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Differential geometric guidance based on the involute of the target's trajectory: 2-D aspects

O. Ariff, R. Żbikowski, A. Tsourdos, and B. A. White
Department of Aerospace, Power & Sensors
Cranfield University (RMCS Shrivenham)
Swindon SN6 8LA, ENGLAND

***Abstract*— This paper presents a novel approach to missile guidance utilising the differential geometry of curves and not relying on the line of sight (LOS) information. The target's trajectory is treated as a smooth curve of known curvature and the new algorithm is based on the involute of the target's curve. The missile's trajectory uses the concept of virtual target to generate the correct involute trace. It is shown that the missile is either on the trace immediately or can reach it through a flying in procedure. Following the trace may require a 3-D manoeuvre, not described here, while the 2-D aspects of the algorithm lead to very simple formulae. The reference scenario is a planar air-to-air engagement of point masses with a manoeuvrable target of the F-16 fighter class. Simulations for perfect target information show intercepts both for the involute law and PN guidance. PN based intercepts occur quicker, but the involute based trajectories are more difficult to evade and always result in a side impact.**

I. Introduction

This paper proposes a fresh look at the generation of intercept trajectories for missile guidance. The reference scenario is a planar air-to-air engagement of point masses with a manoeuvrable target of the F-16 fighter class. The motivation is to overcome, without the complexity of Optimal Control [1], the two shortcomings of the proportional navigation (PN) algorithm [14] family: (i) reliance on the line of sight (LOS) for derivation of intercept geometry and kinematics, (ii) effectiveness against non-manoeuvring targets only. The second limitation can be, to some extent, alleviated through various modifications of the original PN algorithm, e.g. augmented PN. Still, the kinematic rules of the PN algorithm family aim at compensating the deviation of the missile velocity vector from the LOS. In particular, the target manoeuvres manifest themselves indirectly through the motion of the LOS. Such motion becomes very rapid when the missile and target are close to each other, even for benign evasive manoeuvres. This is an inescapable consequence of using *straight* lines (intercept triangles) to express *curved* trajectories (manoeuvres). Indeed the original PN was conceived for a target moving with a constant speed along a straight line. A generalisation to scenarios with manoeuvres leads naturally to freeing the intercept geometry of the rectilinear framework.

This paper is organised as follows. This introductory section continues by presenting a synoptic account of the essential facts from Differential Geometry in Section I-A, followed by a summary of the relevant prior work in Section I-B. The novelty of the proposed approach is contrasted with that work in Section I-C. Brief information on involutes is given in Section II which leads to the heart of the paper, Section III, where the new algorithm is derived in detail. The assumptions on the scenarios used to test the new law are collected in Section IV. The actual results follow in Section V and the paper ends with conclusions in Section VI.

A. Background

How can the curvilinear engagement geometry be reformulated and endowed with appropriate intercept kinematics? We believe that the differential geometry of spatial curves [9], [10], [5] offers an attractive setting for this task. Indeed, it has long been a tool for kinematic analysis of material points, especially in mechanism and machine theory [4].

The essence of differential geometric description of smooth curves in $\mathbb{R}^3$ is the use of Calculus to quantify how a curve deviates locally from its linear approximation. The point mass velocity vector $\mathbf{v} = \mathbf{v}(t)$ over time t is expressed as a multiple $v = v(t)$ of the unit tangent vector $\mathbf{T} = \mathbf{T}(t)$, i.e. $\mathbf{v}(t) = v(t)\mathbf{T}(t)$. At the same moment in time, the normal vector $\mathbf{k} = \mathbf{k}(t)$ is obtained by differentiating $\mathbf{T}$ and is therefore[1] orthogonal to $\mathbf{T}$. Again, $\mathbf{k}(t) = \kappa(t)\mathbf{N}$, where $\mathbf{N}$ is the unit normal vector and κ is the curvature. The curvature measures the local deviation of the curve from the rectilinear progression along the tangent line. Indeed, its inverse $\rho = 1/\kappa$ is the radius of best fitting circle in the plane spanned by the tangent and normal vectors. The right-handed local coordinate frame is completed by the binormal vector $\mathbf{b}(t) = \tau(t)\mathbf{B}$, where $\mathbf{B} \stackrel{\text{def}}{=} \mathbf{T} \times \mathbf{N}$ is the unit binormal vector. The torsion τ measures locally how much the curve deviates from the plane spanned by the tangent and normal vectors. It is convenient to replace the time parameter t with the arc length $s \stackrel{\text{def}}{=} \int_0^t v(t)dt$, for then the reparameterised curve has unit speed, as $ds/dt = v$. The above definitions and relationships are summarised[2] by the Frenet-Serret equations:

