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Tracking of Perturbed Nonlinear Plants using Robust Right Coprime Factorization Approach

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***Abstract*— This paper deals with a plant output tracking design problem of perturbed nonlinear plants by using robust right coprime factorization approach. One of the interesting control system design schemes, which was given by G. Chen and Z. Han, uses robustness of the right coprime factorization for robust stability of the closed-loop system with perturbation. Unfortunately, the robust right coprime factorization cannot easily improve the tracking performance of the control system except for simple cases. In this paper, nonlinear operator-based design method for nonlinear plant output to track a reference input is given. Examples are presented to support the theoretical analysis.**

I. Introduction

Based on the existing research results [1, -, 5], constructions of (left and right) coprime factorizations for some nonlinear plants can be obtained. Then, Bezout identity can be given for the above class of nonlinear plants. Their applications to the stabilization of the nonlinear plant have been an exciting approach. Recently, the relation between the robustness of the right coprime factorization and the robust stability of the perturbed nonlinear plant is shown in [6]. That is, when a condition for the robust right coprime factorization of a nonlinear plant under unknown but bounded perturbations is satisfied, robust stabilization for the nonlinear feedback control system with the perturbation can be ensured. Meanwhile, the perturbed signal of the nonlinear plant does not affect the plant output error signal. In practice, the output feedback signal often includes the part of uncertainties and disturbances. This causes that the desired output tracking performance of the real nonlinear plant is difficult to realize. However, tracking issues for the above control design scheme are not considered in [6].

This paper deals with a plant output tracking problem for nonlinear plants under unknown but bounded perturbations by extending the design scheme in [6]. Based on a nonlinear operator framework, some design conditions for output tracking are discussed. The detailed design method of the operator for plant output tracking is given. The operator is designed without using the information of inverse of the nonlinear system. As a result, the perfect tracking can be obtained for the plant with bounded perturbations.

This paper is organized as follows. In Section 2 we give a brief outline of the result in [6] and state our problem statement. The design method of the proposed operator for plant output tracking is discussed in Section 3. Several examples are illustrated in Section 4 to show the effectiveness of the proposed method. Conclusions are drawn in Section 5.

Notations: When $A : B \rightarrow C$ is an operator, $A[s]$ means that it is causal, but not necessarily linear or bounded, the domain $\bar{D}(A) = B$ with the range $\bar{R}(A) \subseteq C$.

II. Problem statement

The organization of this section is as follows. First, the research result in [6] is outlined. Next, considering the summary, the problem considered in this paper is given.

Consider a nonlinear unstable plant described by the following right coprime factorization:

$$P = ND^{-1} \tag{1}$$

where $D : W \rightarrow U$, and $N : W \rightarrow Y$ are stable operators, respectively, and the space W is called a $quasi - state$ space [6] of P. D is invertible, and $P : U \rightarrow Y$. U and V are linear spaces over the field of real numbers, respectively. Here, we assume that input singal u is in a subset U^* of U and output y is in a subset Y^* of Y. U^* and Y^* are the normed subspaces of U and Y respectively, called the stable subspace. A feedback control system is said to be well-posed if every signal in the control system is uniquely determined for any input signal in U. For the above nonlinear plant (1), under the condition of well-posedness, N and D are said to be right coprime factorization if there exist two stable operators $S : Y \rightarrow U$ and $R : U \rightarrow U$ satisfying the following Bezout identity [3]

$$SN + RD = I \tag{2}$$

where R is invertible and I is the identify operator.

Definition [6]: A feedback control system is said to be overall stable if all the signals of the system are bounded.

Usually, real nonlinear plants must deal with uncertainties and disturbances, and the above perturbation affects D and N. For the case of $D \rightarrow D + \Delta D$ or $N \rightarrow N + \Delta N$, the satisfying design scheme for robust stabilization was given in [6]. Where, ΔD and ΔN are bounded and the operators $D + \Delta D$ and $N + \Delta N$ are stable.

