Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Control Design of Switched LPV Systems Using Multiple Parameter-Dependent Lyapunov Functions

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***Abstract*— For a linear parameter-varying (LPV) plant with a large parameter variation region, it is often conservative to design a single LPV controller over the entire parameter space. This paper studies the control design of switched LPV systems using multiple parameter-dependent Lyapunov functions to improve performance and enhance design flexibility. Two autonomous switching logics, hysteresis switching and switching with average dwell time, are discussed. The control synthesis conditions for both switching logics are formulated, which are generally non-convex but can be convexified under certain conditions. The proposed switched LPV control schemes are applied to a magnetic bearing problem to demonstrate its advantages over existing LPV control approach.**

I. Introduction

Linear parameter-varying (LPV) control theory is a systematic gain-scheduling design technique [14], [2], [1], [18], [19], which has been widely used in the fields ranging from aerospace to process control industries. Different from conventional gain-scheduling techniques, LPV control theory provides stability and performance guarantee over a wide range of changing parameters. An LPV system is characterized as a group of local descriptions of nonlinear dynamics that depend on time-varying parameters. The LPV synthesis condition can be formulated as a linear matrix inequality (LMI) optimization problem using a single Lyapunov function, either quadratic or parameter-dependent, in the entire parameter space [4], [21]. However, for an LPV system with a large parameter variation region, a single Lyapunov function may not exist. If it does exist, it is possible to sacrifice the performance in some parameter subregions in order to obtain a uniform LPV controller representation over the entire parameter region. One reasonable approach to avoid those problems is to design several LPV controllers, each suitable for a specific parameter subregion, and switch among them to achieve the best possible performance. The LPV systems then become a new class of systems, namely, switched LPV systems.

Closely related to LPV systems, the switched systems are described by an interaction between continuous time systems and discrete switching events, which are usually dependent on states or time [8]. Due to their wide applications in adaptive control, air-traffic management, and reconfigurable control, the study of switched systems has become an important research area in recent years. As shown in [9], the dynamic behavior of switched systems is much more complicated than either continuous or discrete dynamics. One useful tool for proving stability of switched systems is based on multiple Lyapunov functions, which are discontinuous [15], [16], [5], [7], [17]. However, most of the past research are focused on switched systems with each subsystem described by linear time invariant (LTI) dynamics with the exception of [17], in which nonlinear subsystems are considered. Moreover, the performance issue of switched systems has not been adequately addressed.

The results of switched LTI systems has been extended to the analysis and control of switched LPV systems [10], which is later generalized in [11] by introducing average dwell time switching logic [6]. The stability of switched LPV systems is analyzed using multiple parameter-dependent Lyapunov functions, which are allowed to be discontinuous at the switching surfaces. Switched LPV control technique permits using different controllers in different parameter subregions, and switching among them according to the evolution of parameters. For an LPV system, it is conceivable that parameter-dependent switching is more practical than state-dependent or time-dependent switching. Switched LPV control is also beneficial to improve controlled performance and enhance design flexibility.

The paper is organized as follows: Section II provides a brief introduction of switched LPV systems. In Section III, we study switched LPV control design problems under hysteresis switching and switching with average dwell time logics. The switching control synthesis conditions will be formulated as matrix optimization problems. Section IV uses a magnetic bearing example to demonstrate the advantages of the newly proposed switching LPV control techniques. Finally, the paper concludes in Section V. All the proofs have been omitted to save space.

The notation is standard. $\mathbf{R}$ stands for the set of real numbers and $\mathbf{R}_+$ for the non-negative real numbers. $\mathbf{R}^{m\times n}$ is the set of real $m \times n$ matrices. The transpose of a real matrix M is denoted by M^T. The orthogonal complement of matrix M is denoted by Ker(M). We use $\mathbf{S}^{n\times n}$ to denote the real symmetric $n \times n$ matrices and $\mathbf{S}_+^{n\times n}$ to denote positive definite matrices. If $M \in \mathbf{S}^{n\times n}$, then $M > 0$ ($M \geq 0$) indicates that M is positive definite (positive semidefinite) and $M < 0$ ($M \leq 0$) denotes a negative definite (negative semidefinite) matrix. For $x \in \mathbf{R}^n$, its norm is defined as $\|x\| := (x^T x)^{\frac{1}{2}}$. The space of square integrable functions is denoted by $\mathcal{L}_2$, that is, for any $u \in \mathcal{L}_2$, $\|u\|_2 := \left[\int_0^\infty u^T(t)u(t)dt\right]^{\frac{1}{2}}$ is finite.

