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Nonstationary Robust Control for Time-Varying System

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***Abstract*— This paper presents a synthesis method of nonstationary robust controller for a time-varying system considering various uncertainties. For the uncertainties, a scaled structured uncertainty is regarded as weightings for variations of parameters and a design function in the time domain. Meanwhile, an unstructured uncertainty is counted by a filter enveloping model errors of a control object in the frequency domain. A wire changing its length is adopted as the controlled object and then its vibration control is discussed as a practical problem for solution. The performances of controllers are demonstrated in the time and the frequency domains through the numerical calculations for the case that the object is subjected to disturbance and variation of parameters. The proposed controller designed by considering both uncertainties shows the advantages for the robustness.**

I. INTRODUCTION

We mention the nonstaionary robust control for a time-varying system such as a wire changing its length. For the robust control of time-varying or -invariant system, DGKF paper [1] is known very well as a proposal to design a robust controller for a part of time-invariant system in the time-domain. And also Sanpei et al. [2] proposed to design the output-feedback robust controller for all the time-invariant system based on H_∞ theory. Furthermore, Limebeer et al. [3] established to design a time-varying robust controller for a time-varying system based on the differential game theory, and then they utilize the constant γ which means the index of robust stabilizing performance. However, for the control of time-varying system, its parameters vary in the time domain, that is, it is certainly that the γ is also varying with time. Hence, the γ becomes a function of time, conversely, it is effective for the enhancement of robust stabilizing ability to utilize the $\gamma(t)$ as a design function of nonstationary controller. Moreover, if the $\gamma(t)$ is assigned huge value, the designed controller is H_2 controller [4]. Resultingly, it is also feasible to implement the H_2 and H_∞ switching controller using the $\gamma(t)$.

Meanwhile, for the vibration control of flexible structure, the spillover due to uncertainties is a serious problem on the active control. The uncertainties are categorized into scaled structured and unstructured ones. The scaled structured uncertainty arises from parameter variation and error of controlled system in the time domain. And the unstructured uncertainty is also caused by model errors due to ignored mode of model in the frequency domain. Hence, the vibration control of flexible structure is the good subject matter to verify the robust stabilizing performance. In this paper, we mention the vibration control of wire changing its length which is a time-varying system and a kind of flexible structure. On the control for its vibrations, Takagi and Nishimura [5] in 1998 proposed a gain-scheduled control method based on LMI for a tower crane considering variation of length of crane-rope. And authors [6], [7] investigated for the control of transverse vibrations of wire such as the elevator cable caused by the resonance with sway of a high-rise building.

In this paper, the main objective is to present a synthesis method of nonstationary robust controller considering uncertainties positively. Besides the proposed control method is verified through numerical calculations simulating the practical problem which is the vibration control for the wire changing its length.

II. NONSTATIONARY ROBUST CONTROL METHOD

A. Construction of generalized plant

In this paper, we mention the vibration control for the following time-varying system.

$$\dot{x}_n(t) = A_n(t)x_n(t) + B_n(t)u(t) + D_n(t)z_d(t) \tag{1}$$

where $x_n(t)$ is the state values, $A_n(t)$, $B_n(t)$ and $D_n(t)$ are the time-varying matrices, $u(t)$ the control input, $z_d(t)$ the system disturbance. In this research, the controller having the robustness for uncertainties is proposed. Besides, the control object is a system having flexibility and time-varying parameters. Therefore, the formulation of its model is very important for the reason that it needs the exact model or the model based on complex formulation to obtain the numerical realization of control object completely. Conversely, the numerical model with lower dimension is reasonable for the design of controller. Based on the lower model for it, the model errors due to the ignored high order modes cause the control and observation spillovers. Moreover, the complete realization of parameter identification for the control object is also difficult, especially, a nonparametric identification of time-varying system is complicated. Consequently, we considered that the parameter errors and variations of control object are time-domain uncertainties. From the above arguments, we frame the model errors of ignored high order modes as an unstructured uncertainty and the parameter variation and error as a scaled structured uncertainty. For designing the robust controller, we construct the augmented system including both uncertainties in the equation.

$$\dot{x}_z(t) = \begin{bmatrix} \dot{x}_n(t) \\ \dot{x}_r(t) \end{bmatrix} = \begin{bmatrix} A_n(t)+\Delta A_n(t) & 0 \\ 0 & A_r(t)+\Delta A_r(t) \end{bmatrix} x_z(t) + \begin{bmatrix} B_n(t)+\Delta B_n(t) \\ B_r(t)+\Delta B_r(t) \end{bmatrix} u(t) + \begin{bmatrix} D_n(t) \\ 0 \end{bmatrix} z_d(t) \tag{2}$$

