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Quadratic stabilization of a switched affine system about a nonequilibrium point

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Abstract— **This paper deals with the problem of quadratic stabilization of switched affine systems, where the state of the switched system has to be driven to a point (the "switched equilibrium") which is not in the set of subsystems equilibria. Quadratic stability of the switched equilibrium is assessed using a continuous Lyapunov function, having piecewise continuous derivative. A necessary and sufficient condition is given for the case of two subsystems and a sufficient condition is given in the general case. Two switching rules are presented: a state feedback, in which sliding modes may occur, and an hybrid feedback, in which sliding modes can be avoided. Two examples illustrate our results.**

Keywords: **switched affine system; quadratic stabilization; Lyapunov function; switched equilibrium.**

I. INTRODUCTION

Switched systems are a class of hybrid systems consisting of a family of continuous-time subsystems and a criterion that rules the switching among them. The switching signal may just depend on time or state, or may be generated by more complex techniques. In the last years, several results for Lyapunov stability and stabilization of switched and hybrid continuous-time systems have been proposed in the literature [1], [2], [3], [4], [5].

One of the problems is the stabilization of a switched system using a single Lyapunov function whose derivative is negative and bounded by a quadratic function within the activation regions of each subsystem, and may eventually be positive outside these regions. In the field of switched systems, this stability property is usually defined as *quadratic stability* [6], [1].

The approach proposed by Wicks et al. [7] points out that the existence of a stable convex combination of the linear subsystems A_i implies the possibility of quadratic stabilization of the origin through a suitable switching rule. Feron [8] demonstrated that this condition is not only sufficient but also necessary for quadratic stability in the case of two subsystems. In order to avoid the occurrence of fast switching, a hybrid switching strategy has been proposed (see [9], [6]).

Another approach is given in the nonlinear case by Pettersson and Lennartson [10]. Sufficient conditions guaranteeing quadratic stability can be formulated as a nonlinear optimization problem, where the existence of a stable convex combination of dynamical matrices is required.

The problem of quadratic stabilization of a plant with a set of given state-feedback controllers has also been considered by Skafidas et al. [11]. A sufficient condition for quadratic stabilization of a switched control system is the existence of a convex combination of the controllers which stabilizes the plant. Results on quadratic stabilization have also been extended to the case of uncertain linear systems.

In [12], Xu et Antsaklis used a geometric approach to give necessary and sufficient conditions for the stabilization of second-order switched systems.

However, all the above papers consider the case of autonomous switched systems, in which the state of the switched system must be driven to a point which is an equilibrium point for all the component subsystems (or at least for some of them).

The aim of this paper is the extension of some of the previous results (mainly from [7], [8]) to the case of switched affine systems, where the state of the switched system must be driven to a point which is not in the set of subsystems equilibria. This point is denoted *switched equilibrium*, since switching is used to construct a new equilibrium which is not present in the original subsystems. It should be noted that there is not a trivial solution to the problem, even if all subsystems are individually stable. Switching among subsystems is thus a necessary condition to reach and stay in the neighborhood of the switched equilibrium.

The proposed methodology offers a useful tool for stabilization of a system with piecewise constant inputs. This situation is typical of many practical applications (e.g. temperature control), where technological or economical constraints dictate the use of actuators which are capable of generating a discrete set of controls rather than a continuous range of controls. Moreover, this approach can be used conversely to design a set of input values which allow quadratic stabilization of the origin for a system with piecewise constant inputs and with one or more operating modes (one or more dynamical matrices A_i).

The outline of the paper is as follows. The switched equilibrium problem is formulated in the next section. Conditions guaranteeing quadratic stabilization are given in Section III. Two different switching rules are discussed in Section IV, namely a state feedback and a hybrid feedback. Section V presents two illustrative examples.