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II. SWITCHED LINEAR PARAMETER-VARYING SYSTEMS

Consider an open-loop LPV system governed by the equation

$$\begin{bmatrix}\dot{x}\\ e\\ y\end{bmatrix} = \begin{bmatrix}A(\rho) & B_1(\rho) & B_2(\rho)\\ C_1(\rho) & D_{11}(\rho) & D_{12}(\rho)\\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho)\end{bmatrix}\begin{bmatrix}x\\ d\\ u\end{bmatrix} \tag{1}$$

where $x, \dot{x} \in \mathbf{R}^n$, $e \in \mathbf{R}^{n_e}$, $d \in \mathbf{R}^{n_d}$, $u \in \mathbf{R}^{n_u}$ and $y \in \mathbf{R}^{n_y}$. All of the state-space data are continuous functions of the parameter ρ. It is assumed that ρ evolves in a compact set $\mathcal{P} \subset \mathbf{R}^s$ with its parameter variation rate bounded by $\underline{\nu}_k \leq \dot{\rho}_k \leq \overline{\nu}_k$ for $k = 1, 2, \cdots, s$. In the interests of notational compactness, the parameter dependence will not always be shown in the sequel.

To simplify the presentation, we also assume that

(A1) (A, B_2, C_2) triple is parameter-dependent stabilizable and detectable for all $\rho \in \mathcal{P}$,

(A2) The matrix functions $\begin{bmatrix}B_2^T & D_{12}^T\end{bmatrix}$ and $\begin{bmatrix}C_2 & D_{21}\end{bmatrix}$ have full row ranks for all $\rho \in \mathcal{P}$,

(A3) $D_{22} = 0$.

Given the open-loop LPV system (1), an LPV controller working for the entire parameter region can be computed using well-known LPV control theory [4], [21], which is based on single Lyapunov function (quadratic or parameter-dependent). However, the control design requirements are often different and even conflicting for different parameter regions. This could complicate the LPV control design problem.

Suppose that the parameter set $\mathcal{P}$ is covered by a finite number of closed subsets $\{\mathcal{P}_i\}_{i \in Z_N}$, where the index set $Z_N = \{1, 2, \ldots, N\}$, and $\mathcal{P} = \bigcup \mathcal{P}_i$. The adjacent parameter subsets are separated by a family of switching surfaces, and they have either overlapped or disjointed interiors.

In this paper, we are interested in the problem of designing a family of LPV controllers in the form of

$$\begin{bmatrix}\dot{x}_k\\ u\end{bmatrix} = \begin{bmatrix}A_{k,i}(\rho,\dot{\rho}) & B_{k,i}(\rho)\\ C_{k,i}(\rho) & D_{k,i}(\rho)\end{bmatrix}\begin{bmatrix}x_k\\ y\end{bmatrix}, \qquad i \in Z_N \tag{2}$$

each suitable for a specific parameter subset $\mathcal{P}_i$. The dimension of controller state is $x_k \in \mathbf{R}^{n_k}$. Each controller stabilizes the open-loop system with best achievable performance in a specific parameter region, and meanwhile build a switching logic to keep the closed-loop system stable when switching among the controllers.

The switching occurs when the parameter trajectory hits one of the switching surfaces. A switching signal is defined as a piecewise constant function σ. It is assumed that σ is continuous from the right everywhere. The switching signal for the case of two parameter subsets, i.e., $Z_N = \{1, 2\}$, is depicted in Fig. 1.

Under switched LPV control, the closed-loop LPV system can be described by

$$\begin{bmatrix}\dot{x}_{cl}\\ e\end{bmatrix} = \begin{bmatrix}A_{cl,\sigma}(\rho,\dot{\rho}) & B_{cl,\sigma}(\rho)\\ C_{cl,\sigma}(\rho) & D_{cl,\sigma}(\rho)\end{bmatrix}\begin{bmatrix}x_{cl}\\ d\end{bmatrix} \tag{3}$$

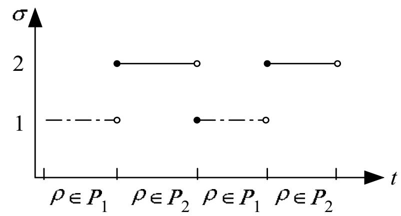

Fig. 1. A switching signal in the case of two parameter subsets

where $x_{cl} \in \mathbf{R}^{n+n_k}$, and

$$\begin{bmatrix}A_{cl,\sigma} & B_{cl,\sigma}\\ C_{cl,\sigma} & D_{cl,\sigma}\end{bmatrix} = \left[\begin{array}{cc|c} A & 0 & B_1\\ 0 & 0 & 0\\ \hline C_1 & 0 & D_{11}\end{array}\right] + \left[\begin{array}{cc} 0 & B_2\\ I & 0\\ \hline 0 & D_{12}\end{array}\right]\begin{bmatrix}A_{k,\sigma} & B_{k,\sigma}\\ C_{k,\sigma} & D_{k,\sigma}\end{bmatrix}\left[\begin{array}{cc|c} 0 & I & 0\\ C_2 & 0 & D_{21}\end{array}\right] \tag{4}$$

Note that the resulting closed-loop system is a switched LPV system, which could have discontinuity and multiple values at switching surfaces due to the use of multiple LPV controllers.