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$$y(t) = \begin{bmatrix} C_n(t)+\Delta C_n(t) & C_r(t)+\Delta C_r(t) \end{bmatrix} x_z(t) + D_r(t)u(t) + V(t)w_o(t) \quad (3)$$

which are based on the time-varying nominal system expressed by matrices $A_n(t)$, $B_n(t)$, $D_n(t)$ and $C_n(t)$, they include the scaled structured uncertainty Δ and the unstructured uncertainty system $x_r(t)$, $A_r(t)$, $B_r(t)$, $C_r(t)$ and $D_r(t)$, $w_o(t)$ and $V(t)$ are the observation noise and its power vector, respectively. The unstructured uncertainty is expressed by a shaped filter such as a high pass filter. In addition, the nominal system is given by the system (1). This augmented system indicates the set of model of control object, which has a fluctuation band in the time domain described by the scaled structured uncertainty and the one in the frequency domain performed by the unstructured uncertainty.

By the previous augmented system and weighting functions, we synthesize a generalized plant to build a robust controller. For the scaled structured uncertainty, the closed loop expression of system substitutes for the open loop one by using virtual disturbance input $w_a(t)$, $w_b(t)$, $w_c(t)$ and weighted performance output $z_a(t)$, $z_b(t)$, $z_c(t)$. And also, the unstructured uncertainty derived from a shaped filter is turned from being closed loop expression into being open loop one with a virtual disturbance input $w_g(t)$ and a weighted performance output $z_g(t)$. Moreover, the state values and the control input are open looped from the system noise $z_d(t)$ and the observation noise $w_o(t)$ to the performance outputs $z_Q(t)$ and $z_R(t)$. As an illustration, the open loop expression of generalized plant is shown in Fig.1 which abbreviates the description of time. In this control problem, the state space description of generalized plant is described by the followings.

$$\dot{x}_z(t) = A_z(t)x_z(t) + B_1(t)w(t) + B_2(t)u(t) \quad (4)$$
$$z_h(t) = C_1(t)x_z(t) + D_{12}(t)u(t) \quad (5)$$
$$y_z(t) = C_2(t)x_z(t) + D_{21}(t)w(t) \quad (6)$$

$$w(t) = \begin{bmatrix} z_d(t) & w_o(t) & w_a(t) & w_b(t) & w_c(t) & w_g(t) \end{bmatrix}^T$$
$$z_h(t) = \begin{bmatrix} z_Q(t) & z_R(t) & z_a(t) & z_b(t) & z_c(t) & z_g(t) \end{bmatrix}^T$$

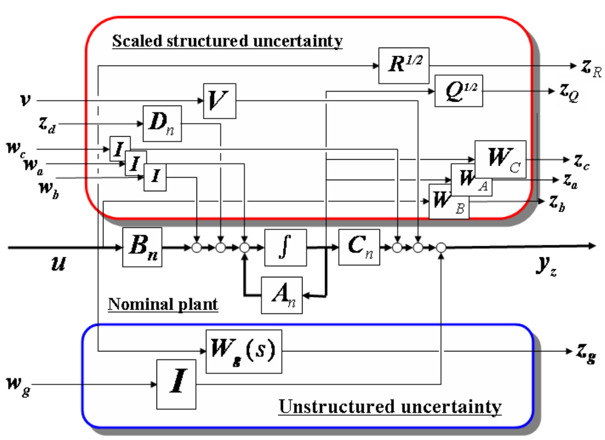


Fig. 1. Schematic diagram of structure of generalized plant

$$A_z(t) = \begin{bmatrix} A_n(t) & 0 \\ 0 & A_r(t) \end{bmatrix}, B_1(t) = \begin{bmatrix} \begin{bmatrix} D_n(t) \\ 0 \end{bmatrix} & 0 & I & I & 0 & 0 \end{bmatrix}$$
$$B_2(t) = \begin{bmatrix} B_n(t) \\ B_r(t) \end{bmatrix}, C_2(t) = \begin{bmatrix} C_n(t) & 0 \end{bmatrix}$$
$$C_1(t) = \begin{bmatrix} Q^{1/2}(t) & 0 \\ 0 & 0 \\ W_A(t) & 0 \\ 0 & 0 \\ W_C(t) & 0 \\ 0 & C_r(t) \end{bmatrix}, D_{12}(t) = \begin{bmatrix} 0 \\ R^{1/2}(t) \\ 0 \\ W_B(t) \\ 0 \\ D_r(t) \end{bmatrix}$$
$$D_{21}(t) = \begin{bmatrix} 0 & V(t) & 0 & 0 & I & I \end{bmatrix}$$

where the $W_A(t)$, $W_B(t)$ and $W_C(t)$ are the weighting matrices of scaled structured uncertainties, $Q(t) = Q^{1/2\,T}(t)Q^{1/2}(t)$ and $R(t) = R^{1/2\,T}(t)R^{1/2}(t)$. Therefore, this paper presents the synthesis method of controller to implement the robust stabilization of the above augmented system considering the scaled structured and unstructured uncertainties.

