+\epsilon_2 I)+(A_2+\epsilon_2 I)^TP_2-P_2B_2B_2^TP_2+Q_2=0\\ &S_2=B_2^TP_2\end{aligned}$$

Next, the control law which swings up the second pendulum is considered. The energy control method is used as the controller which swings up the second pendulum as a same way to Step 1. In Step 2, the pendulum to be swung up is the second one and the control input to swing up is the displacement of the joint of the two pendulums (Joint 2), whereas, in Step 1, the pendulum is the first one and the control input is the cart position. Hence, in Step 2, the joint position is controlled in the same way as the cart position in Step 1.

The coordinate of horizontal component z_2 of Joint 2 is given by

$$z_2 = z + L\sin\theta$$

Thus, the acceleration of z_2 is

$$\ddot{z}_2 = \ddot{z} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta \tag{19}$$

From (1) and (2), it is obtained as

$$\ddot{\theta} = \frac{A_{12}(f_{21}+f_{22}\ddot{z})-A_{22}(f_{11}+f_{12}\ddot{z})}{A_{11}A_{22}-A_{12}A_{21}} \tag{20}$$

where

$$\begin{aligned}A_{11} &= I_\theta, \quad A_{22}=I_\phi\\ A_{12} &= A_{21} = M_1\cos(\theta-\phi)\\ f_{11} &= M_1\dot{\phi}^2\sin(\theta-\phi)+(c_1+c_2)\dot{\theta}\\ &\qquad -c_2\dot{\phi}-M_2 g\sin\theta\\ f_{12} &= M_2\cos\theta\\ f_{21} &= -M_1\dot{\theta}^2\sin(\theta-\phi)-c_2(\dot{\theta}-\dot{\phi})-M_3 g\sin\phi\\ f_{22} &= M_3\cos\phi\end{aligned}$$

From (19) and (20), the relation between $\ddot{z}_2$ and $\ddot{z}$ is derived as

$$\begin{aligned}\ddot{z} &= \frac{(\ddot{z}_2+L\dot{\theta}^2\sin\theta)(A_{11}A_{22}-A_{12}A_{21})}{A_{11}A_{22}-A_{12}A_{21}+(A_{12}f_{22}-A_{22}f_{12})L\cos\theta}\\ &\quad - \frac{(A_{12}f_{21}-A_{22}f_{11})L\cos\theta}{A_{11}A_{22}-A_{12}A_{21}+(A_{12}f_{22}-A_{22}f_{12})L\cos\theta}\end{aligned} \tag{21}$$

When u_{z_2} is assumed to be the acceleration of the joint and the second pendulum is assumed to be single pendulum, the dynamics of the second pendulum is as follows.

$$\begin{aligned}\ddot{\phi} &= \frac{M_3}{I_\phi}(g\sin\phi-\ddot{z}_2\cos\phi)\\ &= \frac{m_2l_2}{I_2}(g\sin\phi-u_{z_2}\cos\phi)\end{aligned} \tag{22}$$

Therefore, the control input which swings up the second pendulum using the energy control method as in Step1 is

$$u_{z_2} = u_{a2}\mathrm{sign}(E_2\dot{\phi}\cos\phi) \qquad (u_{a2}>0) \tag{23}$$

where, E_2 is energy of the second pendulum.

When the angle of the second pendulum is large, the influence of the first pendulum is large and complex, and deciding a swing-up input to the second pendulum is difficult. Therefore the control input is applied during a small moment just after the second pendulum passes through the vertical line, as shown below.

$$u_{z_2} = \begin{cases} u_{a2}\mathrm{sign}(E_2\dot{\phi}\cos\phi) & (\sin\phi\cdot\dot{\phi}<0 \\ & \quad \text{and} \quad \cos\phi<\cos\mu)\\ 0 & (\sin\phi\cdot\dot{\phi}\geq 0\\ & \quad \text{and} \quad \cos\phi<\cos\mu)\\ 0 & (\cos\phi\geq\cos\mu)\end{cases} \tag{24}$$

From (21), acceleration u_2 of the cart which makes acceleration of the joint position to be u_{z_2} which swings up the second pendulum is

$$\begin{aligned}u_2 &= \frac{(u_{z_2}+L\dot{\theta}^2\sin\theta)(A_{11}A_{22}-A_{12}A_{21})}{A_{11}A_{22}-A_{12}A_{21}+(A_{12}f_{22}-A_{22}f_{12})L\cos\theta}\\ &\quad - \frac{(A_{12}f_{21}-A_{22}f_{11})L\cos\theta}{A_{11}A_{22}-A_{12}A_{21}+(A_{12}f_{22}-A_{22}f_{12})L\cos\theta}\end{aligned} \tag{25}$$

By combining u_1 of (17) and u_2 of (25), the following controller is obtained, which swings up the second pendulum while stabilizing the first pendulum.

$$u = u_1 + u_2 \tag{26}$$

The sliding mode control input u_1 which stabilizes the first pendulum has strong robustness to disturbances and the swing-up input u_2 can be treated as disturbance, Therefore, it is not influential to stabilization.