III. SWITCHING CONTROL VIA MULTIPLE PARAMETER-DEPENDENT LYAPUNOV FUNCTIONS

A discontinuous Lyapunov function consisting of multiple parameter-dependent Lyapunov functions is useful for stability analysis and control design of switched LPV systems. If there exist a family of positive-definite matrix functions $\{X_i(\rho)\}_{i \in Z_N}$, and each of them is smooth over the corresponding parameter subset $\mathcal{P}_i$. The multiple parameter-dependent Lyapunov functions can then be defined as

$$V_\sigma(x, \rho) = x^T X_\sigma(\rho) x \tag{5}$$

where the value of switching signal σ represents the active operating region $\mathcal{P}_i$ and thus determines the corresponding matrix function $X_i(\rho)$.

Generally speaking, for a switched LPV system to be stable, the value of the discontinuous Lyapunov function V_σ is not necessarily to decrease over the entire parameter trajectory. In fact, it is often enough to require that the value of V_σ decreases in the active parameter region $\mathcal{P}_i$ provided proper switching logic is adopted. This will lead to relaxed stability condition and provides enhanced design flexibility. In this section, we consider the synthesis conditions of switched LPV systems with two different switching logics, both of which rely on multiple parameter-dependent Lyapunov functions.

A. Hysteresis Switching

For the hysteresis switching logic, it is assumed that any two adjacent parameter subsets are overlapped, as shown in Fig. 2. Thus there are two switching surfaces between two adjacent parameter subsets. We use $\mathcal{S}_{ij}$ to denote the switching surface specifying the one-directional move from subset $\mathcal{P}_i$ to $\mathcal{P}_j$.

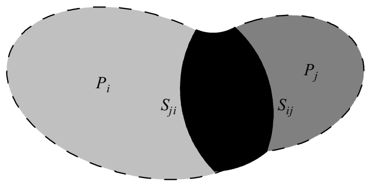

Fig. 2. Hysteresis switching regions

The switching event occurs when the parameter trajectory hits one of the switching surfaces $\mathcal{S}_{ij}$ or $\mathcal{S}_{ji}$. The evolution of the switching signal σ is described as follows: Let $\sigma(0) = i$ if $\rho(0) \in \mathcal{P}_i$. For each $t > 0$, if $\sigma(t^-) = i$ and $\rho(t) \in \mathcal{P}_i$, keep $\sigma(t) = i$. On the other hand, if $\sigma(t^-) = i$ but $\rho(t) \in \mathcal{P}_j$, i.e., hitting the switching surface $\mathcal{S}_{ij}$, let $\sigma(t) = j$. Repeating this procedure, we generate a piecewise constant signal σ which is continuous from the right everywhere. Since σ changes its value only after the continuous trajectory has passed through the intersection of adjacent subsets $\mathcal{P}_i$ and $\mathcal{P}_j$, chattering is avoided. Also due to bounded parameter variation rates, only finite number of switches will happen in any finite time interval.

Now we consider the switched closed-loop system (3). Assume the matrix function $X_i(\rho)$ is related to the Lyapunov function of the closed-loop system when the ith controller is active. If on the switching surface $\mathcal{S}_{ij}$, we have

$$X_i(\rho) \geq X_j(\rho) \tag{6}$$

i.e. the Lyapunov function of the closed-loop system (3) is non-increasing when switching from $\mathcal{P}_i$ to $\mathcal{P}_j$. Then the jth controller is activated. We will partition the Lyapunov function matrices of the closed-loop system (3) according to the plant and controller state dimensions as

$$X_i(\rho) = \begin{bmatrix} S_i(\rho) & N_i(\rho) \\ N_i^T(\rho) & ? \end{bmatrix}$$
$$X_i^{-1}(\rho) = \begin{bmatrix} R_i(\rho) & M_i(\rho) \\ M_i^T(\rho) & ? \end{bmatrix}$$

where $M_i(\rho)N_i^T(\rho) = I - R_i(\rho)S_i(\rho)$, and "?" means the elements we don't care. By choosing $M_i(\rho) = R_i(\rho)$ and $N_i(\rho) = R_i^{-1}(\rho) - S_i(\rho)$, the synthesis condition of switched LPV control based on hysteresis switching logic can be stated in the following theorem.