B. Formulation of nonstationary robust control problem

In the priori derived generalized plant, the H_∞ norm of the closed loop transfer function G_{zw} from the virtual worst disturbance $w(t)$ to the performance output $z_h(t)$ is described by L_2 induced norm in the time domain as shown in the following equation.

$$\min_u \|G_{zw}\|_\infty = \min_u \sup_w \frac{\|z_h(t)\|_2}{\|w(t)\|_2} =: \gamma^* \quad (7)$$

where $\|\ \|_\infty$ gives H$_\infty$ norm, $\|\ \|_2$ gives L_2 induced norm and γ^* is the minimum value of H$_\infty$ norm realized by the optimal controller $K(y,t)^*$. With (4)-(6) and (7) given, the H$_\infty$ optimal control problem is to find a controller $K(y,t)^*$. For the $\gamma > \gamma^*$, the control satisfying

$$J_\infty(u^o, w^o) = \min_u \max_w \int_0^\infty [z_h^T(t)z_h(t) - \gamma^2 w^T(t)w(t)]\ dt \leq 0 \quad (8)$$

is called the quasi optimal control. Here, w^o is the worst disturbance which is the solution of maximization problem

$$J_\infty(u, w^o) = \max_w J_\infty(u, w) \quad (9)$$

and it is described by the equation

$$w^o = \gamma^{-2}B_1(t)X_w(t)x_z(t) \quad (10)$$

where $X_w(t)$ is the positive definite symmetric matrix as a solution of the Riccati differential equation derived from (9). Thereupon, u^o is the optimal control input which is the solution of minimization problem

$$J_\infty(u^o, w) = \min_u J_\infty(u, w) \quad (11)$$

and it is given by the equation

$$u^o = -\tilde{R}^{-1}(t)(B_2^T(t)X_u(t) + \tilde{S}(t))x_z(t) \quad (12)$$

where $X_u(t)$ is the positive definite symmetric matrix as a solution of the Riccati differential equation derived from

(11). In this control problem, it is a main objective that this control converges to the H_∞ optimal control by using iteration of solving a controller $K(y,t)$ with decreasing γ. In other words, it is the goal for this control problem to accomplish $\gamma \rightarrow \gamma^*$ or $J_\infty = 0$. In this research, we adopt

$$\gamma(t) > \gamma^*([\,0,\, t^*\,], 0 < t^* < t_f) \tag{13}$$

for γ, and (8) is transformed into the following problem.

$$\begin{aligned} J(u^o, w^o) &= \min_u \max_w \int_0^{t_f} [z_h^T(t) z_h(t) - \gamma(t)^2 w^T(t) w(t)]\ dt \\ &= \min_u \max_w \int_0^{t_f} [x_z^T(t) \tilde{Q}(t) x_z(t) + 2 x_z^T(t) \tilde{S}(t) u(t) \\ &\quad + u^T(t) \tilde{R}(t) u(t) - \gamma(t)^2 w^T(t) w(t)]\ dt \end{aligned} \tag{14}$$

$$\tilde{Q}(t) = C_1^T(t) C_1(t), \tilde{S}(t) = C_1^T(t) D_{12}(t), \tilde{R}(t) = D_{12}^T(t) D_{12}(t)$$

In this paper, we will solve the problem to obtain a controller $K(y,t)$ which minimizes the criterion function of (14) by using optimal control input u^o with the existence of the worst disturbance w_o for the case that this control problem is finished by finite time.

C. Synthesis of controller

We begin by considering a controller which achieves the robust stabilization for the time-varying generalized plant. The solution of (14) is a saddle point solution satisfying the inequality condition:

$$J(u^o, w) \leq J(u^o, w^o) \leq J(u, w^o) \tag{15}$$

and it is implemented with the following controller in the state space description [3].