II. SWITCHED EQUILIBRIUM PROBLEM

In this paper we consider the following switched affine system

$$\dot{x} = A_\sigma x + b_\sigma, \quad (1)$$

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where $x \in \mathbb{R}^n$ and $\sigma(t) : [0,\infty) \rightarrow \mathcal{P} = \{1,\ldots,m\}$ is a piecewise constant function of time called *switching signal*. We say that the subsystem Σ_i is active at time t when $\sigma(t) = i$. Assuming that all A_i are nonsingular, each subsystem Σ_i has a (stable or unstable) equilibrium point $\bar{x}_i = -A_i^{-1}b_i$. For such a system we want to find a switching signal such that a given point $\bar{x}$ is a stable equilibrium.

With a little abuse of terminology, we will call the point $\bar{x}$ a *switched equilibrium*. In fact, discarding the trivial case where $\bar{x} = \bar{x}_i$ for some i, the regulation can be achieved only through switching, even if all the subsystems are asymptotically stable.

Without loss of generality, we consider the case where the switched equilibrium is the origin $\bar{x} = 0$, because any prescribed equilibrium point can be shifted to the origin via a change of variable $\tilde{x} = x - \bar{x}$.

Quadratic stability for a switched affine system can be defined in terms of a Lyapunov function as follows.

Definition 1: The switched equilibrium $\bar{x} = 0$ is quadratically stable if and only if there exist a matrix $P = P^T > 0$ and a constant $\varepsilon > 0$ such that for the quadratic function $V(x) = x^T P x$ we have

$$\dot{V}(x(t)) \leq -\varepsilon x^T x,$$

along all system trajectories.

Remark 1: Discontinuities in the right-hand side of (1), which imply discontinuities in the derivative of the Lyapunov function can be overcome by the introduction of Filippov solutions [13], [11]. Therefore, the derivative $\dot{V}(x(t))$ is defined as follows

- when the system Σ_i is active

$$\dot{V}(x(t)) = \dot{V}_i(x(t)) = x^T(A_i^T P + PA_i)x + 2b_i^T Px,$$

- at the switching point between system Σ_i and system Σ_j

$$\dot{V}(x(t)) = \sup_{\gamma\in[0,1]} \left\{\gamma \dot{V}_i(x(t)) + (1-\gamma)\dot{V}_j(x(t))\right\},$$

The key concept in the paper is the notion of *average system*, a convex combination of the subsystems, defined as follows

$$\Sigma_{eq} : \dot{x} = A_{eq}x + b_{eq}, \tag{2}$$

where

$$A_{eq} = \textstyle\sum_{i=1}^m \alpha_i A_i \quad b_{eq} = \sum_{i=1}^m \alpha_i b_i$$

with $0 < \alpha_i < 1$ and $\sum_{i=1}^m \alpha_i = 1$.

The average system (2) may be approximatively implemented by sequential switching among subsystems with the dwell time of Σ_i proportional to α_i [7]. In order to achieve a good approximation, switching must be fast enough (the largest dwell time must be at least one order of magnitude faster than the smallest time constant of all the subsystems). Clearly asymptotic stability of the origin requires $b_{eq} = 0$ and A_{eq} to be a stability matrix.

Such a time average control involves rapid switching, which may lead to problems always associated to chattering. Moreover, quadratic stability of the switched equilibrium can be achieved only with infinitely fast switching. This suggests searching for a modification of this methodology, where rapid switching can be avoided except when the state approaches the switched equilibrium.

III. QUADRATIC STABILIZATION OF A SWITCHED EQUILIBRIUM

The objective of this section is to construct a continuous Lyapunov function whose derivative along any state trajectory is negative for each subsystem Σ_i within some regions in the state-space. Moreover, these regions must cover the entire state-space. A necessary and sufficient condition for quadratic stabilization in the case of two subsystems and a sufficient condition in the general case will be presented.