So far, we summarized the results of [6]. In the following, the problem statement of this paper is given. We only extend the design scheme to the case of $D \to D + \Delta D$ and $N \to N + \Delta N$ under the assumption of the resulting system being well-posed under the perturbations. Then, suppose that the plant P has a bounded perturbation ΔP, that is, ΔD and ΔN are bounded and the operators $D + \Delta D$ and $N + \Delta N$ are stable (Fig.1, where $M = I$ for the case in [6]) such that

$$P + \Delta P = (N + \Delta N)(D + \Delta D)^{-1} \tag{3}$$

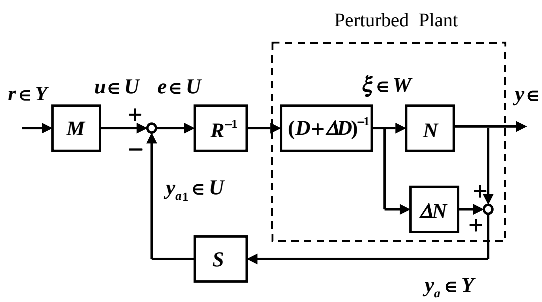

Fig. 1. The proposed control system

We also assume that there exist S_1 and R_1 satisfying the perturbed Bezout identity

$$S_1(N + \Delta N) + R_1(D + \Delta D) = I$$

Therefore, with the above presentation, the perturbed plant retains a right comprime factorization. In Fig.1, $M : Y \to U$ is a designed operator and it is stable.

The merit of the design method described in [6] is that the error signal $e(t)$ is not affected by the perturbed signal. That is, the perturbed signal $\Delta N z$ cannot be transmitted back to the error signal $e(t)$. In practice, in this sense we can avoid the undesired influence from uncertainties, output disturbances and feedback sensor error. However, for the perturbed plant, the tracking problem of the plant output $y(t)$ (or $y_a(t)$ in some cases) is undertaken, namely, we have two kinds of tracking problem, problem 1: $y \to r$ for plant with uncertainty and output disturbance ΔN, problem 2: $y_a \to r$ for internal sensing error ΔN in plant. Since the error signal-based tracking techniques [3, 7] are not valid, somewhat serious problem remains unsolved with regard to the tracking design for this kind of system. In this paper, the objective is to design a nonlinear operator M (see Fig.1) such that plant output $y(t)$ (or $y_a(t)$) tracks to the reference signal $r(t)$ under the perturbations of ΔD and ΔN. Here, the normed linear subspaces of U^* and Y^* are designed based on control objective. Namely, in some cases we can select that Y^* does not belong to Y for satisfying the desired tracking performance.

III. The proposed design scheme

Under the discussion in Section 2, the closed-loop system described in Fig.1 is robust stable in the presence of ΔP represented by ΔD and ΔN provided that $u \in U$ is bounded. The nonlinear system depicted in Fig.1 can be regarded as a system with perturbation depicted in Fig.2. In tracking control problem, reference signal $r \in Y^*$ is

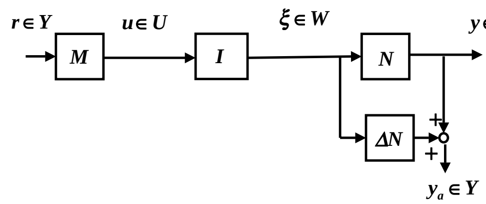

Fig. 2. The equivalent diagram of the proposed control system

considered. The main result of this paper is given as follows.

Theorem: Suppose that the following operator design condition is satisfied for the stable designed operator M.

$$NM(r)(t) = I(r)(t) \tag{4}$$

where $r(t)$ is a given reference input, and $r \in Y^*$. Then, the output $y(t)$ tracks to the reference input $r(t)$. Also, if

$$(N + \Delta N)M(r)(t) = I(r)(t) \tag{5}$$

with stable M, then, the perturbed output $y_a(t)$ tracks to the reference input $r(t)$.

Proof: From Fig.2, we have $u(t) = M(r)(t)$. Then, we can obtain $N(u)(t)$. From condition (4), we have

$$N(u)M(r)(t) = I(r)(t) \tag{6}$$

This fact leads to the desired result under $r(t) \in Y^*$, $y(t) \in Y^*$. That is, $y(t) = I(r)(t) = r(t)$. According to the same argument, if $(N + \Delta N)M(r)(t) = I(r)(t)$, $r(t) \in Y^*$, $y_a(t) \in Y^*$, we can obtain $y_a(t) = I(r)(t) = r(t)$.