Theorem 1: Given an open-loop LPV system (1), the parameter set $\mathcal{P}$ and its overlapped covering $\{\mathcal{P}_i\}_{i\in Z_N}$, if there exist positive-definite matrix functions $R_i(\rho), S_i(\rho) : \mathbf{R}^s \rightarrow \mathbf{S}_+^{n\times n}, i \in Z_N$, such that for any $\rho \in \mathcal{P}_i$,

$$\mathcal{N}_R^T \begin{bmatrix} \left\{ \begin{array}{c} R_iA^T + AR_i \\ -\sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \dfrac{\partial R_i}{\partial \rho_k} \end{array} \right\} & R_iC_1^T & B_1 \\ C_1R_i & -\gamma_i I & D_{11} \\ B_1^T & D_{11}^T & -\gamma_i I \end{bmatrix} \mathcal{N}_R < 0 \tag{7}$$

$$\mathcal{N}_S^T \begin{bmatrix} \left\{ \begin{array}{c} A^TS_i + S_iA \\ +\sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \dfrac{\partial S_i}{\partial \rho_k} \end{array} \right\} & S_iB_1 & C_1^T \\ B_1^TS_i & -\gamma_i I & D_{11}^T \\ C_1 & D_{11} & -\gamma_i I \end{bmatrix} \mathcal{N}_S < 0 \tag{8}$$

$$\begin{bmatrix} R_i & I \\ I & S_i \end{bmatrix} \geq 0 \tag{9}$$

where

$$\mathcal{N}_R = \mathrm{Ker}\begin{bmatrix} B_2^T & D_{12}^T & 0 \end{bmatrix}, \ \mathcal{N}_S = \mathrm{Ker}\begin{bmatrix} C_2 & D_{21} & 0 \end{bmatrix},$$

and for any $\rho \in \mathcal{S}_{ij}$

$$R_i \leq R_j \tag{10}$$
$$S_i - R_i^{-1} \geq S_j - R_j^{-1} \tag{11}$$

then the closed-loop LPV system (3) is exponentially stabilized by switched LPV controllers in the entire parameter set $\mathcal{P}$, and its induced $\mathcal{L}_2$ performance from d to e is less than $\gamma = \max\{\gamma_i\}_{i\in Z_N}$ given initial condition $x(0) = 0$.

Remark 1: The notation $\sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \frac{\partial}{\partial \rho_k}$ in (7)-(8) represents the combination of derivative terms in the form of $\nu_k \frac{\partial}{\partial \theta_k}$ when ν_k is taken as either $\underline{\nu}_k$ or $\bar{\nu}_k$. Therefore each inequality means 2^s different LMIs which must be checked.

Note that the term R_i^{-1} appears in the condition (11), so the synthesis condition for switching LPV controllers is generally non-convex. After solving matrix functions $R_i(\rho)$ and $S_i(\rho)$, the gains of switching LPV controllers can be constructed using the formula in [3]. However, to comply with hysteresis switching logic, we need to choose a particular realization of LPV controllers with $M_i(\rho) = R_i(\rho)$ and $N_i(\rho) = R_i^{-1}(\rho) - S_i(\rho)$.

The non-convex switching LPV synthesis condition is usually difficult to solve. However, if we enforce the matrix variables $R_i(\rho)$ to be continuous on the switching surfaces, then for any $\rho \in \mathcal{S}_{ij}$

$$R_i(\rho) = R_j(\rho) \tag{12}$$
$$S_i(\rho) \geq S_j(\rho) \tag{13}$$

This implies that the dynamic controller on each switching surface has a different state-estimate gain, but has the same state-feedback gain. The equality constraint (12) can be rewritten as an LMI condition through a relaxation process

$$-\epsilon I < R_i(\rho) - R_j(\rho) < \epsilon I \tag{14}$$

where ϵ is a small positive number.

Alternative approach to avoid the non-convex condition on the switching surfaces is to use multiple state-feedback LPV control laws $u_i = F_i(\rho)x$ if all the states are available for feedback control use. Then the closed-loop LPV system is given by

$$\begin{bmatrix} \dot{x}_{cl} \\ e \end{bmatrix} = \begin{bmatrix} A(\rho) + B_2(\rho)F_\sigma(\rho) & B_1(\rho) \\ C_1(\rho) + D_{12}(\rho)F_\sigma(\rho) & D_{11}(\rho) \end{bmatrix} \begin{bmatrix} x_{cl} \\ d \end{bmatrix} \tag{15}$$

The following corollary shows that the switching state-feedback LPV control problem is solvable by convex optimization.