Theorem 1: Given the switched affine system (1) with $m = 2$, the point $\bar{x} = 0$ is a quadratic stabilizable switched equilibrium if and only if there exists $\alpha \in (0,1)$ such that

$$A_{eq} = \alpha A_1 + (1-\alpha)A_2 \text{ is Hurwitz} \tag{3}$$

$$b_{eq} = \alpha b_1 + (1-\alpha)b_2 = 0 \tag{4}$$

Proof: (Sufficiency) If the convex combination A_{eq} is stable, there exist two positive definite symmetric matrices P and Q such that

$$A_{eq}^T P + PA_{eq} = -Q. \tag{5}$$

Using (3), we can rewrite (5) as

$$\alpha x^T(A_1^T P + PA_1)x + (1-\alpha)x^T(A_2^T P + PA_2)x = -x^T Qx,$$

and adding the null term $2(\alpha b_1 + (1-\alpha)b_2)^T Px = 0$ (recall (4)), we obtain

$$\alpha\left((x^T(A_1^T P + PA_1)x + 2b_1^T Px\right) + \\ + (1-\alpha)\left(x^T(A_2^T P + PA_2)x + 2b_2^T Px\right) = -x^T Qx. \tag{6}$$

Let λ_{min} be the smallest (positive real) eigenvalue of Q. Given $0 < \varepsilon \leq \lambda_{min}$, we have $-x^T Qx \leq -\varepsilon x^T x$, so that (6) can be rewritten as

$$\alpha\left((x^T(A_1^T P + PA_1)x + 2b_1^T Px\right) + \\ + (1-\alpha)\left(x^T(A_2^T P + PA_2)x + 2b_2^T Px\right) \leq -\varepsilon x^T x, \tag{7}$$

or equivalently

$$\alpha\left((x^T(A_1^T P + PA_1)x + 2b_1^T Px + \varepsilon x^T x\right) + \\ + (1-\alpha)\left(x^T(A_2^T P + PA_2)x + 2b_2^T Px + \varepsilon x^T x\right) \leq 0. \tag{8}$$

This means that for every nonzero x we have that either $x^T(A_1^T P + PA_1)x + 2b_1^T Px + \varepsilon x^T x \leq 0$ or $x^T(A_2^T P + PA_2)x + 2b_2^T Px + \varepsilon x^T x \leq 0$, or equivalently we have that either $x^T(A_1^T P + PA_1)x + 2b_1^T Px \leq -\varepsilon x^T x$ or $x^T(A_2^T P + PA_2)x + 2b_2^T Px \leq -\varepsilon x^T x$. Now, define the two regions

$$\Omega_i = \left\{x \mid x^T(A_i^T P + PA_i)x + 2b_i^T Px \leq -\varepsilon x^T x\right\}, i \in \{1,2\}. \tag{9}$$

These are two closed regions which overlap and cover $\mathbb{R}^n \setminus \{0\}$. It is easy to show that any strategy where the system

Σ_i is active in region Ω_i assures quadratic stability, using the Lyapunov function $V(x) = x^T Px$ (with P given by (5)). In fact, within the region Ω_i

$$\dot{V}(x(t)) = \dot{V}_i(x(t)) = x^T(A_i^T P + PA_i)x + 2b_i^T Px \le -\varepsilon x^T x,$$

while at the switching points (which are interior to the region $\Omega_1 \cap \Omega_2$)

$$\dot{V}(x(t)) = \sup_{\gamma\in[0,1]} \left\{\gamma \dot{V}_1(x(t)) + (1-\gamma)\dot{V}_2(x(t))\right\} \le \\ \le \max_{i=1,2}\left\{\dot{V}_i(x(t))\right\} \le -\varepsilon x^T x.$$

Then, there exist $\varepsilon > 0$ and $P = P^T > 0$ satisfying Definition 1.

(Necessity) If the switched equilibrium is quadratically stable, for every $x \neq 0$ one of the following conditions must be satisfied $x^T(A_1^T P + PA_1)x + 2b_1^T Px \le -\varepsilon x^T x$ or $x^T(A_2^T P + PA_2)x + 2b_2^T Px \le -\varepsilon x^T x$, or stated otherwise it is necessary that

$$x^T(-A_1^T P - PA_1 - \varepsilon I)x - 2b_1^T Px \ge 0 \\ \text{when } x^T(A_2^T P + PA_2 + \varepsilon I)x + 2b_2^T Px \ge 0 \quad (10)$$

and

$$x^T(-A_2^T P - PA_2 - \varepsilon I)x - 2b_2^T Px \ge 0 \\ \text{when } x^T(A_1^T P + PA_1 + \varepsilon I)x + 2b_1^T Px \ge 0 \quad (11)$$

We do not consider the case in which $b_i = 0$ and A_i is stable for some i. In this case the condition is trivially true because one of the two inequalities is always satisfied (system Σ_i is quadratically stable and has an equilibrium in the origin).