Based on the proposed theorem, the detailed procedure of checking the design conditions is as follows. To apply the system to the reference signal $r(t) \in Y^*$, we will design the operator $M(r)$ to satisfy

$$NM(r)(t) = I(r)(t) = r(t) \in Y^* \tag{7}$$

Then we have $y(t) = r(t)$. In the same manner, if the reference signal $r(t) \in Y^*$ and

$$(N + \Delta N)M(r)(t) = I(r)(t) = r(t) \in Y^* \quad (8)$$

Then we have $y_a(t) = r(t)$. As shown in the above theorem, the plant output $y(t)$ tracks to the reference input $r(t)$. Even if $y_a(t)$ is the plant output in some cases, the perturbed signal does not affect the tracking performance.

We next present an example to illustrate the efficacy of the design conditions of the proposed operator in tracking performance.

Remark: In this paper, operator N is nonlinear, and not all $r(t)$ satisfies the proposed theorem, namely, we have to consider whether the space Y^* satisfies the condition. Then, M is different to the inverse of N. Further, in some cases, for satisfying the theorem we have to design an approximated $r(t)$ for the need of plant output. That is, for satisfying the desired tracking performance $y(t) = r(t)$ (or $y_a(t) = r(t)$), we should select that Y^* does not belong to Y.

IV. Design procedure explanation and simulation

The purpose of this example is to demonstrate the benefit of the proposed method. Simulation study is conducted using the following nonlinear plant given in [6].

$$\begin{aligned} P(\bar{u})(t) &= \int_0^t \bar{u}^{1/3}(\tau)d\tau + e^{t/3}\bar{u}^{1/3} \\ &= ND^{-1}(\bar{u})(t) \\ N(w)(t) &= \int_0^t e^{-\tau/3}w^{1/3}(\tau)d\tau + w^{1/3}(t) \\ D(w)(t) &= e^{-t}w(t) \end{aligned} \quad (9)$$

where $\bar{u} \in U$, and $P(\bar{u}) \in Y$. U and Y are two linear spaces, where $U \in C_{[0,\infty]}$ and $Y = \{u + e^{t/3}u' | u \in C^1_{[0,\infty]}\} \subset U$. $C_{[0,\infty]}$ is the space of continuous functions and $C^1_{[0,\infty]}$ is the space of pieacewise continuously differentiable functions. From Figs. 3-5, the step response of D and N is stable, the step response of D^{-1} is unstable. From (3), suppose that the plant P has a perturbation ΔP such that

$$\begin{aligned} (P + \Delta P)(\bar{u})(t) &= (1 + \Delta_1)\int_0^t (1 + \Delta_3)^{-1/3}\bar{u}^{1/3}(\tau)d\tau \\ &\quad + (e^{t/3} + \Delta_2)(1 + \Delta_3)^{-1/3}\bar{u}^{1/3} \\ (N + \Delta N)(w)(t) &= (1 + \Delta_1)\int_0^t e^{-\tau/3}w^{1/3}(\tau)d\tau \\ &\quad + (1 + e^{-t/3}\Delta_2)w^{1/3}(t) \\ (D + \Delta D)(w)(t) &= (1 + \Delta_3)e^{-t}w(t) \end{aligned} \quad (10)$$

where the linear space and normed linear space of $P(\bar{u})(t)$ and $(P+\Delta P)(\bar{u})(t)$ are same. Δ_1, Δ_2 and Δ_3 are bounded functions in $U = W$. Based on the proposed design scheme, we design two operators R and S as

$$\begin{aligned} R(\bar{u})(t) &= (1 + \Delta_3)^{-1}\bar{u}(t) \\ S(y_a)(t) &= A(y_a)(t) - \Delta_s \end{aligned} \quad (11)$$

and

$$A(y_a)(t) = [Y_1^3 - X_1^3](h(t))^3 \quad (12)$$

$$\begin{aligned} \Delta_s &= -3\Delta_1 - 3(\Delta_1)^2 - (\Delta_1)^3 + 3e^{2/3t}\Delta_2 \\ &\quad + 3e^{t/3}(\Delta_2)^2 + (\Delta_2)^3 \end{aligned} \quad (13)$$

$$h(t) = e^{-t/3}w^{1/3}(t) \quad (14)$$

$$N(w)(t) = X_1\int_0^t h(\tau)d\tau + Y_1 h(\tau) \quad (15)$$

where $X_1 = 1$ and $Y_1 = e^{t/3}$.