Corollary 1: The closed-loop LPV system (15) is exponentially stabilized by state-feedback switching LPV controllers in the entire parameter set $\mathcal{P}$ and $\|e\|_2 \leq \gamma\|d\|_2$ with $\gamma = \max\{\gamma_i\}_{i\in Z_N}$, if there exist positive-definite matrix functions $R_i(\rho)$ such that for any $\rho \in \mathcal{P}_i$,

$$\mathcal{N}_R^T \begin{bmatrix} \left\{ \begin{array}{c} R_iA^T + AR_i \\ -\sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \dfrac{\partial R_i}{\partial \rho_k} \end{array} \right\} & R_iC_1^T & B_1 \\ C_1R_i & -\gamma_i I & D_{11} \\ B_1^T & D_{11}^T & -\gamma_i I \end{bmatrix} \mathcal{N}_R < 0 \tag{16}$$

where $\mathcal{N}_R = \text{Ker}\begin{bmatrix} B_2^T & D_{12}^T & 0 \end{bmatrix}$, and for any $\rho \in \mathcal{S}_{ij}$

$$R_i \leq R_j \tag{17}$$

Furthermore, the switching state-feedback LPV gains are given by

$$F_i = -\left(D_{12}^TD_{12}\right)^{-1}\left[\gamma_i B_2^T R_i^{-1} + D_{12}^T C_1\right]$$

for any $i \in Z_N$.

B. Switching with Average Dwell Time

If the overlapped region between two adjacent parameter subsets shrinks, it eventually becomes a single switching surface, as shown in Fig. 3. Different from hysteresis switching, here $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$ represent the same switching surface between subsets $\mathcal{P}_i$ and $\mathcal{P}_j$ no matter which direction the parameter trajectory moving from. It is obvious that there must be a continuous Lyapunov function if the condition (6) on the switching surface is to be satisfied. To relax continuity requirement of Lyapunov functions across the switching surfaces, we will consider another switching logic with average dwell time [6], [11]. However, only restricted number of switchings is allowed between a finite time interval.

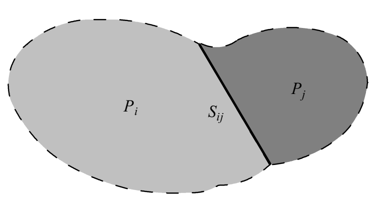

Fig. 3. Switching regions with dwell time

Denote $N_\sigma(T,t)$ as the number of switchings among subsets $\mathcal{P}_i$ on an interval (t,T). The switching signal σ has average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that

$$N_\sigma(T,t) \leq N_0 + \frac{T-t}{\tau_a} \qquad \forall T \geq t \geq 0 \tag{18}$$

where N_0 is called the chatter bound. This idea relaxes the concept of dwell time, allowing the possibility of switching fast when necessary and then compensating for it by switching sufficiently slow later on. If we choose the particular structure of $X_i(\rho)$ similar to that in Section III-A, then the next theorem gives the synthesis condition of switching LPV control with average dwell time.

Theorem 2: Given scalars $\lambda_0 > 0, \mu > 1$, an open-loop LPV system (1), the parameter set $\mathcal{P}$ and its partition $\{\mathcal{P}_i\}_{i\in Z_N}$, if there exist positive-definite matrix functions $R_i(\rho), S_i(\rho) : \mathbf{R}^s \to \mathbf{S}_+^{n\times n}$, such that for any $\rho \in \mathcal{P}_i$,

$$\mathcal{N}_R^T \begin{bmatrix} R_iA^T + AR_i - \sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \dfrac{\partial R_i}{\partial \rho_k} + \lambda_0 R_i & R_iC_1^T & B_1 \\ C_1R_i & -\gamma_i I & D_{11} \\ B_1^T & D_{11}^T & -\gamma_i I \end{bmatrix} \mathcal{N}_R < 0 \tag{19}$$

$$\mathcal{N}_S^T \begin{bmatrix} A^TS_i + S_iA + \sum_{k=1}^{s} \{\underline{\nu}_k, \bar{\nu}_k\} \dfrac{\partial S_i}{\partial \rho_k} + \lambda_0 S_i & S_iB_1 & C_1^T \\ B_1^TS_i & -\gamma_i I & D_{11}^T \\ C_1 & D_{11} & -\gamma_i I \end{bmatrix} \mathcal{N}_S < 0 \tag{20}$$

$$\begin{bmatrix} R_i & I \\ I & S_i \end{bmatrix} \geq 0 \tag{21}$$

where

$$\mathcal{N}_R = \text{Ker}\begin{bmatrix} B_2^T & D_{12}^T & 0 \end{bmatrix}, \; \mathcal{N}_S = \text{Ker}\begin{bmatrix} C_2 & D_{21} & 0 \end{bmatrix},$$

and for any $\rho \in \mathcal{S}_{ij}$

$$\frac{1}{\mu} R_j \leq R_i \leq \mu R_j \tag{22}$$

$$\frac{1}{\mu}\left(S_j - R_j^{-1}\right) \leq S_i - R_i^{-1} \leq \mu\left(S_j - R_j^{-1}\right) \tag{23}$$

then the closed-loop LPV system (3) is asymptotically stabilized by switching LPV controllers in the entire parameter set $\mathcal{P}$ for every switching signal σ with average dwell time

$$\tau_a > \frac{\ln \mu}{\lambda_0} \tag{24}$$

and its induced $\mathcal{L}_2$ performance from d to e is less than $\gamma = \max\{\gamma_i\}_{i\in Z_N}$ given initial condition $x(0) = 0$.