By applying the S-procedure [14] to one of the previous conditions (e.g. to (10)), we conclude that for some $\beta \ge 0$ the following relation holds

$$x^T(-A_1^T P - PA_1 - \varepsilon I)x - 2b_1^T Px + \\ -\beta(x^T(A_2^T P + PA_2 + \varepsilon I)x + 2b_2^T Px) \ge 0, \quad (12)$$

or equivalently

$$x^T\left((A_1 + \beta A_2)^T P + P(A_1 + \beta A_2)\right)x + \\ + 2(b_1 + \beta b_2)Px \le -\varepsilon(1+\beta)x^T x. \quad (13)$$

We can rewrite (13) in terms of a convex combination of A_i and b_i as follows

$$x^T\left(\frac{(A_1 + \beta A_2)^T}{1+\beta}P + P\frac{(A_1 + \beta A_2)}{1+\beta}\right)x + \\ + 2\frac{(b_1 + \beta b_2)}{1+\beta}Px \le -\varepsilon x^T x, \quad (14)$$

This means that $\bar{x} = 0$ must be an asymptotically stable equilibrium for the average system $\dot{x} = (\alpha A_1 + (1-\alpha)A_2)x + (\alpha b_1 + (1-\alpha)b_2)$, where $\alpha = \frac{1}{1+\beta}$. Thus the condition is necessary, too. □

When there are more than two subsystems, it is possible to search for a pair of subsystems satisfying (3) and (4). Moreover, Theorem 1 can be generalized to the case of m subsystems as a sufficient condition only.

Theorem 2: Given the switched affine system (1), if there exist $\alpha_i \in (0,1),\ i = 1,\ldots,m$ such that

$$\sum_{i=1}^m \alpha_i = 1 \quad (15)$$

$$A_{eq} = \sum_{i=1}^m \alpha_i A_i \text{ is Hurwitz} \quad (16)$$

$$b_{eq} = \sum_{i=1}^m \alpha_i b_i = 0 \quad (17)$$

then the point $\bar{x} = 0$ is a quadratic stabilizable switched equilibrium.

Proof: The proof immediately follows from the proof of sufficiency for Theorem 1 with minor modifications. □

Remark 2: For the class of switched system considered it is impossible to achieve uniform quadratic stability, that is quadratic stability under arbitrary switching signals. In fact, it requires that for any $x \neq 0$

$$\dot{V}_i(x) = x^T(A_i^T P + PA_i)x + 2b_i^T Px \le -\varepsilon x^T x, \qquad \forall i \in \mathcal{P}$$

However, in the subsystem equilibrium $\bar{x}_i = -A_i^{-1}b_i$ we have

$$\dot{V}_i(\bar{x}_i) = 2\bar{x}_i^T A_i^T P\bar{x}_i + 2b_i^T P\bar{x}_i = \\ = -2b_i^T A_i^{-T} A_i^T P\bar{x}_i + 2b_i^T P\bar{x}_i = 0$$

Remark 3: Conditions (4) and (17) may appear as very strong constraints on the system structure. However, it should be remembered that a switched affine system is often used to model a linear system with a quantized input variable, namely a system $\dot{x} = Ax + Bu$ where

$$u(t) : [0,\infty) \to \{\bar{u}_1,\ldots,\bar{u}_m\}.$$

In this case, the switched system is defined by

$$A_i = A \quad b_i = B\bar{u}_i$$

and, provided that the input values can be chosen arbitrarily, condition (17) can be always met with by using no more than 2 input values. Moreover, condition (16) reduces to the stability of A.