When the second pendulum is swung up in the neighborhood $\phi=0$ by controller (26), the controller will be switched to the controller of Step 3.

V. STABILIZE TWO PENDULUMS (STEP 3)

The state $(\theta, \dot{\theta}, \phi, \dot{\phi}) = (0, 0, 0, 0)$ is an unstable equilibrium point of the both pendulums to which both pendulums will be stabilized. Choose x_3 as the following state variables

$$\begin{aligned} x_3 &= [x_{31},\ x_{32},\ x_{33},\ x_{34},\ x_{35},\ x_{36}]^T \\ &= [\theta,\ \dot{\theta},\ \phi,\ \dot{\phi},\ z,\ \dot{z}]^T \end{aligned} \tag{27}$$

With (3) and (27), the linearized state-space equation of (1) and (2) around $x_3 = 0$ is

$$\begin{aligned} \dot{x}_3 &= A_3 x_3 + B_3 u \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \\ 0 \\ 1 \end{bmatrix} u \end{aligned} \tag{28}$$

where the elements of matrices A_3 and B_3 are given as follows.

$$\begin{aligned} a_{21} &= -\frac{M_2 I_\phi g}{I_\phi I_\theta - M_1^2} \\ a_{22} &= -\frac{M_1 c_2 + I_\phi (c_1 + c_2)}{I_\phi I_\theta - M_1^2} \\ a_{23} &= \frac{M_1 M_3 g}{I_\phi I_\theta - M_1^2} \\ a_{24} &= \frac{I_\phi c_1 + M_1 c_2}{I_\phi I_\theta - M_1^2} \\ a_{41} &= \frac{M_1 M_2 g}{I_\phi I_\theta - M_1^2} \\ a_{42} &= \frac{I_\theta c_2 + M_1 (c_1 + c_2)}{I_\phi I_\theta - M_1^2} \\ a_{43} &= -\frac{M_3 I_\theta g}{I_\phi I_\theta - M_1^2} \\ a_{44} &= -\frac{I_\theta c_2 + M_1 c_1}{I_\phi I_\theta - M_1^2} \\ b_2 &= \frac{M_1 M_3 - M_2 I_\phi}{I_\phi I_\theta - M_1^2} \\ b_4 &= \frac{M_1 M_2 - M_3 I_\theta}{I_\phi I_\theta - M_1^2} \end{aligned}$$

By the result of the control in Step 2, the state variables $(x_{31},\ x_{32},\ x_{33})$ are in the neighborhood of $(0, 0, 0)$. However, the state variable x_{34} is not in the neighborhood of 0, because the second pendulum is swinging. Therefore, a sliding mode controller keeping robustness is used as stabilizing controller for the two pendulums. The input control u which stabilizes (28) is

$$u = -(S_3 B_3)^{-1} (S_3 A_3 x_3 + R_3 \mathrm{sign}(\sigma_3) + K_3 \sigma_3) \tag{29}$$

$$\sigma_3 = S_3 x_3 \tag{30}$$

where $R_3 > 0, K_3 > 0$ and S_3 is the solution of the following Riccati equation with $\epsilon_3 > 0$

$$\begin{aligned} & P_3 (A_3 + \epsilon_3 I) + (A_3 + \epsilon_3 I)^T P_3 - P_3 B_3 B_3^T P_3 + Q_3 = 0 \\ & S_3 = B_3^T P_3 \end{aligned}$$

VI. NUMERICAL SIMULATION

In order to show the performance of the proposed scheme in this paper, computer simulation of the swing-up control of the serial double inverted pendulum is conducted. The parameters in (1) and (2) are selected to be those of an experimental system of serial double inverted pendulums in our laboratory. These are given in Table II. And the parameters of the control law are given in Table III. Simulation result is shown in Fig. 2. Fig. 2 illustrates the responses of θ, $\dot{\theta}$, ϕ, $\dot{\phi}$, z, u and the number of steps. The initial states of the system are given by ($\theta(0)$, $\phi(0)$, $\dot{\theta}(0)$, $\dot{\phi}(0)$)=$(0, 0.1, 0, 0.01)$. And the times when controllers are switched are