Using average dwell time switching logic, the Lyapunov function is not required to monotonically decrease over switching surfaces. In fact, it allows the change of Lyapunov function by $\mu(>1)$ times of its value before switching. As a consequence, the average switching frequency over a finite time interval is limited to $\frac{1}{\tau_a}$ to compensate for possible increase of Lyapunov functions. The hybrid LPV control with average dwell time switching logic is also studied in [12]. In comparison, our synthesis condition not only guarantees the stability of the closed-loop system, but also provides an upper bound of the induced $\mathcal{L}_2$ performance over the entire parameter space. However, our synthesis condition for switching control with average dwell time is non-convex. Since there are coefficients μ and $\frac{1}{\mu}$ involved in (22), the synthesis conditions with average dwell time switching logic cannot be convexified by simply setting $R_i = R_j$ on the switching surfaces. But the non-convex condition can be avoided by switching state feedback control law.

Note that the switching LPV synthesis condition for this switching logic is different from hysteresis switching LPV control results. The $(1,1)$ term in (19 – 20) implies that the open-loop plant can be thought as a shifted system with its A matrix changing to $A + \frac{\lambda_0}{2} I$. It is the same for controller A_k matrix. Therefore, if the matrix functions $R_i(\rho)$ and $S_i(\rho)$ can be solved, then the gains of the switching LPV controllers will be constructed by replacing A and A_k in the standard LPV controller formula [3] by $A + \frac{\lambda_0}{2} I$ and $A_k + \frac{\lambda_0}{2} I$.

IV. Example

In this section, we apply the proposed switching LPV control synthesis technique to an active magnetic bearing (AMB) system and demonstrate its advantages over conventional LPV control designs.

Owing to the linear dependence of the rotor speed in the plant dynamics, the nonlinear gyroscopic equations of AMB can be simplified to a set of linear time-varying differential equations as [13], [20]

$$\ell\ddot{\theta} = -\frac{\rho J_a}{J_r}\ell\dot{\psi} + \frac{1}{m}\left(-4c_2\ell\theta + 2c_1\phi_\theta + f_{d\theta}\right) \quad (25)$$

$$\ell\ddot{\psi} = \frac{\rho J_a}{J_r}\ell\dot{\theta} + \frac{1}{m}\left(-4c_2\ell\psi + 2c_1\phi_\psi + f_{d\psi}\right) \quad (26)$$

$$N\dot{\phi}_\theta = e_\theta + 2d_2\ell\theta - d_1\phi_\theta \quad (27)$$

$$N\dot{\phi}_\psi = e_\psi + 2d_2\ell\psi - d_1\phi_\psi \quad (28)$$

where ρ denotes the rotor speed. θ, ψ are the Euler angles denoting the orientation of rotor centerline. J_a, J_r are the moment of inertia of the rotor in axial and radial directions, respectively. ϕ_θ, ϕ_ψ are the differential magnetic flux from electromagnetic pairs, e_θ, e_ψ are the corresponding differences of electric voltage. $f_{d\theta}$, $f_{d\psi}$ are disturbance forces caused by gravity, modeling errors, imbalances, etc. The constants c_1, c_2, d_1, d_2 and m depend on the AMB's geometry and parameters, which are given in [20].

Let $x^T = \begin{bmatrix} \ell\theta & \ell\psi & \ell\dot{\theta} & \ell\dot{\psi} & \phi_\theta & \phi_\psi \end{bmatrix}$, $d^T = \begin{bmatrix} f_{d\theta} & f_{d\psi} \end{bmatrix}$, and $u^T = \begin{bmatrix} e_\theta & e_\psi \end{bmatrix}$, then the linearized equations (25)–(28) can be written as an LPV system P_ρ

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1 d + B_2 u \\ e &= C_1 x + D_{11} d + D_{12} u \\ y &= C_2 x + D_{21} d + D_{22} u \end{aligned}$$

where the state-space data are

$$A(\rho) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{4c_2}{m} & 0 & 0 & -\frac{\rho J_a}{J_r} & \frac{2c_1}{m} & 0 \\ 0 & -\frac{4c_2}{m} & \frac{\rho J_a}{J_r} & 0 & 0 & \frac{2c_1}{m} \\ \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} & 0 \\ 0 & \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} \end{bmatrix}$$

$$B_1 = \frac{1}{m}\begin{bmatrix} 0_{2\times 2} \\ I_2 \\ 0_{2\times 2} \end{bmatrix}, \qquad B_2 = \frac{1}{N}\begin{bmatrix} 0_{4\times 2} \\ I_2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} I_2 & 0_{2\times 4} \\ \multicolumn{2}{c}{0_{2\times 6}} \end{bmatrix}, \; D_{11} = 0_{4\times 2}, \; D_{12} = \begin{bmatrix} 0_{2\times 2} \\ I_2 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} I_2 & 0_{2\times 4} \end{bmatrix}, \qquad D_{21} = 0_{2\times 2}, \qquad D_{22} = 0_{2\times 2}$$