IV. SWITCHING RULES

Since the activation regions Ω_i partially overlap, different switching rules can be adopted. In this paper we focus on two simple strategies which take the form of a state feedback and a hybrid feedback, respectively. These strategies mainly differ since sliding modes may occur in the first strategy, while they are avoided in the second one.

A. State Feedback Switching Rule

In the first switching rule, the subsystem with the highest rate of decrease of $V(x)$ is activated

$$\sigma(x) = \arg\min_{i\in\mathcal{P}}\{\dot{V}_i(x)\}. \quad (18)$$

Therefore, this rule takes the form of a state-feedback, where the *activation region* of the i-th system is defined as

$$\Psi_i = \{x \mid \dot{V}_i(x) < \dot{V}_j(x), \forall j \neq i\}, \quad (19)$$

The state feedback strategy assures the highest decrease rate $\dot{V}(x(t))$. Unfortunately this rule does not avoid the occurrence of sliding modes, which may appear in some region of the state space, even when the state is far away from the switched equilibrium.

B. Hybrid Feedback Switching Rule

The second strategy is a hybrid feedback, which avoids the occurrence of sliding modes introducing some hysteresis. The basic idea is to give a lower bound on the decay rate $\dot{V}_{min}(x(t)) = -\varepsilon x^T x$ with $\varepsilon > 0$ and to switch off the active system only when it no more satisfies the required constraint.

The strategy can be summarized as follows in the general case of m subsystems

1) (*initialization*) at time $t = 0$ activate the system Σ_{i_0} with
$$i_0 = \arg\min_{i \in \mathcal{P}} \{(\dot{V}_i(x_0))\};$$
2) (*switching off rule*) if system Σ_i is active and $x^T(A_i^T P + PA_i)x + 2b_i^T Px > -\varepsilon x^T x$ switch to system Σ_j with
$$j = \arg\min_{j \in \mathcal{P}} \{(\dot{V}_j(x))\}.$$
3) (*equilibrium neighbourhood rule*) if $\|x\| \le \rho_{off}$ stop switching until $\|x\| \ge \rho_{on}$ (with $\rho_{off} < \rho_{on}$)

We define the *admissible region of activation* (where system Σ_i *can* be active) as follows
$$\Phi_i = \{x \mid \dot{V}_i(x) + \varepsilon x^T x < 0\}. \quad (20)$$

This strategy assures that the interval between two consecutive switching is always bounded away from zero.

V. EXAMPLES

The following examples illustrate some of the ideas presented in this paper.

Example 1: We first consider the problem of constructing a quadratically stable switched equilibrium in the origin by switching between two individually stable affine systems.

$$A_1 = \begin{bmatrix} 4 & 5 \\ -7 & -7 \end{bmatrix} \qquad b_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -4 & -1 \\ 1 & -2 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

The eigenvalues of A_1 are $\{-1.50 \pm 2.18i\}$ and the eigenvalues of A_2 are $\{-3, -3\}$. It should be noted that even if both systems are individually stable there is not a trivial solution to the problem of switched equilibrium stabilization, since none of the two systems has an equilibrium in the origin. Condition (4) is satisfied for $\alpha = 0.8$ and the corresponding convex combination (3) is stable (eigenvalues in $\{-1.80 \pm 1.70i\}$). Selecting $Q = I_2$ and applying the state feedback switching rule, the switching surface is given by
$$\psi_{12} = 1.58x_1^2 + 2.94x_1x_2 - 0.247x_2^2 - 16.2x_1 - 11.9x_2 = 0$$
with $\Psi_1 = \{x \mid \psi_{12} < 0\}$ and $\Psi_2 = \{x \mid \psi_{12} > 0\}$.