$$\begin{bmatrix} \mathbf{T}'(s) \\ \mathbf{N}'(s) \\ \mathbf{B}'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T}(s) \\ \mathbf{N}(s) \\ \mathbf{B}(s) \end{bmatrix}, \quad (1)$$

[1] Since $\mathbf{T} \cdot \mathbf{T} = 1$, differentiation gives $\dot{\mathbf{T}} \cdot \mathbf{T} = 0$.
[2] Note that each of $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$ is a vector in $\mathbb{R}^3$, in general.

where $'$ means differentiation with respect to s. Parameterisations of (1) differ by the factor of speed $v = ds/dt$, as for any vector $\mathbf{x}$, we obviously have $d\mathbf{x}/dt = (d\mathbf{x}/ds)(ds/dt)$.

It should be emphasised that differential geometry of curves has nothing new to say about straight lines, as $\kappa = \tau = 0$ then. Indeed, such case can be handled by PN, and hence our focus on manoeuvring targets.

B. Prior work

There have not been many attempts to use the differential geometric formulation for missile guidance.

The notable work [3], [6], [7] is a recent example. In those papers the 3-D kinematics of missile-target point masses were resolved with respect to the LOS. The missile trajectory arc length s_m was used as the common parameter for both curves. The target/missile speed ratio $m = v_t/v_m = \text{const} < 1$ was assumed, together with $v_m = \text{const}$. The resulting kinematics formulae combined the unit vector $\mathbf{e}_r$ along the LOS and the unit rotational vector $\mathbf{e}_\theta$ of the LOS with (1), so that the vectors $\mathbf{T}_t$, $\mathbf{T}_m$ (with the speed ratio m) and $\mathbf{N}_t$, $\mathbf{N}_m$ (with the curvatures κ_t, κ_m) were involved. Since the missile speed was assumed constant, the guidance command only changed the direction of the velocity vector. The command was expressed in terms of the desired curvature κ_{mp} obtained from the requirement that the LOS rate ω is zero. For this to work, the desired closing velocity r'_{mp} would be needed. In the actual command r'_{mp} was replaced with the real closing speed r' and a heuristic gain $A > 2$ was introduced:

$$\kappa_m = m^2 \kappa_t \frac{\mathbf{N}_t \cdot \mathbf{e}_\theta}{\mathbf{N}_m \cdot \mathbf{e}_\theta} - A \frac{r' \omega}{\mathbf{N}_m \cdot \mathbf{e}_\theta}. \qquad (2)$$

If $A > 2$, then the LOS rate ω decreases, provided $r' < 0$, since substituting (2) to the target-missile kinematics, resolved on $\mathbf{e}_r$ and $\mathbf{e}_\theta$, yields:

$$\omega = \omega_0 (r/r_0)^{A-2}. \qquad (3)$$

An extra command for torsion τ_m was necessary for the law (2) to be well-defined, and the overall scheme worked only for a certain set of initial conditions. The end result was a generalisation of PN for arbitrarily manoeuvring (but constant speed) targets, valid for some initial conditions. The algorithm was based on the LOS information: $\mathbf{e}_r$, $\mathbf{e}_\theta$, r' and ω, and also differential geometric quantities: κ_t, $\mathbf{N}_t$ and $\mathbf{N}_m$.