The design objective of switching LPV control is to stabilize the system over large range of rotor speeds and to minimize the disturbance effect. The weighted open-loop interconnection is given in Fig. 4, where the weighting functions are chosen as

$$W_z(s) = \frac{200(s+100)}{s+0.01} I_2, \quad W_u(s) = \frac{0.001s}{0.05s+1} I_2$$

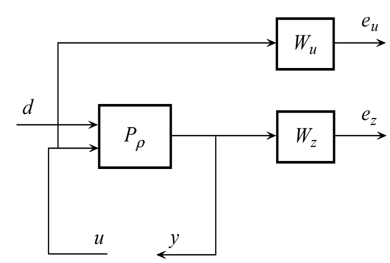

Fig. 4. Weighted open-loop interconnection for the AMB system

The rotor speed is assumed to vary between 315 rad/s to 1100 rad/s, and its variation rate is less than 100 rad/s^2. The rotor dynamics exhibits strong gyroscopic effects in this speed range. Due to large variations of rotor speed, it is conservative to use a single LPV controller over the whole parameter region. For hysteresis switching logic, the parameter space is divided into two overlapped subsets, $[315 \;\; 720]$ and $[700 \;\; 1100]$.

As mentioned before, the synthesis condition (7)–(11) for hysteresis switching control is nonconvex. To avoid solving non-convex problem, we assume all states are available for feedback control use, and the resulting synthesis condition (16)–(17) becomes convex and globally solvable. The multiple parameter-dependent Lyapunov functions are specified as affine functions of scheduling parameters. That is, we assume for each parameter subset

$$R_i(\rho) = R_i^0 + R_i^1 \rho$$

where matrices R_i^k with $k = 0, 1$ are new optimization variables to be determined. The performance level γ_i in each parameter subsets are 3.2868×10^{-3} and 3.2730×10^{-3}, respectively. Then the γ value over the entire parameter set is $\max\{\gamma_1, \gamma_2\}$, which represents the "worst-case" LPV control performance. As a comparison, the performance level achieved using single parameter-dependent Lyapunov function is 3.3098×10^{-3}, which is slightly worse than the switching control.

We then conduct simulation for AMB using switching LPV control. A time-varying rotor speed profile is chosen as shown in Fig. 5. Note that the rotor speed trajectory is deliberately chosen to cross the intersection of two parameter subsets $[315 \quad 720]$ and $[700 \quad 1100]$ back and forth to illustrate the effect of LPV control switching. Disturbances $f_{d\theta}$ and $f_{d\psi}$ are chosen as unit step inputs with opposite signs. As shown in Fig. 5, the switching occurs at 1.9 s and 5.5 s, respectively. The simulation result is presented in Fig. 6, where the subplots are zoomed view of responses around the switching time. Although there are small glitches during controller switching, the performance for entire time history is nevertheless acceptable.

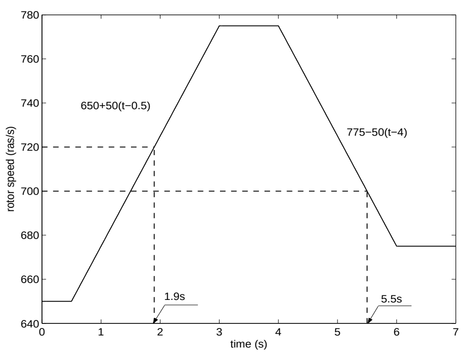

Fig. 5. Time histories of parameter ρ

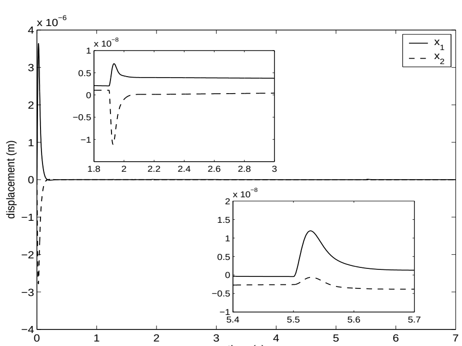

Fig. 6. Time history of rotor displacement x_1 and x_2

V. CONCLUSION

In this paper, control design of switched LPV systems using multiple parameter-dependent Lyapunov functions is proposed. A family of LPV controllers are designed, each suitable for a specific parameter region. The possible transient instability caused by switching among controllers is avoided by choosing suitable switching logics. The synthesis conditions for two switching rules, hysteresis switching and switching with average dwell time, are derived. They are generally non-convex and can be convexified for two special cases, 1) output-feedback control with same control gain and different estimate gain at the switching surface, 2) state-feedback control problem. The state-feedback control with hysteresis switching logic is then applied to a magnetic bearing control problem, and promising simulation results are obtained.