In the following examples, tracking problem is to track output $y(t)$(or $y_a(t)$) to a step function. But for the step function, there does not exist a solution M, hence, reference input r(t) which has a solution M and approximates the step function is chosen. For obtaining the tracking of $y(t) = r(t)$ (problem 1), from the design condition (4) of M described in the theorem, $M : Y \to U$ is given as follows.

$$M(r) = \frac{1}{1 - e^{-t}}A(r), t > 0 \quad (16)$$

where $\Delta_s = 0$. In the simulation, for satisfying condition (4), we select $\bar{w}(t) = \frac{1}{64}$, $r(t) = \int_0^t e^{-\tau/3}\bar{w}^{1/3}(\tau)d\tau + \bar{w}^{1/3}(t)$, $\Delta_1 = -1$, $\Delta_2 = 1/3e^{-2/3t}$, and $\Delta_3 = 0$ such that we can obtain M and N from (15) and (16), where $w(t)$ is the output signal of M. In this case, the perturbed Bezout identity remains unchanged, namely, $S(N + \Delta N) + R(D + \Delta D) = SN + RD = I$.

Fig.6 shows the reference input $r(t)$ (dashed line: almost overlapped by the solid line) and plant output $y(t)$ (solid line) using the proposed method, plant control input is given in Fig.7, and better tracking performance is obtained. The simulation results of $r(t)$ (dashed line) and $y_a(t)$ (solid line) are also given in Fig.8. From Fig. 8, $y_a(t)$ is perturbed. In this case, for satisfying the theorem we have to design an approximated $r(t) = 1 - 3e^{-t/3}$ for the need of plant output.

Concerning the tracking of $y_a(t) = r(t)$, from the design condition (5) of M, $M : Y \to U$ is also given as (16) but $r(t)$ is different to the former case. We select $\bar{w}(t) = 1 - e^{-20t}$, $r(t) = \int_0^t e^{-\tau/3}\bar{w}^{1/3}(\tau)d\tau + \bar{w}^{1/3}(t)$. Δ_s, Δ_1, Δ_2, and Δ_3 are same as in the former case. For satisfying the theorem, we can also obtain M and N as former case, where $w(t)$ is the output signal of M. Fig.9 shows the reference input $r(t)$ (dashed line) and plant output $y_a(t)$ (solid line) and control input is given in Fig.10, satisfying tracking performance is obtained. The simulation results of the reference input $r(t)$ (dashed line) and perturbed $y(t)$ (solid line) are shown in Fig.11. Meanwhile, when $\Delta_1 = \Delta_2 = \Delta_3 = 0$, we have $\Delta N = \Delta D = 0$, the perfect tracking of the reference input $r(t)$ (dashed line: almost overlapped by the solid line) and $y(t)$ (solid line) are shown in Fig.12, where $\bar{w}(t) = \frac{1}{64}$.