Previous noteworthy work includes the kappa guidance [8, Section 8.6], [11], [12] which builds upon the classical work on the E guidance by Cherry [2]. The original algorithm of Lin [8] was 2-D and based on the simple reasoning of Cherry that in inertial coordinates the missile acceleration is $\ddot{\mathbf{x}} = \mathbf{g} + \mathbf{a}_c$, where $\mathbf{g}$ is the gravity vector and $\mathbf{a}_c$ the command vector. If $\ddot{\mathbf{x}}$ is integrated twice in succession on $[t_0, t_f]$, two independent equations are obtained. They are expressed in terms of the current (at t_0) and final (at t_f; predicted intercept point) velocities and positions ($\mathbf{v}_0$, $\mathbf{r}_0$ and $\mathbf{v}_f$, $\mathbf{r}_f$), and the time-to-go $t_g = t_f - t_0$. The unknown acceleration $\ddot{\mathbf{x}}$ can be expanded in a functional series, but only two coefficients can be determined, as the successive integrations of $\ddot{\mathbf{x}}$ yield two independent equations. Cherry chose for this approximate representation $\ddot{\mathbf{x}} \approx C_1 + C_2 t_g$ which yielded:

$$\mathbf{a}_c = \frac{K_1}{t_g}(\mathbf{v}_f - \mathbf{v}_0) + \frac{K_2}{t_g^2}(\mathbf{r}_f - \mathbf{r}_0 - \mathbf{v}_0 t_g) - \mathbf{g}, \qquad (4)$$

where the first error term is the velocity-to-go and the second the position-to-go, while $K_1 = -2$ and $K_2 = 6$. In seeker coordinates, the curvature corresponding to (4) was approximately:

$$\kappa_c = \frac{K_1}{R} \sin\delta \cos\sigma - \frac{K_2}{R} \sin\sigma - \mathbf{g}_\perp, \qquad (5)$$

where $\mathbf{g}_\perp$ is the component of $\mathbf{g}$ along $\mathbf{N}_m$, R the LOS range, δ the angle error between $\mathbf{v}_0$ and $\mathbf{v}_f$, and σ the heading error angle. Lin's generalisation of Cherry's approach [8, Section 8.6] consisted in recalculation of K_1 and K_2 via optimal control, resulting from maximisation of the final (impact) speed v_f with $\delta(t_f) = \sigma(t_f) = 0$. Nonlinear flight dynamics of the missile were included both for the vertical and horizontal plane cases. Since the overriding consideration was to obtain closed-form formulae for K_1 and K_2, several simplifications had to be introduced into the solution. The end result was thus suboptimal. The analytical expressions obtained were rather involved functions of the aerodynamic coefficients and engagement geometry. The fundamental formula (5) was explicitly based on the LOS information, as indeed was the whole setting, which also requires independent determination of the predicted intercept point. The same was also true in 3-D, when a torsion command was included [12].

C. Novelty

This paper differs from the work described in Section I-B in two main aspects. Firstly, it focuses on deriving an algorithm for 2-D intercept trajectory generation that is *not* based on a LOS framework. Secondly, an explicit aim is to tackle manoeuvrable targets in a way that leads to a conceptually and computationally *simple* guidance law.

The scenario considered is a planar air-to-air engagement of an F-16 class manoeuvring target. It is assumed that both the target and the missile speeds (v_t and v_m) are constant and the target's curvature κ is known. The missile trajectory is then derived from these data and is an appropriate involute of the target's curve. An intercept algorithm is given by constructing a kinematically correct involute, i.e. one which is consistent with the time-to-go requirement, resulting from the constant ratio v_t/v_m. This algorithm utilises the concept of *virtual target* which is an auxiliary means to compute the required involute.

II. Involutes

A curve C is a (thrice) differentiable mapping $\mathbf{x}\colon [0, c] \to \mathbb{R}^3$ and its trace is the set $\mathcal{T}(\mathbf{x}) = \{\mathbf{x}(\int) \in \mathbb{R}^{\ni} \mid \int \in [\prime, \rfloor]\}$. This definition is kinematic in nature: the curve $\mathbf{x}$ is formed by a point traversing $\mathbb{R}^3$ and leaving the trace $\mathcal{T}(\mathbf{x})$. The same trace may correspond to many curves, e.g. each of the circles $x_{1,k}(s) = \cos ks, x_{2,k}(s) = \sin ks,\ s \in [0, 2\pi]$, $k = 1, 2, \ldots$ is a distinct curve, but their common trace is $x_1^2 + x_2^2 = 1$.