$$\dot{\hat{x}}(t) = \hat{A}(t) \hat{x}(t) + \hat{B}(t) y_z(t) \tag{16}$$

and the output

$$u(t) = F(t) \hat{x}(t) \tag{17}$$

is constructed by

$$\begin{aligned} \hat{A}(t) =& A_z(t) + B_2(t) F(t) + \gamma^{-2}(t) B_1(t) B_1^T(t) X(t) \\ & - \hat{B}(t)(C_2(t) + \gamma^{-2}(t) D_{21}(t) B_1^T(t) X(t)) \\ \hat{B}(t) =& (I - \gamma^{-2}(t) Y(t) X(t))^{-1} (Y(t) C_2^T(t) + B_1(t) D_{21}^T(t)) \\ & \times (D_{21}(t) D_{21}^T(t))^{-1} \\ F(t) =& -\tilde{R}(t)^{-1} (B_2^T(t) X(t) + \tilde{S}^T(t)) \end{aligned}$$

where $X(t)$ and $Y(t)$ is given by solving the following Riccati differential equation with an explicit method such as Runge-Kutta method.

$$\begin{aligned} -\dot{X}(t) &= X(t) A_z(t) + A_z^T(t) X(t) + X(t) B_1(t) B_1^T(t) X(t) / \gamma^2(t) \\ & - (X(t) B_2(t) + \tilde{S}(t)) \tilde{R}^{-1}(t) (B_2^T(t) X(t) + \tilde{S}^T(t)) + \tilde{Q}(t) \qquad (18) \\ & X(t_f) = 0 \qquad (19) \end{aligned}$$

$$\begin{aligned} \dot{Y}(t) &= Y(t) A_z^T(t) + A_z(t) Y(t) + Y(t) C_1^T(t) C_1(t) Y(t) / \gamma^2(t) \\ & - (Y(t) C_2^T(t) + B_1(t) D_{21}^T(t)) (D_{21}(t) D_{21}^T(t))^{-1} \\ & \times (C_2(t) Y(t) + D_{21}(t) B_1(t)) + B_1(t) B_1^T(t) \qquad (20) \\ & Y(0) = 0 \qquad (21) \end{aligned}$$

where (19) is the final condition and (21) the initial condition. Therefore, (18) is solved by using Runge-Kutta method in the inverse direction of time from (19) and then (20) is performed in the forward direction of time from (21). Furthermore, we utilize a polynomial function for the expression of nonstationary robust controller changing with time. The time-variation of the controller is smooth, hence, the order of polynomial interpolation is enough $5 \sim 10$ to give a description of the time-varying controller.

D. Design of $\gamma(t)$, the weightings and the shaped filter for uncertainties

For the nonstationary robust control method, the design function $\gamma(t)$ on the worst disturbance is constructed depending on the variation of uncertainty. To perform H_∞ control, the $\gamma(t)$ is designed by using discrete γ-iteration through all the time. If it is assumed that the time-varying parameters of controlled system is continuous on time, the $\gamma(t)$ is based on the combination of discrete minimum $\gamma(t_i)^*$ given by γ-iteration at the arbitrary time t_i. However, the $\gamma(t)$ derived by the iteration can be utilized directly to design a robust controller, because the Riccati differential equation has no stable solution based on it. Resultingly, we use a larger $\gamma(t)$ referring the $\gamma(t_i)^*$ derived from γ-iteration. Secondly, we estimate the quantity of the scaled structured uncertainty considering the parameter variation by the followings.

$$\begin{aligned} & \Delta A_n(t) = A_{max} - A_n(t), \Delta B_n(t) = B_{max} - B_n(t), \\ & \Delta C_n(t) = C_{max} - C_n(t) \end{aligned} \tag{22}$$

In this equation, the subscript 'max' means the matrix including the absolute maximum values through all the time, and all the symbols expresses the matrix. Thus the scaled structured uncertainties are substituted into the following equation.

$$\begin{aligned} & \Delta A_n(t) = I_A \delta_A W_A(t), \Delta B_n(t) = I_B \delta_B W_B(t), \\ & \Delta C_n(t) = I_C \delta_C W_C(t) \end{aligned} \tag{23}$$

where I_i is an identity matrix, $\delta_i I$ a repeated scalar block and W_i the weighting matrix expressing the quantitative scale. Finally, the shaped filter is applied to note the unstructured uncertainty, and it then is designed to envelop the model errors due to ignored high order modes.