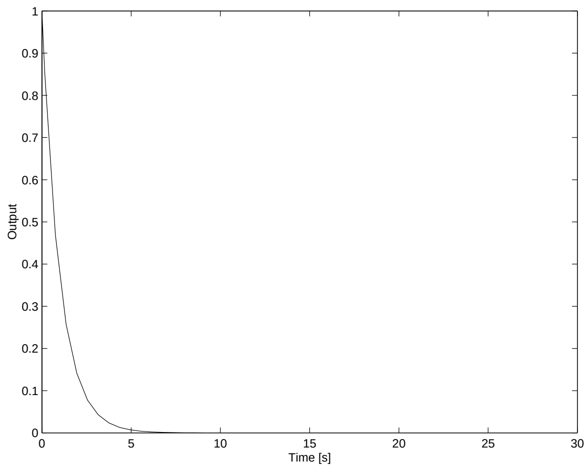

Fig. 3. Step response of D

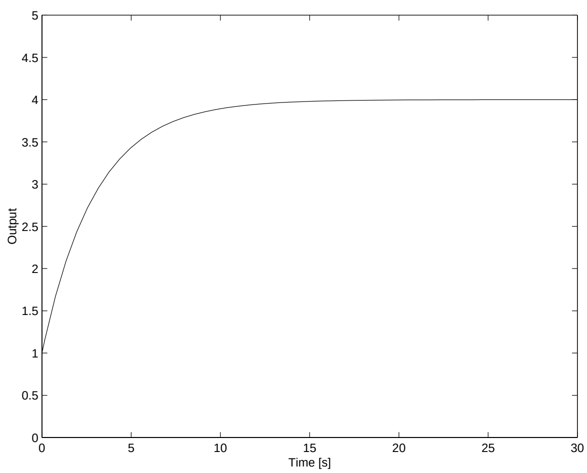

Fig. 4. Step response of N

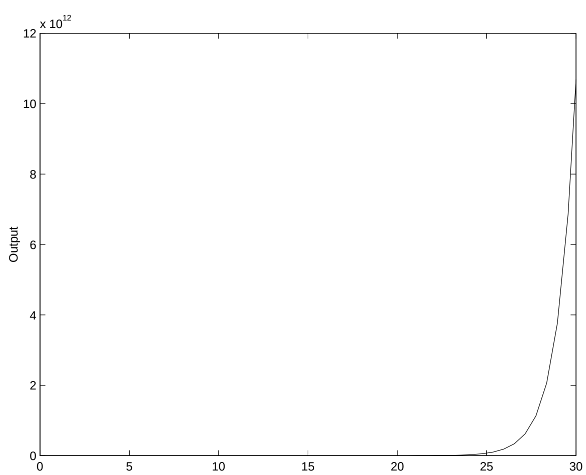

Fig. 5. Step response of D^{-1}

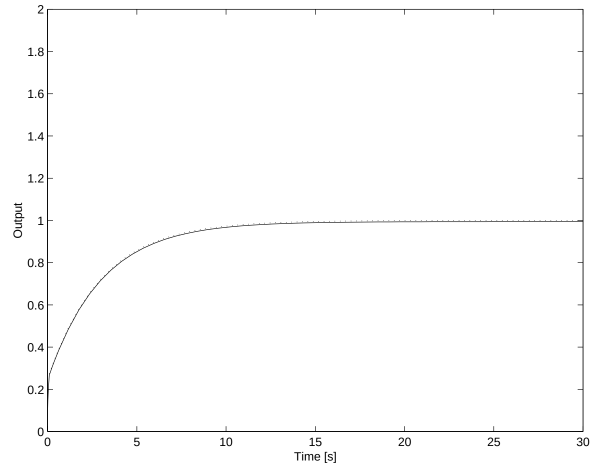

Fig. 6. The plant output $y(t)$ and the reference input $r(t)$

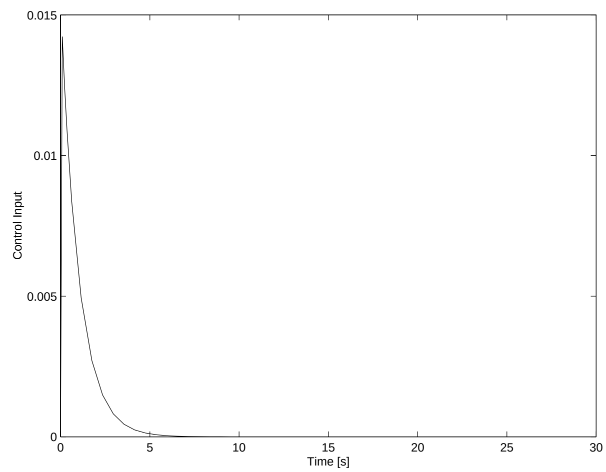

Fig. 7. The plant input for the case of Fig.6

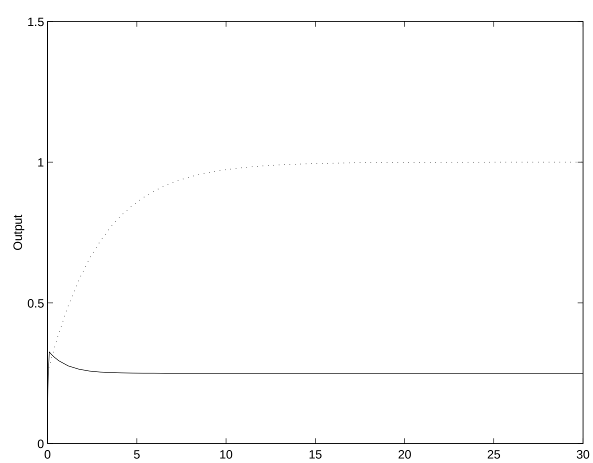

Fig. 8. $y_a(t)$ and the reference input $r(t)$ when $y(t) = r(t)$

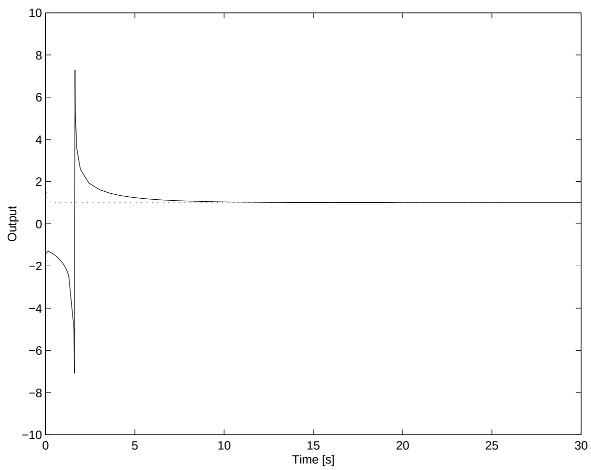

Fig. 9. The plant output $y_a(t)$ and the reference input $r(t)$

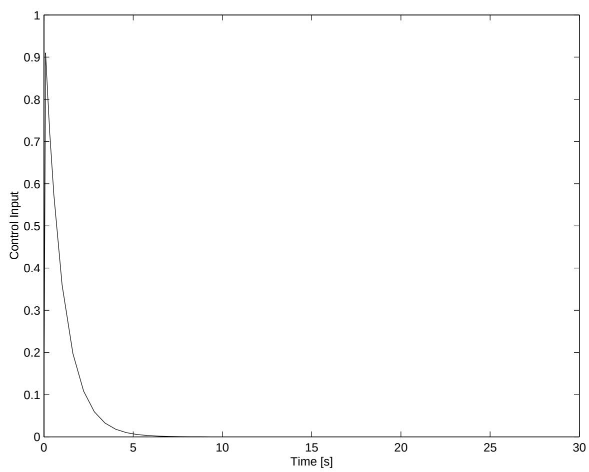

Fig. 10. The plant input for the case of Fig.9

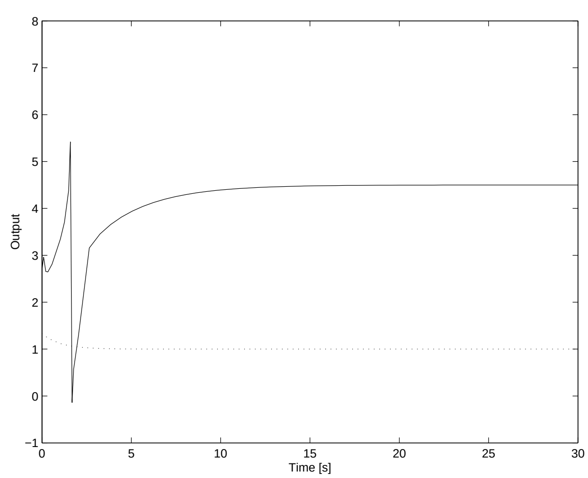