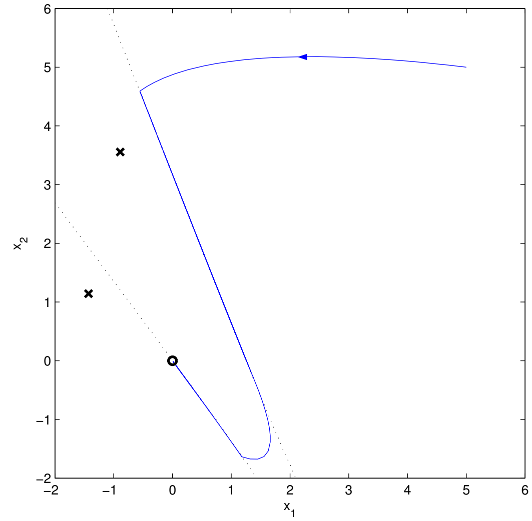

Fig. 1. Trajectory simulation with the state feedback switching rule (solid), boundaries of activation region Ψ_i (dotted), subsystems equilibria (crosses) and switched equilibrium (circle) for Example 1.

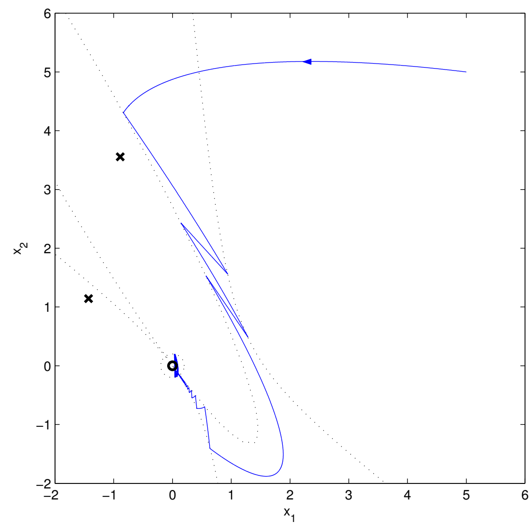

Fig. 2. Trajectory simulation with the hybrid feedback switching rule (solid), boundaries of admissible region Φ_i (dotted), subsystems equilibria (crosses) and switched equilibrium (circle) for Example 1.

The boundaries of the activation regions and a sample trajectory are reported in Fig. 1. It should be noted that sliding motions occur along two portions of the switching surface.

With the hybrid feedback switching rule, by setting $\varepsilon = 0.5$, $\rho_{off} = 0.1$ and $\rho_{on} = 0.2$ we obtain the following admissible regions of activation
$$\Phi_1 : 2.08x_1^2 + 2.94x_1x_2 + 0.253x_2^2 - 3.25x_1 - 2.39x_2 \le 0$$
$$\Phi_2 : -10.8x_1^2 - 11.8x_1x_2 - 3.51x_2^2 + 13.0x_1 + 9.56x_2 \le 0$$

The result of the simulation of the system with this switching rule and with the same initial state is depicted in Fig. 2, together with the commutation boundaries and the ball inside which switching is stopped.

Example 2: The second example consider four individually unstable third order subsystems, which share a stable convex combination.

$$A_1 = \begin{bmatrix} 4.15 & -1.06 & -6.70 \\ 5.74 & 4.78 & -4.68 \\ 26.38 & -6.38 & -8.29 \end{bmatrix} \qquad b_1 = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -3.20 & -7.60 & -2.00 \\ 0.90 & 1.20 & -1.00 \\ 1.00 & 6.00 & 5.00 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 5.75 & -16.48 & 2.41 \\ 9.51 & -9.49 & 19.55 \\ 16.19 & 4.64 & 14.05 \end{bmatrix} \qquad b_3 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -12.38 & 18.42 & 0.54 \\ -11.90 & 3.24 & -16.32 \\ -26.50 & -8.64 & -16.60 \end{bmatrix} \qquad b_4 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

The eigenvalues of matrices A_i are respectively $\{2.67 \pm 5.31i,\ 5.30\}$, $\{-1,\ 2 \pm i\}$, $\{12.27 \pm 10.88i,\ -14.23\}$ and $\{-18.44 \pm 16.57i,\ 11.13\}$. Condition (17) is satisfied for

$$\alpha_1 = 0.15 \quad \alpha_2 = 0.2 \quad \alpha_3 = 0.3 \quad \alpha_4 = 0.35$$

and the corresponding convex combination (16) is Hurwitz (eigenvalues in $\{-2.59,\ -0.96,\ -0.17\}$). The regions Ψ_i and Φ_i can be computed as in the previous example.