III. CONTROL OBJECT

In this study, we consider the following system: a flexible structure contains a wire and mass points as shown in Fig.2 which illustrates a length-varying wire system. This system has time-varying parameters such as length or moving velocity, namely, this is classified into time-varying system. It is assumed in this research that the structure, the wire and mass points have horizontal deflections and then longitudinal vibrations are ignored, because we focus on the proposal of control method and it is easy to understand intuitively the vibration control of transverse deflection of wire caused by the resonance between the structure and the wire. And a control force is added directly into one of mass points to displace at the boundary of wire. First, we construct the equation of motion of wire. A wave equation of length-varying wire is expressed by the following equation.

$$\rho A\left(\frac{\partial}{\partial t}+v(t)\frac{\partial}{\partial s}\right)u(s,t)-\frac{\partial}{\partial s}T(s)\frac{\partial u(s,t)}{\partial s} + c(s)\left(\frac{\partial}{\partial t}+v(t)\frac{\partial}{\partial s}\right)u(s,t)=0 \quad (24)$$

where s is the coordinate along the wire, $u(s,t)$ the distributed parameter of deflection of wire in the transverse direction, t the arbitrary time, ρA the line density of wire, $T(s)$ the tension of wire depending on the up-and-down position, $v(t)$ the velocity of variation of length of wire and $c(s)$ the damping coefficient per unit length of wire. The model of wire is constructed by using Galerkin's method as $u(s,t)=N(s)r(t)$. From the previous wave equation, the motion of equation of an element is derived.

$$\begin{aligned}&\begin{bmatrix}2\alpha(t) & \alpha(t)\\ \alpha(t) & 2\alpha(t)\end{bmatrix}\begin{bmatrix}\ddot{r}_i(t)\\ \ddot{r}_{i+1}(t)\end{bmatrix}\\ &+\begin{bmatrix}2\gamma_r(t)-d_c(t) & \gamma_r(t)+d_c(t)\\ \gamma_r(t)-d_c(t) & 2\gamma_r(t)+d_c(t)\end{bmatrix}\begin{bmatrix}\dot{r}_i(t)\\ \dot{r}_{i+1}(t)\end{bmatrix}\\ &+\begin{bmatrix}\beta(t)-d_k(t) & -\beta(t)+d_k(t)\\ -\beta(t)-d_k(t) & \beta(t)+d_k(t)\end{bmatrix}\begin{bmatrix}r_i(t)\\ r_{i+1}(t)\end{bmatrix}=0 \qquad (25)\\ &\alpha(t)=\rho Al(t)/6, l(t)=(l(0)+v(t)t)/n, d_c(t)=\rho Av(t),\\ &\gamma_r(t)=c(s)l(t)/6, \beta(t)=(T(s)-\rho Av^2(t))/l(t),\\ &d_k(t)=0.5c(s)v(t)\end{aligned}$$

where n is the discrete number and it is assumed that the wire is not interacted directly with an external force. In addition, $d_c(t)$ and $d_k(t)$ are the advection terms depending on the velocity of length variation of wire. For the case that the variation of its velocity is smaller than the other parameter's ones, the advection terms are neglected because the influence of wave propagation has little effect on the vibration of wire. Based on the superposition of equation of motion of an element in n times and the construction of dynamics equation of structure and mass points, we derive a time-varying equation of motion of controlled system as the following.

$$M(t)\ddot{x}_d(t)+C(t)\dot{x}_d(t)+K(t)x_d(t)=F(t) \qquad (26)$$

$$x_d(t)=\begin{bmatrix}x_{b1}(t)\cdots x_{bM}(t) & x_m(t) & r_1(t)\cdots r_{n-1}(t) & x_e(t)\end{bmatrix}^T \qquad (27)$$

where $M(t)$, $C(t)$, $K(t)$ and $F(t)$ are the inertial, damping, stiffness and external force matrices, respectively. x_{bi} is the absolute displacement of each story of structure, M the discrete number of structure, x_m the relative displacement of mass point at one end from the top of structure, x_e the relative displacement of mass point at the other end, r_i the relative displacement of i-th discrete element, and then x_m and x_e coincide with r_0 and r_n , respectively. Besides, we non-dimensionalize the coupled equation of motion based on the normalizing values of length and time. The details of non-dimensional model are found in reference [7]. Finally, the state equation (1) is obtained by the coupled and non-dimensionalized equation.

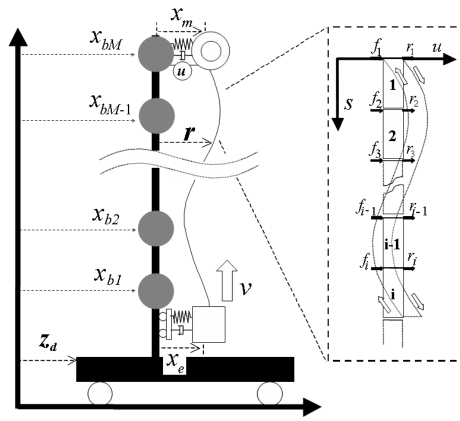


Fig. 2. Schematic diagram of controlled system

For the wire and mass points, their state values are expressed by the relative displacement, velocity and acceleration from the structure, because they are generally measured by using non-contact or built-in displacement sensor fixed on the structure. Thus the disturbances into the wire and mass points are the absolute accelerations of structure, and then they depend on the position of mass points of wire. Therefore, we will express their disturbances by using dynamics of structure under the assumption that the structure is a cantilever in the numerical calculation.