- Step 1 $\longrightarrow$ Step 2
  At the condition of '$\cos\theta > 0.8$'
- Step 2 $\longrightarrow$ Step 3
  At the condition of '$\cos\phi > 0.94$'

The figure shows that, first, only the first pendulum has swung up by (12) of the controller of Step 1, and next, the second pendulum has swung up while the first pendulum is stabilized at the upright position by (26) of the controller of Step 2, finally both pendulums have been stabilized at the upright position by (29) of the controller of Step 3. Especially, at Step 2, stabilizing the first pendulum is not influenced by the input which swings up the second pendulum. And the cart does not move greatly from the center of a rail at each step.

TABLE II
VALUE OF SYSTEM PARAMETERS

m_1	0.18 [kg]	m_2	0.10 [kg]
n_1	0.078 [kg]	n_2	0.05 [kg]
l_1	0.19 [m]	l_2	0.115 [m]
J_{n_1}	2.8×10^{-5} [kgm^2]	J_{n_2}	2.0×10^{-6} [kgm^2]
I_1	0.0089 [kgm^2]	I_2	0.0018 [kgm^2]
c_1	0.0001 [kgm^2/s]	c_2	0.002 [kgm^2/s]
L	0.38 [m]	g	9.8 [m/s^2]

TABLE III
VALUE OF CONTROL PARAMETERS

Step 1	$u_{a1} = 14,\ \ K_1 = [6.5, 6.5]$
Step 2	$u_{a2} = 50,\ \ \mu = \pi/6,$
	$R_2 = 3,\ \ K_2 = 1.0,\ \ \epsilon_2 = 1.0$
Step 3	$R_3 = 3,\ \ K_3 = 10,\ \ \epsilon_3 = 1.0$

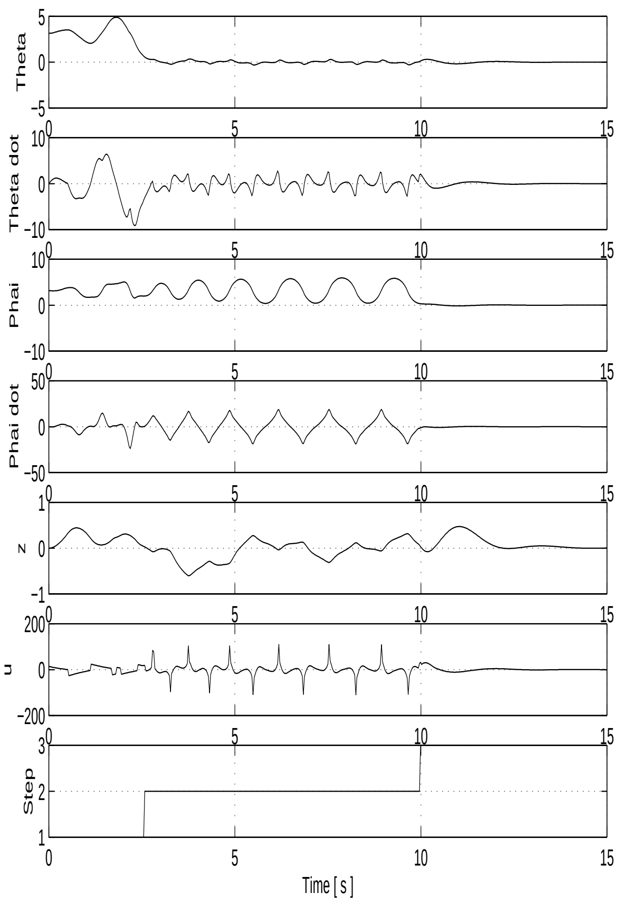

Fig. 2. Time responses of a swing-up control of a serial double inverted pendulum

VII. CONCLUSIONS

In this paper, a control scheme which swings up a serial double inverted pendulum is proposed. This problem is divided into three steps, Step 1 is to swing up the first pendulum, Step 2 is swinging up control of the second pendulum while stabilizing the first pendulum and Step 3 is stabilizing the both pendulums at the upright position, and the control laws for the three steps are proposed. In Step 1, the energy based controller for swing-up the first pendulum is used and added by the position control of a cart to the energy based controller. In Step 2, a controller which combines the controller to swing up the second pendulum by the energy control method and the sliding mode controller to stabilize the first pendulum is proposed. In Step 3, a controller is proposed to stabilize both pendulums by using a sliding mode control method keeping strong robustness. Finally, a numerical simulation is given to show the effectiveness of the proposed scheme.

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