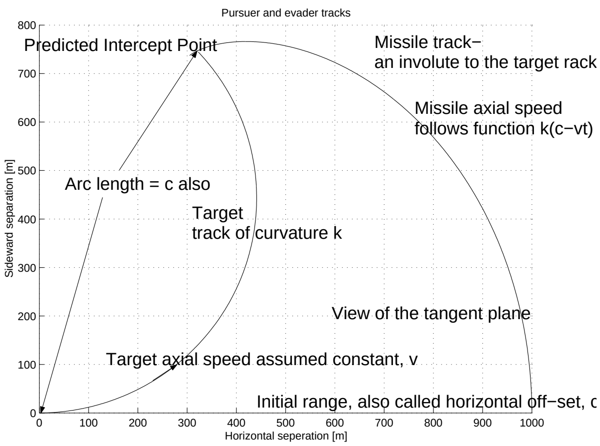

Fig. 1. Trajectory produced by involute guidance law.

The tangent line to a curve C generates the tangent surface which for planar curves coincides with the defining plane. A curve C^* which lies on the tangent surface of C and intersects the tangent lines orthogonally is called an *involute* of C, see Figure 1. The equation of an involute $\mathbf{x}^*$ of the curve $\mathbf{x}$, is:

$$\mathbf{x}^*(s) = \mathbf{x}(s) + (c - s)\mathbf{T}(s), \tag{6}$$

so that $\mathbf{x}^*(c) = \mathbf{x}(c)$ is the intercept for $s = c$ and

$$\mathbf{v}^*(s) = (c - s)\kappa\mathbf{N}(s), \tag{7}$$

Here $\mathbf{v}^* \stackrel{\text{def}}{=} d\mathbf{x}^*/ds$, while s, $\mathbf{T}(s)$, $\mathbf{N}(s)$ and κ are parameters of the *original* curve C. This is the kinematic model of an involute based missile-target engagement.

The involute's curvature is expressed in terms of the original curve's curvature κ and torsion τ:

$$(\kappa^*)^2 = \frac{\kappa^2 + \tau^2}{(c - s)^2\kappa^2}. \tag{8}$$

For 2-D $\tau = 0$ and in our case $s = vt$, so that

$$\kappa^* = \frac{1}{c - vt}. \tag{9}$$

Reparameterisation of the above formulae with missile arc length $s_m = v_m t = (v_m/v)s$ is obvious, as $v_m = \text{const}$.

III. Involute based guidance law

There are three main problems in application of the mathematical principles of involutes to practical guidance:

1) **Position** The missile's position $\mathbf{x}^*(s)$ has to be on the tangent line of $\mathbf{T}(s)$, i.e. the target's velocity, see (6).
2) **Speed** The speed should decrease linearly, reaching zero at the impact point: $v^*(s) = (c - s)\kappa$, see (7).
3) **Orientation** The target's tangent vector $\mathbf{T}(s)$ and the missile's normal vector $\mathbf{N}^*(s)$ must be collinear for all s, as C^* intersects the tangent lines of C orthogonally.

Solution to problem 2 is possible after removing restriction 1 which is done using the concept of virtual target described in Section III-A. Problem 3 is solved in Section III-B.

A. Correct involute generated by virtual target

A way to overcome restriction 1 is to postulate existence of previous positions of the target, i.e. where it could have been *before* the engagement commenced. This leads to a virtual trajectory obtained by concatenation of the previous positions with the current ones, thus implicitly expanding the observed target trajectory. The expanded trajectory can be traversed by the virtual target at a constant speed, different from the real target's speed. Mathematically, this is an extension of the domain of the target's curve, combined with its reparameterisation from s to s_{vrt}. For the real target, equation (6) exists on $[0, c]$, and hence so do all the involutes as well. If (6) is reparameterised by s_{vrt} and extended to $[0, c']$, the same happens to its involutes, thus enriching the set of possible missile intercept trajectories.