IV. NUMERICAL CALCULATIONS

A. Conditions of numerical calculations

In this chapter, we examine the reduction and the robust stabilization performances of the proposed controllers through the numerical calculations. The control methods for the examination are six as shown in the left side of Table I. Basically, we examine the nonstationary robust controllers designed by various weightings, $\gamma(t)$ and shaped filter. The first is the basic nonstationary robust controller based on the constant γ, the second the one designed by $\gamma(t)$ derived from discrete γ-iteration through all the time, the third one based on the generalized plant including the shaped filter, the fourth one designed by the generalized plant including the weightings describing the scaled structured uncertainty, the sixth the gain-scheduled robust controller (G.S.) based on the generalized plant considering both uncertainties and the sixth one the nonstationary robust controller designed on the generalized plant considering both uncertainties. These examinations are performed for the case that the structure is subjected to the basement disturbance, the value of tension of wire shifts from -50% to +200% in the time domain while the wire descends. For the synthesis of controller, it is supposed that the discrete number of wire is 10, the number of mass points of structure model is 5 and the verification model of wire has the discrete number n equal to 40. In addition, we adopt the signal shown in Figs.3 and 4 as the disturbance into the controlled system.

TABLE I

THE NOMINAL CONTROL RESULTS (R.R.:REDUCTION RATE)

#	shape of γ	Max u	R.M.S. u	R.R. of Max x	R.R. of R.M.S. x
1	Const.	31.5	13.0	0.524	0.580
2	γ-it.	28.6	12.5	0.518	0.566
3	γ-it.+filter	24.9	11.5	0.585	0.419
4	γ-it.+delta	23.9	9.3	0.539	0.498
5	G.S.+γ-it.+delta+filter	32.5	11.0	0.538	0.548
6	γ-it.+delta+filter	29.2	10.7	0.517	0.525

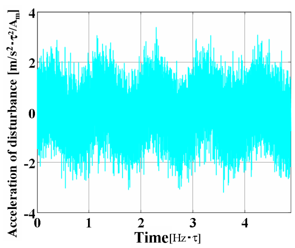


Fig. 3. Time history of acceleration of disturbance

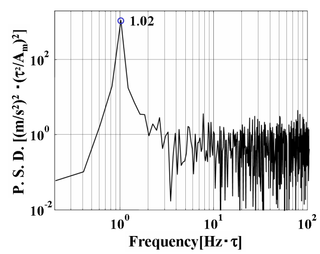


Fig. 4. Power spectrum density of acceleration of disturbance

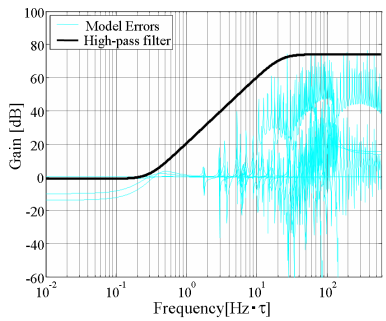


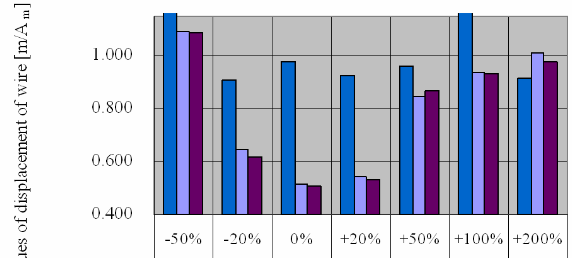
Fig. 5. The frequency characteristics of model errors from u to y and shaped filter enveloping them

The second order high-pass filter is applied to note the unstructured uncertainty, and it is then designed to envelop all the model errors in the frequency domain as shown in Fig.5. The model errors come of the ignored high order modes of controlled object, besides, they are expressed by the differences in the frequency responses under control and non-control which are the responses from the control input u to all the observed outputs y. In addition, since the incrementation of order of controller by accompanying the high-pass filter is only two and it is very small for the entire order of controller, the controller performs the practical use without problems.

Finally, for the gain-scheduled robust controller, its synthesis is established by the discrete γ-iteration and the solutions of the Riccati differential equations (18) and (20) based on the controlled system which has fixed parameters at the arbitrary time. In this examination, the eighteen gain-scheduled robust controllers are prepared at regular intervals during the control period, hence, the other controllers are given by their smooth connection using spline interpolation.