The results of the simulation of the system with the two switching rules are depicted in Fig. 3. The effectiveness of the hybrid feedback in reducing the occurrence of fast switching with respect to state feedback can be appreciated in Fig. 4. In fact, state feedback involves sliding motions on the switching surfaces, regardless of the proximity to the switched equilibrium. On the contrary, hybrid feedback assures a comparably slow switching when the state is far enough from the switched equilibrium (e.g. interval $(0 \div 0.4)$). When the state approaches the switched equilibrium, the commutations become faster (e.g. interval $(0.4 \div 0.9)$), since switching surfaces are closer to each other. However, thanks to rule 3 in the neighbourhood of the equilibrium (e.g. interval $(0.9 \div 1)$), switching time is always bounded away from zero.

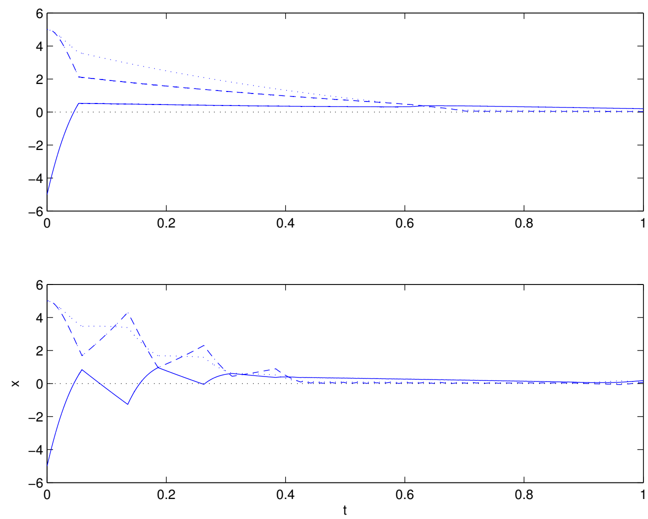

Fig. 3. Trajectory simulation for Example 2, with the state feedback switching rule (above) and with the hybrid feedback switching rule (below).

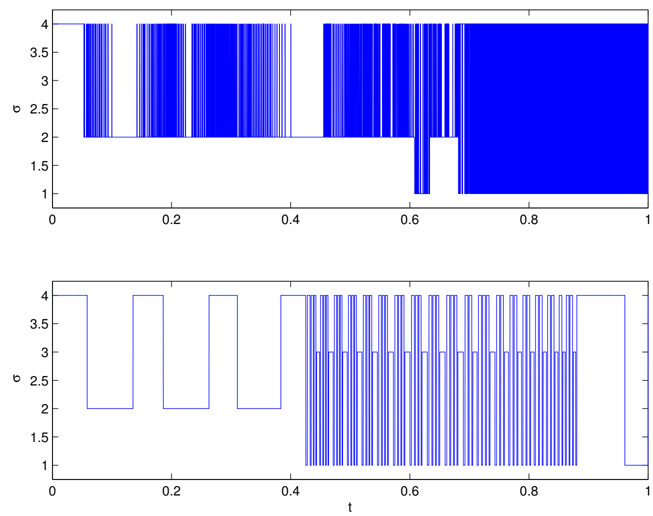

Fig. 4. Switching signals for Example 2, with the state feedback switching rule (above) and with the hybrid feedback switching rule (below).

VI. CONCLUSIONS

This paper studies quadratic stabilization of a switched affine system about a nonequilibrium point. Switching is thus used to construct a stable equilibrium which is not in the set of subsystems equilibria. Some conditions for quadratic stability of the origin are presented, together with a switching rule which explicitly avoid fast switching. An area of further work involves the application of this method to the case of systems with piecewise constant inputs, where constant terms can be arbitrarily designed.

VII. ACKNOWLEDGMENTS

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