The virtual target is an imaginary point mass, moving at a constant speed v_{vrt}, which will reach the intercept point *at the same time* as the real target, but starting at a point *behind*[3] the real target. This new starting point is obtained by reparameterising and extending the real target's trajectory, so that restriction 1 is satisfied for the virtual target. Hence, the missile position at the beginning of the engagement $s = 0$ is on the virtual target's tangent at $s_{\text{vrt}} = 0$.

Assuming that restriction 3 is satisfied, this enables the intercept of the virtual target. Importantly, this can be done by following the trace of the involute the missile is *already* traversing (at launch). Moreover, this will also lead to the intercept of the real target. Indeed, both targets will arrive at the same time at the point ($s = c$ for the real and $s_{\text{vrt}} = c'$ for the virtual), where the involute trace meets both virtual and real trajectories. Figure 2 illustrates this concept.

In order to facilitate easy computation of c' and v_{vrt}, we postulate the extension of the observed trajectory by concatenation of a circular arc. The radius of the arc will be computed from the curvature of the target at the beginning of the engagement

$$\rho_{\text{vrt}} = 1/\kappa(0), \tag{10}$$

where $\kappa = \kappa(s)$ is parameterised with s; see Section I-A for the notion of radius of curvature. The virtual trajectory is parameterised by s_{vrt} and at $s_{\text{vrt}} = 0$ its Frenet frame $(\mathbf{T}_{\text{vrt}}(0), \mathbf{N}_{\text{vrt}}(0))$ must be orthogonal to the missile's frame[4] at the beginning of the engagement $(\mathbf{T}^*(0), \mathbf{N}^*(0))$,

[3] For a forward quarter engagement this should read *in front of*.

[4] Note that the missile's frame is parameterised by s, not s_{vrt}; see (6)–(9).

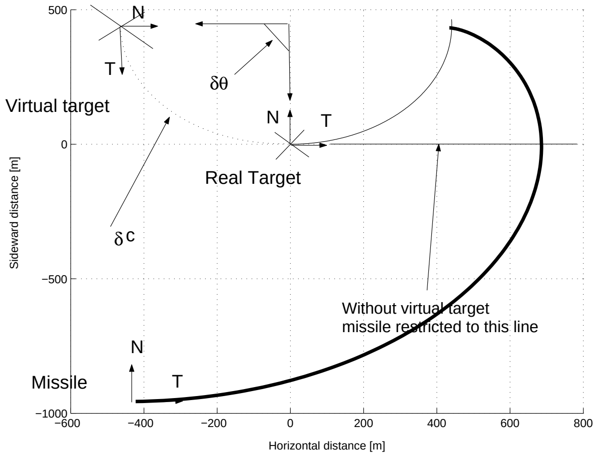

Fig. 2. Finding the correct involute.

i.e. at $s = 0$. Since the orientations of both the missile's and the target's frames (both parameterised by s) are assumed to be known at 0, the angular difference between the real and virtual frames $\delta\theta$ is readily computed. Indeed it is the angle through which the target's frame $(\mathbf{T}(0), \mathbf{N}(0))$ has to be rotated to be orthogonal to $(\mathbf{T}^*(0), \mathbf{N}^*(0))$. Hence the arc length of the extension of the observed trajectory is

$$\delta c = \rho_{\rm vrt}\, \delta\theta. \tag{11}$$

The target moves with the constant speed v and the intercept occurs at $s = c$, so that

$$t_{go} = \frac{c}{v}, \tag{12}$$

while, using (11), we have

$$c' = c + \delta c. \tag{13}$$

Hence, the constant speed of the virtual target must be

$$v_{\rm vrt} = \frac{c'}{t_{go}} = \Big(1 + \frac{\delta c}{c}\Big) v \tag{14}$$

and, finally,

$$s_{\rm vrt} = v_{\rm vrt} t = \Big(1 + \frac{\delta c}{c}\Big) s \tag{15}$$

which, together with (13), completes the virtual extension and reparameterisation of the target's trajectory. Reparameterising (6) with $s_{\rm vrt}$ generates the trace of the correct involute $\mathcal{T}(\mathbf{x}^*)$ thus solving problem 1.