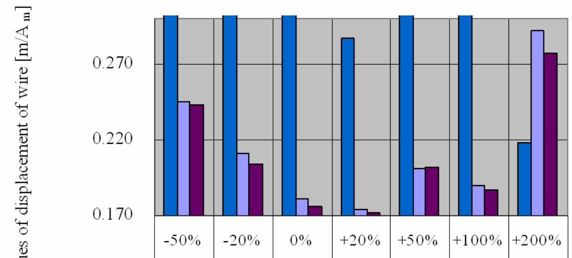
B. Consideration about numerical calculations

Table I indicates the nominal performances and the control inputs by all the controllers which are made almost equal quantitatively. For the examination in the time domain, Figs.6 and 7 show the maximum and root mean



	-50%	-20%	0%	+20%	+50%	+100%	+200%
Noncontrol	1.284	0.910	0.980	0.925	0.959	1.301	0.916
No.1	1.092	0.647	0.514	0.541	0.845	0.936	1.013
No.2	1.088	0.620	0.508	0.533	0.868	0.931	0.977

Fig. 6. The maximum values of displacement of wire with controllers No.1 and 2

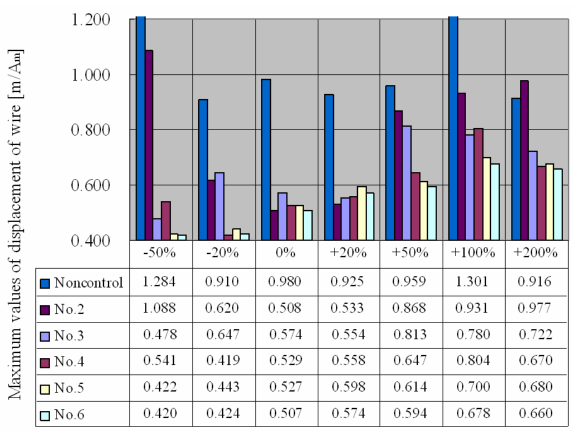


	-50%	-20%	0%	+20%	+50%	+100%	+200%
Noncontrol	0.336	0.315	0.311	0.287	0.306	0.335	0.218
No.1	0.245	0.211	0.181	0.174	0.201	0.190	0.293
No.2	0.243	0.204	0.176	0.172	0.202	0.187	0.277

Fig. 7. The root mean square values of displacement of wire with controllers No.1 and 2

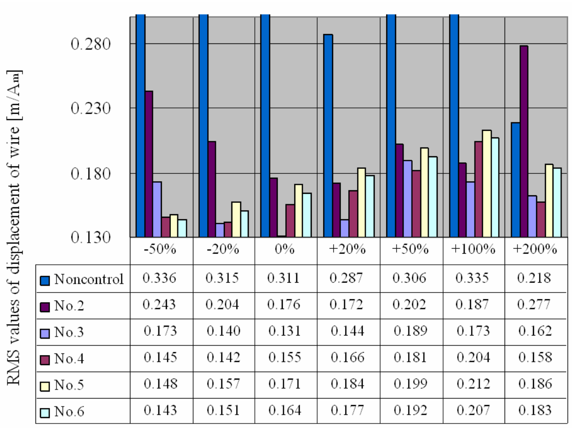
square (R.M.S.) values of displacement of wire through the entire time for the controllers No.1 and 2. From these results in the time domain, the controller No.2 gets the good performance when the tension of wire varies. As a result, it is clarified that the variation of $\gamma(t)$ influences the robust stabilizing performances of controller, and then the optimal $\gamma(t)$ exists. Here the γ of No.1 is 90.0 and the minimum $\gamma(t)$ of No.2 is 41.8. The reason why the γ of No.1 is larger than No.2 is that the Riccati differential equation of No.1 is not able to be solved by using the minimum value of $\gamma(t)$ of No.2. This result to utilize the large value for γ implies that it is possible to implement the more robust control based on the optimal $\gamma(t)$ derived by the other method than the discrete γ-iteration in this paper.

Additionally, Figs.8 and 9 illustrate the same results of the controllers No.2 to 6 as Figs.6 and 7. The controllers No.3 to 6 bring out better robust stabilizing performances than previous ones. Although No.6 controller obtains the best regulation of vibrations in the maximum value, No.3 gets the best one in the R.M.S. value. To reduce the maximum values is more important than to reduce the R.M.S. ones because the wire gets the fatal damage during the bouts causing its maximum displacement. Meanwhile, to suppress the rms value is only related to extend the life span of wire. Moreover, No.5 gets the same performance as No.6, however, the more control input is required to get it. Finally, the results in the time domain imply to select the synthesis method of controller according to desired performance and condition of uncertainty.