Fig. 11. $y(t)$ and the reference input $r(t)$ when $y_a(t) = r(t)$

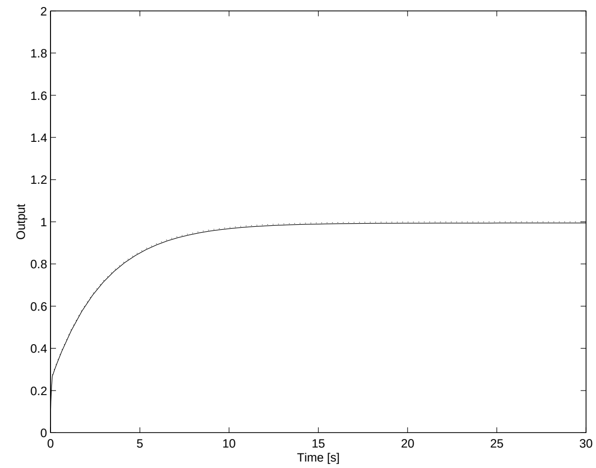

Fig. 12. $y(t)$ and the reference input $r(t)$ when $\Delta N = \Delta D = 0$

V. CONCLUSIONS

In this paper, from the viewpoint of robustness of the right coprime factorization, a plant output tracking design problem of perturbed nonlinear plants was considered by using robust right coprime factorization approach. Under the condition of robust stability of the closed-loop system with perturbation, tracking conditions are derived and nonlinear operator-based design method for real nonlinear plant output to track a reference input is given. The effectiveness of the proposed method is also confirmed through simulations.

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