The trace is a reference trajectory, but the missile may not arrive at the intercept point at $t_f = t_{go}$, as it flies at a constant speed. This is the essence of problem 2. If the engagement begins too late, $t_f > t_{go}$ and the missile will never make it. If the engagement begins too early, $t_f < t_{go}$, so it will have to loiter above the tangent plane, while following the trace. This requires a 3-D manoeuvre and is thus outside the remit of this paper; a way to do it is described elsewhere.

B. *Flying in on the correct involute*

As explained in Section III-A, the virtual trajectory is obtained by adding a circular arc to the real trajectory. The radius of this arc $\rho_{\rm vrt}$ is given by (10) and the corresponding arc length δc by (11), obtained from the angular difference $\delta\theta$ between the real and virtual frames.

Figure 2 shows that for $\delta\theta = \pi/2$ the virtual target's and missile's frames can be aligned, so that restriction 3 is obeyed. If the missile were farther in the negative direction of horizontal separation, there would still exist $\delta\theta > \pi/2$ satisfying requirement 1, but not 3. While the missile position would then indeed be on the corresponding tangent line of the virtual target, the virtual target's and missile's frames could *not* be aligned, i.e. restriction 3 would be violated. Such situation is illustrated in Figure 3, where the initial missile position is $\mathbf{x}_m(0) = (0, 4000)$, while the target is at $\mathbf{x}(0) = (0, 0)$. The circular arc with radius $\rho_{\rm vrt} = R_v$ is extended by $-(3/2)\pi$ and there exists a tangent line l of this arc on which $\mathbf{x}_m(0)$ lies. However, the tangent vector $\mathbf{T}_l$ of this line is not collinear with the missile's normal vector which is $\mathbf{N}_m = (0, 1)$, i.e. parallel to the direction of sideward separation. If the missile keeps flying straight on (zero manoeuvre effort), there will come a moment (captured in Figure 3 for $-\pi/2$), when $\mathbf{T}_l$ and $\mathbf{N}_m$ are indeed collinear. At that moment the situation is like in Figure 2, so that the virtual target algorithm can be activated, and the missile will start flying along the correct involute, while obeying restrictions 1 and 3.

Therefore, the 2-D aspects of the algorithm are as follows.

Case 1 If, at the beginning of the engagement, the missile is close enough to the target, a virtual target can be computed satisfying 1 and 3. This also means that the missile is immediately on the correct involute and can follow its trace.

Case 2 If, at the beginning of the engagement, the missile is too far for the alignment of its frame with the virtual target's frame to occur, it first executes the flying in phase. This means that the missile continues flying in a straight line until the alignment occurs, i.e. when it attains the correct involute and the situation reduces to Case 1.

The criterion for distinguishing the cases follows from the definition of the virtual trajectory. The extension of the real trajectory is an arc of the circle with radius and centre derived from the position $\mathbf{x}(s)$ and curvature $\kappa(s)$ of the target, see (10) and Figure 3. The criterion tests whether the current normal of the missile is collinear with a tangent of the circle. If this is true, then this is Case 1; otherwise it is Case 2.

The vector connecting the target's centre of curvature with the missile's position at s, see Figure 3, is

$$\mathbf{R}_{nc}(s) = \Big(\mathbf{x}(s) + \kappa(s)\mathbf{N}(s)\Big) - \mathbf{x}_m(s) \tag{16}$$

and its length is $R_{nc}(s) = \|\mathbf{R}_{nc}(s)\|$. The angle between the missile's unit normal vector $\mathbf{N}_m(s)$ and $\mathbf{R}_{nc}(s)$ is