REFERENCES

[1] P. Apkarian and R.J. Adams, Advanced Gain-Scheduling Techniques for Uncertain System, *IEEE Trans. Contr. Syst. Tech.*, vol. 6, 1997, pp. 21-32.

[2] P. Apkarian and P. Gahinet, A Convex Characterization of Gain-Scheduled $\mathcal{H}_\infty$ Controllers, *IEEE Trans. Automat. Contr.*, vol. 40, 1995, pp. 853-864.

[3] G. Becker, "Additional Results on Parameter-Dependent Controllers for LPV Systems," in *Proc. 13th IFAC World Congress*, 1996, pp. 351-356.

[4] G. Becker and A. Packard, Robust Performance of Linear Parametrically Varying Systems using Parametrically-Dependent Linear Feedback, *Syst. Contr. Letts.*, vol. 23, 1994, pp. 205-215.

[5] M. Branicky, Multiple Lyapunov Functions and Other Analysis Tools for Switched and Hybrid Systems, *IEEE Trans. Automat. Contr.*, vol. 43, 1998, pp. 475-482.

[6] J.P. Hespanha and A.S. Morse, "Stability of Switched Systems with Average Dwell-Time," in *Proc. 38th IEEE Conf. Dec. Contr.*, 1999, pp. 2655-2660.

[7] M. Johansson and A. Ranzer, Computation of Piecewise Quadratic Lyapunov Functions, *IEEE Trans. Automat. Contr.*, vol. 43, 1998, pp. 555-559.

[8] D. Liberzon, *Switching in Systems and Control*, Birkhäuser, Boston, MA:2003.

[9] D. Liberzon and A.S. Morse, Basic Problems in Stability and Design of Switched Systems, *IEEE Contr. Syst. Mag.*, vol. 19, 1999, pp. 59-70.

[10] S. Lim, *Analysis and Control of Linear Parameter-Varying Systems*, PhD Dissertation, Stanford University, Palo Alto, CA:1999.

[11] S. Lim and K. Chan, "Analysis of Hybrid Linear Parameter-Varying Systems," in *Proc. 2003 Amer. Contr. Conf.*, 2003, pp. 4822-4827.

[12] S. Lim and J. Shin, "Fault Tolerant Controller Design using Hybrid Linear Parameter-Varying Control," in *Proc. AIAA Guidance, Nav. Contr. Conf.*, AIAA paper 2003-5492, 2003.

[13] A.M. Mohamed and I. Busch-Vishmiac, Imbalance Compensation and Automation Balancing in Magnetic Bearing Systems using the Q-Parameterization Theory, *IEEE Trans. Contr. Syst. Tech.*, vol. 3, 1995, pp. 202-211.

[14] A.K. Packard, Gain Scheduling via Linear Fractional Transformations, *Syst. Contr. Letts.*, vol. 22, 1994, pp. 79-92.

[15] P. Peleties and R. Decarlo, "Asymptotic Stability of m-Switched Systems using Lyapunov-Like Functions," in *Proc. 1991 Amer. Contr. Conf.*, 1991, pp. 1679-1684.

[16] S. Pettersson and B. Lennartson, "Stability and Robustness of Hybrid Systems," in *Proc. 35th IEEE Conf. Dec. Contr.*, 1996, pp. 1202–1207.

[17] S. Prajna and A. Papachristodoulou, "Analysis of Switched and Hybrid Systems–Beyond Piecewise Quadratic Methods," in *Proc. 2003 Amer. Contr. Conf.*, 2003, pp. 2779–2784.

[18] C.W. Scherer, Robust Mixed Control and LPV Control with Full Block Scalings, in *Recent Advances of LMI Methods in Control* (Ghaoui, L. El, Niculescu, S. ed.), SIAM, 1999.

[19] F. Wu, A Generalized LPV System Analysis and Control Synthesis Framework, *Int. J. Contr.*, vol. 74, 2001, 745-759.

[20] F. Wu, "Switching LPV Control Design for Magnetic Bearing Systems," in *Proc. 10th IEEE Conf. Contr. Appli.*, 2001, pp. 41-46.

[21] F. Wu, X.H. Yang, A. Packard, and G. Becker, Induced $\mathcal{L}_2$ Norm Control for LPV Systems with Bounded Parameter Variation Rates, *Int. J. Robust Non. Contr.*, vol. 6, 1996, pp. 983-998.