	-50%	-20%	0%	+20%	+50%	+100%	+200%
Noncontrol	1.284	0.910	0.980	0.925	0.959	1.301	0.916
No.2	1.088	0.620	0.508	0.533	0.868	0.931	0.977
No.3	0.478	0.647	0.574	0.554	0.813	0.780	0.722
No.4	0.541	0.419	0.529	0.558	0.647	0.804	0.670
No.5	0.422	0.443	0.527	0.598	0.614	0.700	0.680
No.6	0.420	0.424	0.507	0.574	0.594	0.678	0.660

Fig. 8. The maximum values of displacement of wire with controllers No.2 to 6



	-50%	-20%	0%	+20%	+50%	+100%	+200%
Noncontrol	0.336	0.315	0.311	0.287	0.306	0.335	0.218
No.2	0.243	0.204	0.176	0.172	0.202	0.187	0.277
No.3	0.173	0.140	0.131	0.144	0.189	0.173	0.162
No.4	0.145	0.142	0.155	0.166	0.181	0.204	0.158
No.5	0.148	0.157	0.171	0.184	0.199	0.212	0.186
No.6	0.143	0.151	0.164	0.177	0.192	0.207	0.183

Fig. 9. The root mean square values of displacement of wire with controllers No.2 to 6

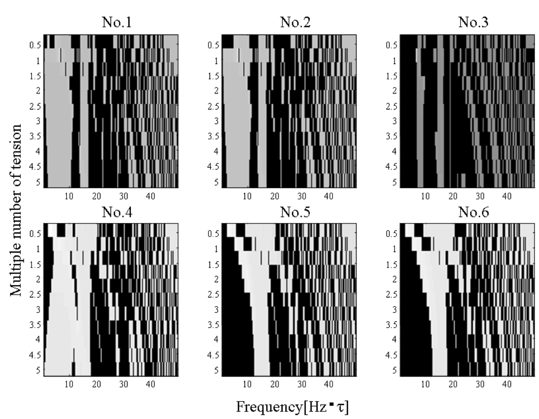


Fig. 10. The digitalized differences of frequency response between non-control and control at $t = t_f/2$ in the case that the tension varies from 0.4 to 5 times (Black $\sim$ white : bad $\sim$ good)

Furthermore, Figs. 10 and 11 illustrate the results in the frequency domain. As an example of results in the frequency domain, Fig.10 shows the digitalized values for differences of frequency responses under control and non-control which are the responses from the disturbance to the center displacement of wire at $t = t_f/2$ in the case that the tension varies from 0.4 to 5 times. The result from white to gray in the figure expresses the suppression performance for the vibrations of wire, that is, the amplitude of frequency response is less than non-control one. Thus the black one illustrates completely bad result. Besides, Fig.11 implies

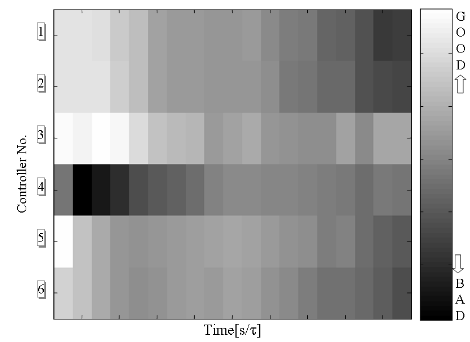


Fig. 11. The time histories of summation of digitalized values in the reduced frequency response within the high frequency range

the time histories of summation of digitalized values in the reduced frequency responses which are limited within the high frequency range more than 1 Hz. Although No.3 controller shows the best suppression performance shown in the result of Fig.11 through all the time, from the result shown in Fig.10 its quantitative reduction is very poor on the ground that it is designed to get conservative due to the filter. Meanwhile, No.4 controller derives the satisfied quantitative suppression of vibrations without the high frequency range. Consequently, No.5 and 6 controllers show the good regulation for vibrations of wire with avoiding the worst case, that is, reducing the maximum displacement of wire. However, since these results in the frequency domain are evaluated discretely in the time domain, the gain-scheduled controller indicates the similar performance to the nonstationary one.

V. CONCLUSIONS

This study presented a synthesis method of nonstationary robust vibration controller considering uncertainties. The proposed method permits the design function $\gamma(t)$ to be designed on the worst disturbance and the time-variation of parameter. The uncertainties were categorized by the scaled structured and unstructured ones in the time and frequency domains, and then the controllers considering both uncertainties obtained the advantages for the vibration control of wire changing its length for the case that the controlled system is subjected to the disturbance and the parameter variation in comparison with the gain-scheduled and other controllers. In future study, we implement a robust stabilizing control based on the optimized value of $\gamma(t)$ and the quantitative evaluation of uncertainties.

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