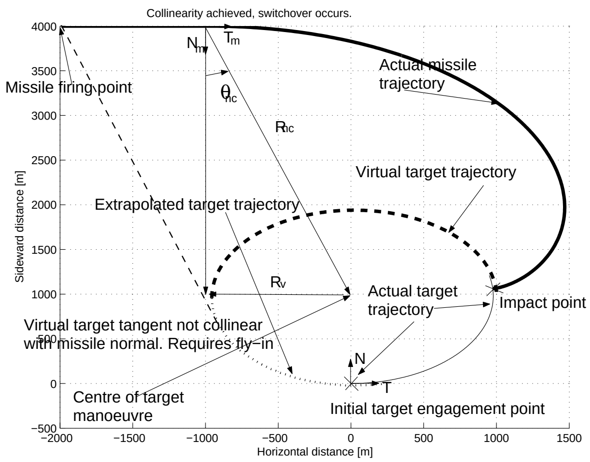

Fig. 3. Forward quarter target engagement and intercept.

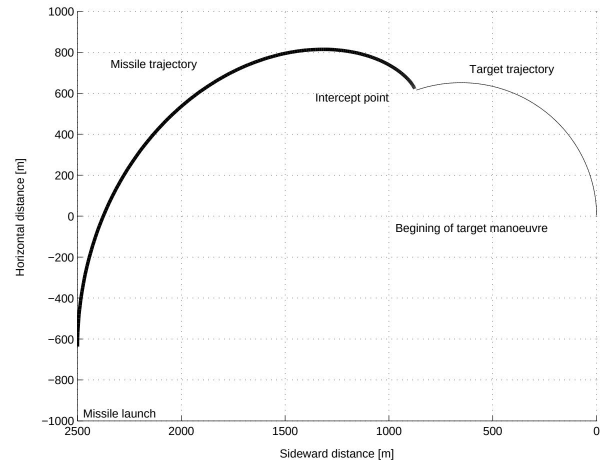

Fig. 4. Forward quarter involute target quasi-planar manoeuvre engagement.

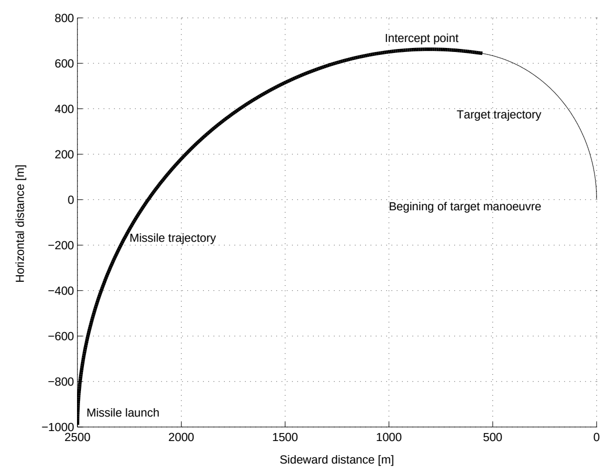

Fig. 5. Forward quarter PN target quasi-planar manoeuvre engagement.

$\theta_{nc}(s)$. The candidate circle for the extending arc at s has radius

$$R_v(s) = 1/\kappa(s). \tag{17}$$

Hence, the flying in criterion is

$$\begin{aligned} \sin\theta_{nc}(s) \leqslant R_v(s)/R_{nc}(s) \quad &\Rightarrow \quad \text{Case 1} \\ \sin\theta_{nc}(s) > R_v(s)/R_{nc}(s) \quad &\Rightarrow \quad \text{Case 2}. \end{aligned} \tag{18}$$

The value $s = s_0$ for which Case 1 occurs is used to define the virtual target with $\rho_{\text{vrt}} = R_v(s_0)$, see (10) and (17).

IV. Simulated scenarios

The following assumptions[5] are made regarding the nature and capabilities of the target:

- the target is an F-16 class fighter;
- the target will manoeuvre in the end-game;
- manoeuvres up to $9g$, sustainable for < 10 seconds;
- the missile-target engagements are all aspect.

The evasive manoeuvre assumed here is a tightly banked turn or loop [13].

The above capabilities will now be described terms of differential geometric parameters, namely $\mathbf{T}$, $\mathbf{N}$, κ. The principal plane of the manoeuvre, or the tangent plane, is horizontal in the earth Cartesian coordinate system. In this context, the following assumptions are made for the tightly banked turn manoeuvre:

1) the curvature of the target's trajectory is known;
2) the curvature is constant $\kappa = 0.34$;
3) the turn is less than 1 complete cycle;
4) the target's speed is 300 m/s (Equivalent Air Speed).

Four engagements are simulated:

1) involute law: forward quarter engagement, Figure 4;
2) PN law: forward quarter engagement, Figure 5;
3) involute law: rear quarter engagement, Figure 6;
4) PN law: rear quarter engagement, Figure 7.

[5]http://www.fas.org/man/dod-101/sys/ac/f-16.htm

V. Results and discussion

Both the PN (which is a benchmark) and the new algorithm intercept the target for perfect target information. However, the engagement times and trajectories vary considerably.

The involute engagement model guarantees a perpendicular impact. The PN guidance law varies in the impact angle, depending on the engagement. In the two scenarios displayed, it evolves into a head-on or tail-chase manoeuvre. In general, the benchmark provides a shorter engagement time. This is because it takes a more direct approach to the intercept point.

These results can be interpreted in two ways. One is that the best missile trajectory should be a straight line to the impact point. Another is that a straight line trajectory allows the pilot to plan an evasive action. On the other hand, an unexpected, curved trajectory adds uncertainty to the evasion strategy, especially that pilots tend to base the strategy on the LOS information. Furthermore, a perpendicular impact angle from the outside ensures that the missile

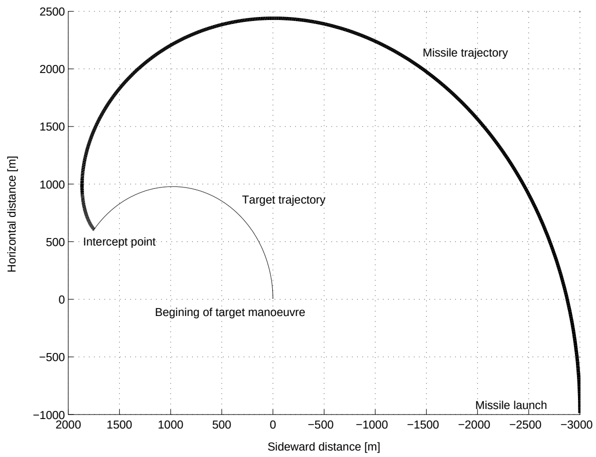

Fig. 6. Rear quarter involute target quasi-planar manoeuvre engagement.

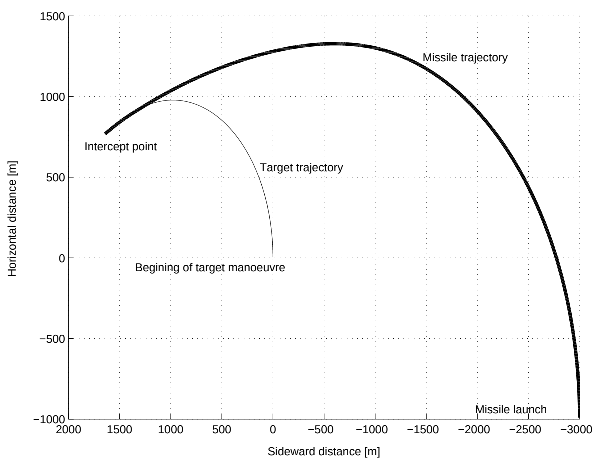

Fig. 7. Rear quarter PN target quasi-planar manoeuvre engagement.

is approaching the target from its blind side, if it is pulling maximum g in a level turn. In terms of lethality, hitting the cockpit increases target kill probability when compared to head-on or tail-chase impacts.

VI. Conclusions

This paper presented a basic model of intercept engagement derived from the differential geometric concept of involute, focusing on 2-D aspects. Assuming that the target's curvature was available, the resulting guidance law was shown to be viable and did not rely on LOS information.

Future work requires derivation of effective estimators of the target's curvature and torsion. Also, the guidance law needs further analysis, especially quantification of the capture region and generalisation to 3-D engagements. Performance should be tested under uncertainty in target information.

It seems that the differential geometry of curves is a productive tool for guidance against manoeuvring targets and should be exploited further.

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